Robust Output Tracking Control of a Laboratory Helicopter for Automatic Landing*

Hao Liu1, Geng Lu2, and Yisheng Zhong3

Abstract—Robust output tracking control problem of a lab helicopter for automatic landing in high seas is dealt with. The motion of the helicopter is required to synchronize with that of an oscillating platform, e.g., the deck of a vessel. A robust linear time-invariant output controller consisting of a nominal controller and a robust compensator is proposed. The robust compensator is introduced to restrain the influences of uncertainties. It is shown that robust stability and robust tracking property can be achieved. Experimental results demonstrate the effectiveness of the designed control approach.

I. INTRODUCTION

Unmanned helicopters have received much attention in the last two decades due to their versatile applications. The flight controller is an important part of an unmanned helicopter and many works have been done on its design. In [1], a controller was employed to track the desired references for a quadrotor helicopter. A robust controller was designed in [2] for a helicopter. Furthermore, fuzzy control strategy [3], backstepping-based control approach [4], and \( H_\infty \) control method [5] have also been investigated for unmanned helicopters.

Automatic landing is a challenging problem for unmanned helicopters. In [6], feedback linearization technique was employed to achieve the vertical flight for a reduced-order model helicopter. A composite nonlinear feedback controller was designed in [7] for automatic landing of a small-scale unmanned aerial vehicle helicopter. Nonlinear \( H_\infty \) control method was applied in [8] for an unmanned helicopter to carry out automatic landing tasks. Moreover, trajectory linearization control approach and sliding mode control method were applied for helicopter automatic landing in [9] and [10] respectively.

The control problem for helicopter automatic landing on an oscillating platform is more challenging because of complex motions the deck is subjected to. Besides, the helicopter is a nonlinear system which contains uncertainties and disturbances. Thus, a robust controller against these uncertainties is needed for the helicopter to carry out automatic landing tasks. Combining the feedforward control technique and the nonlinear adaptive output regulation method, Isidori, Marconi, and Serrani [11] designed an autopilot for helicopters to follow the wave-induced vertical oscillations while hold the longitudinal and lateral positions. The desired reference in the vertical channel is the sum of a constant and a number of sinusoidal signals. Tracking errors and their rates of change are assumed to be available. Furthermore, experimental results on arbitrary vertical and lateral/lateral-longitudinal reference tracking were shown in [12].

In this paper, a robust output tracking control method based on linear quadratic regulation (LQR) approach and robust compensation technique is proposed to address the automatic landing problem of unmanned helicopters under the effects of uncertainties, which consists of parametric uncertainties, unmodeled dynamics, nonlinearity, and external disturbances. A controller designed by this method consists of a nominal controller and a robust compensator. The nominal controller is designed for the nominal linear system to achieve the desired nominal tracking performances, and the effects of uncertainties are restrained by the robust compensator. The helicopter used in this paper is a laboratory-scale helicopter which has gained much attention as shown in [13], [14], [15], [16]. We also have done some works to this kind of helicopter (see, e.g., [17], [18], [19]).

Compared with previous research on the automatic landing control problem of unmanned helicopters, the exosystem which generates the exogenous signals (see, e.g., [11], [18]) is not required in this article. Another advantage is that the designed controller does not depend on the rates of change of tracking errors, which can be unavailable in practical applications. Furthermore, tracking performances can be specified by the nominal LQR controller according to specified requirements; the influences of uncertainties can be reduced by the robust compensator. In addition, the resulted controller is generally easy to be realized in practical applications, since it is a linear time-invariant one.

The following part of this paper is organized in five parts. Section II describes the dynamical model of the laboratory helicopter. The design procedure of the robust output tracking controller is presented in Section III. Section IV proves the main properties of the designed controller. In Section V, experimental setup and results are shown. Finally, Section VI draws the conclusion remarks.

II. PROBLEM STATEMENT

The lab helicopter used in this paper is depicted in Fig. 1. It is a model helicopter with three degrees of freedom (3DOF): the elevate angle \( \alpha(t) \), the travel angle \( \beta(t) \), and

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*This work was supported by National Natural Science Foundation of China under the Grants 61203071 and 61174067, as well as Tsinghua National Laboratory for Information Science and Technology (TNList) Basic Research Foundation.

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the pitch angle $\gamma(t)$. The front and rear motors can generate the driving forces proportional to the applied voltages $u_f(t)$ and $u_r(t)$ respectively. Define $u_1(t) = u_f(t) + u_r(t)$ and $u_2(t) = u_f(t) - u_r(t)$. The system dynamics of the laboratory helicopter can be described by the following linear equations [20]

$$
\begin{align*}
\dot{\alpha}(t) &= a_1 \alpha(t) + a_2 \dot{\alpha}(t) + b_1 u_1(t) + q_1(t) \\
\dot{\beta}(t) &= a_3 \dot{\beta}(t) + b_2 \gamma(t) + q_2(t) \\
\dot{\gamma}(t) &= a_4 \gamma(t) + a_5 \dot{\gamma}(t) + b_3 u_2(t) + q_3(t)
\end{align*}
$$

where $a_i (i = 1, 2, \cdots, 5)$ and $b_i (i = 1, 2, 3)$ are the nominal parameters of the 3DOF helicopter and $q_i(t) (i = 1, 2, 3)$ are called equivalent disturbances, which represent the uncertainties including parameter perturbations and model uncertainties.

**Assumption 1:** The equivalent disturbances $q_i(t) (i = 1, 2, 3)$ satisfy the following equations

$$
\begin{align*}
q_1(t) &= (\ddot{\alpha}_1 - a_1 \alpha(t)) + (\ddot{\alpha}_2 - a_2 \dot{\alpha}(t)) + (\ddot{b}_1 - b_1) u_1(t) + u_1(t) \\
q_2(t) &= (\ddot{\alpha}_3 - a_3 \dot{\beta}(t)) + (\ddot{b}_2 - b_2) \gamma(t) + w_2(t) \\
q_3(t) &= (\ddot{\alpha}_4 - a_4 \gamma(t)) + (\ddot{a}_5 - a_5) \dot{\gamma}(t) + (\ddot{b}_3 - b_3) u_2(t) + u_3(t)
\end{align*}
$$

where $\ddot{\alpha}_i (i = 1, 2, \cdots, 5)$ and $\ddot{b}_i (i = 1, 2, 3)$ indicate the actual values of parameters $a_i$ and $b_i$ respectively and $w_i(t) (i = 1, 2, 3)$ are external bounded disturbances.

**Assumption 2:** The uncertain parameters $\ddot{a}_i (i = 1, 2, \cdots, 5)$ and $\ddot{b}_i (i = 1, 2, 3)$ are bounded and $b_i (i = 1, 2, 3)$ satisfy that $|\ddot{b}_i - b_i| < b_i$ and $|\ddot{b}_2 b_3 - b_2 b_3| < b_2 b_3$ where the nominal parameters $b_i (i = 1, 2, 3)$ are positive.

Define $\rho_1 = \max b_1^{-1} |\ddot{b}_1 - b_1|, \rho_2 = \max b_2^{-1} |\ddot{b}_2 - b_2|$, and $\rho_3 = \max b_3^{-1} |\ddot{b}_3 - b_3|$

**Remark 1:** If Assumptions 1 and 2 hold, one can see that $0 \leq \rho_i < 1 (i = 1, 2, 3)$.

**Assumption 3:** The external disturbances $w_i(t) (i = 1, 2, 3)$ and their derivatives $w_i^{(k)}(t) (k = 1, 2)$ are bounded.

From (1), one can see that the pitch motion can result in the travel motion. Therefore, the robust output control problem considered here focusing on the elevation and travel channels involves the dynamics of the three angles. Let $d_1(t)$ and $d_2(t)$ denote the reference signals for the elevation and travel channels respectively.

**Assumption 4:** The desired elevation and travel angles and their derivatives $d_1^{(k)}(t) (k = 0, 1, 2)$ and $d_2^{(k)}(t) (k = 0, 1, \cdots, 4)$ are piecewise uniformly bounded.

Define

$$
\begin{align*}
x_1(t) &= \begin{bmatrix} x_{11}(t) & x_{12}(t) \end{bmatrix}^T, \\
x_2(t) &= \begin{bmatrix} x_{21}(t) & x_{22}(t) & x_{23}(t) & x_{24}(t) \end{bmatrix}^T
\end{align*}
$$

where $x_{11}(t) = \alpha(t) - d_1(t), x_{12}(t) = \dot{\alpha}(t), x_{21}(t) = \beta(t) - d_2(t), x_{22}(t) = \dot{\beta}(t), x_{23}(t) = \gamma(t)$, and $x_{24}(t) = \dot{\gamma}(t)$. Let $e_1(t) = x_{11}(t), e_2(t) = x_{21}(t), r_1(t) = -a_1 b_1^{-1} d_1(t)$, and $r_2(t) = 0$, then the error system can be described by the following state-space model

$$
\dot{x}_1(t) = A_1 x_1(t) + B_1 (u_1(t) - r_1(t)) + \Delta_1(t) \\
e_1(t) = C_1 x_1(t), i = 1, 2
$$

where $\Delta_1(t) = \begin{bmatrix} -d_1(t) & q_1(t) \end{bmatrix}^T, \Delta_2(t) = \begin{bmatrix} -d_2(t) & q_2(t) & 0 & q_3(t) \end{bmatrix}^T, A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\
0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 \\
0 \end{bmatrix}^T, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\
0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 \\
0 \end{bmatrix}^T$.

**Remark 2:** Since it is difficult to measure the derivatives of the reference signals $d_1(t)$ and $d_2(t)$ accurately for the automatic landing mission in high seas, they are considered as uncertainties in $\Delta_i(t)(i = 1, 2)$.

The control problem this paper is addressed is that for the helicopter system (3), under the Assumptions 1 through 4, a linear time-invariant controller is designed so that the closed-loop control system has the following properties:

1. Robust stability: For any initial condition, all the states involved are bounded.

2. Robust output tracking: For given constants $\varepsilon_i > 0 (i = 1, 2)$, if the initial conditions are bounded, there exist constants $T_i \geq 0 (i = 1, 2)$ so that $\lim_{t \to T_i} |e_i(t)| < \varepsilon_i (i = 1, 2)$.

III. ROBUST CONTROLLER DESIGN

The helicopter model can be described as a nominal one added with equivalent disturbances. A robust output tracking controller is designed in two steps: firstly, a nominal LQR controller is designed to achieve the desired tracking for the nominal system; then, a robust output compensator is introduced to reduce the influences of the uncertainties $\Delta_i(t)(i = 1, 2)$. Therefore the control inputs contain two parts as follows

$$
u_i(t) = u_{0i}(t) + \hat{u}_i(t), i = 1, 2
$$
where $u_0(t) (i= 1, 2)$ are the nominal LQR control inputs and $\hat{u}_i(t) (i= 1, 2)$ are the robust compensating inputs.

Firstly, consider the following cost functions

$$J_i = \frac{1}{2} \int_0^\infty e^{2\sigma t} \left[ \phi_e \dot{e}_i^2 (t) + \theta_e (u_i(t) - r_i(t))^2 \right] dt, \quad i = 1, 2$$

for the nominal systems

$$\begin{align*}
\dot{x}_i(t) &= A_ix_i(t) + B_i(u_i(t) - r_i(t)) \\
e_i(t) &= C_ix_i(t), \quad i = 1, 2
\end{align*}$$

where $\phi_i (i = 1, 2)$ and $\theta_i (i = 1, 2)$ are positive constants and $\sigma_i (i = 1, 2)$ are nonnegative constants which specify the minimum decay rates of the outputs $y_i(t) (i = 1, 2)$ respectively. The state feedback gains $K_1 = \begin{bmatrix} k_{11} & k_{12} \end{bmatrix}$ and $K_2 = \begin{bmatrix} k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix}$ of the nominal controller can be obtained by $K_i = -\theta_i^{-1}B_i^TP_i (i = 1, 2)$, where $P_i (i = 1, 2)$ are the positive definite solutions to the following Riccati equations [21]

$$P_iA_{\sigma i} + A_{\sigma i}^TP_i - \theta_i^{-1}P_iB_iB_i^TP_i + \phi_iC_i^TC_i = 0, \quad i = 1, 2$$

where $A_{\sigma 1} = A_1 + diag(\sigma_1, 0)$ and $A_{\sigma 2} = A_2 + diag(\sigma_2, 0, 0, 0)$. The nominal controller can be given by

$$u_0i(t) = K_ix_i(t) + r_i(t), \quad i = 1, 2 \quad (5)$$

Then, consider the real system which contains the equivalent disturbances. Substituting (4) and (5) into (3), one has that

$$\begin{align*}
\dot{x}_i(t) &= A_iHx_i(t) + B_i\hat{u}_i(t) + \Delta_i(t) \\
e_i(t) &= C_ix_i(t), \quad i = 1, 2 \quad (6)
\end{align*}$$

where $A_iH = A_i + B_iK_i (i = 1, 2)$. Define

$$G_i(p) = C_i(pI_i - A_iH)^{-1}B_i = M_i^{-1}(p)N_i(p), \quad i = 1, 2$$

where $p$ is the Laplace operator, $I_i (i = 1, 2)$ are the corresponding unit matrices, and $M_i^{-1}(p)N_i(p)$ ($i = 1, 2$) are left matrix fraction descriptions of $G_i(p)$ ($i = 1, 2$) which are irreducible respectively. It follows from (6) that

$$e_i(p) = M_i^{-1}(p)N_i(p)\hat{u}_i(p) + C_i(pI_i - A_iH)^{-1}(x_i(0) + \Delta_i(p)), \quad i = 1, 2 \quad (7)$$

The effects of the uncertainties $\Delta_i(p) (i = 1, 2)$ in (7) are needed to be restrained in order to achieve the robust tracking performances of the closed-loop system. From (6), one can see that $\Delta_i(t) (i = 1, 2)$ depend on $e_i(t)(i = 1, 2)$, which are unavailable. Then, the robust filters are introduced, which have the following forms [22]

$$\begin{align*}
F_1(p) &= \frac{f_{11}f_{12}}{(p + f_{11})(p + f_{12})}, \\
F_2(p) &= \frac{f_{21}f_{22}}{(p + f_{21})(p + f_{22})} \quad (8)
\end{align*}$$

with parameters $f_{ij} > 0 (i = 1, 2; j = 1, 2)$. If the robust controller parameters $f_{ij} (i = 1, 2; j = 1, 2)$ are sufficiently large and satisfy $f_{11} \gg f_{12} > 0 (i = 1, 2)$, the robust filters have sufficiently wide frequency bandwidths within which the filter gains approximate 1. Then from (7), one can obtain the robust compensating inputs $\hat{u}_i(p) (i = 1, 2)$ with the forms

$$\begin{align*}
\hat{u}_i(p) &= -F_i(p)N_i^{-1}(p)M_i(p)C_i(pI_i - A_iH)^{-1} \\
&\quad (x_i(0) + \Delta_i(p)), \quad i = 1, 2 \quad (9)
\end{align*}$$

Remark 3: It should be noted that the uncertainties $\Delta_i(p)$ $(i = 1, 2)$ in (9) cannot be measured in practical applications. Thus, one needs to obtain the expressions of the control inputs $\hat{u}_i(p) (i = 1, 2)$, which are independent of $\Delta_i(p) (i = 1, 2)$.

From (6) and (9), one can obtain the robust compensating inputs $\hat{u}_i(p) (i = 1, 2)$ as

$$\hat{u}_i(p) = -(1 - F_i(p))^{-1}F_i(p)N_i^{-1}(p)M_i(p)e_i(p), \quad i = 1, 2 \quad (10)$$

Remark 4: One can see that the feedback controller, as shown in (5) and (10), depend on the elevation, pitch, and travel angles, the rates of change of the three angles, and the tracking errors, while the rates of change of the tracking errors are not required in the control laws. In addition, (4), (5), and (10) result in a linear time-invariant controller which is easily to be implemented in practical applications.

IV. ROBUST OUTPUT TRACKING PROPERTY ANALYSIS

The output tracking properties presented in the second section will be analyzed here. Define $x_{1H} = \begin{bmatrix} x_{1H1} & x_{1H2} \end{bmatrix}^T$ and $x_{2H} = \begin{bmatrix} x_{2H1} & x_{2H2} & x_{2H3} & x_{2H4} \end{bmatrix}^T$, where $x_{1H1} = x_{11}$, $x_{1H2} = x_{12}$, $x_{2H1} = x_{21}$, $x_{2H2} = x_{22}$, $x_{2H3} = x_{23} + b_2^{-1}F_2q_2$, and $x_{2H4} = x_{24} + b_2^{-1}F_2q_2$.

From (6) and (9), one has that

$$\begin{align*}
\dot{x}_{iH} &= A_iHx_{iH} + (1 - F_i)\Delta_i \\
e_i &= C_i(x_{iH}, i = 1, 2) \quad (11)
\end{align*}$$

Lemma 1: If the Assumptions 1 through 4 hold, there exist positive constants $\zeta_{\Delta i}$ and $\zeta_{\Delta i c} (i = 1, 2)$ such that

$$\|\Delta_i\|_{\infty} \leq \zeta_{\Delta i c}\|x_{iH}\|_{\infty} + \zeta_{\Delta i c}, \quad i = 1, 2 \quad (12)$$

Proof: See the APPENDIX.

Theorem 1: Under the Assumptions 1 through 4, there exist sufficiently large parameters $f_{ij} (i = 1, 2; j = 1, 2)$ satisfying $f_{11} \gg f_{12} > 0 (i = 1, 2)$ such that the closed-loop control system has robust stability and robust output tracking properties.

Proof: From (11), one has that

$$\|x_{iH}\|_{\infty} \leq \zeta_{\Delta i c}(0) + \pi_i\|\Delta_i\|_{\infty}, i = 1, 2 \quad (13)$$

where $\pi_i = \max_{j \geq 0} \| (1 - F_i(p)C_i(pI_i - A_iH)^{-1}x_{iH}(0) \|_{1}$, $\zeta_{\Delta i c}(0) = \max_{j \geq 0} |c_j^Te^{A_iHt}x_{iH}(0)| (i = 1, 2)$, and $c_{ij}$ and $c_{2j}$ are $2 \times 1$ and $4 \times 1$ vectors respectively with ones on the $j$th row and zeros elsewhere. From [19], one can obtain that if the parameters $f_{ij} (i = 1, 2; j = 1, 2)$ have sufficiently large values and satisfy $f_{11} \gg f_{12} > 0, \pi_i (i = 1, 2)$ can be
made as small as desired. Thus if \( \pi_i < \zeta_{\Delta i}^{-1} (i = 1, 2) \), one can obtain from (12) and (13) that
\[
\| \Delta i \|_\infty \leq \zeta_{\Delta i} \zeta_{x_{iH}}(0) + \zeta_{\Delta i c} \frac{1}{1 - \pi_i \zeta_{\Delta i}} , i = 1, 2. 
\] (14)
Combining (13) and (14), one can have that
\[
\| x_{iH} \|_\infty \leq \zeta_{x_{iH}}(0) + \pi_i \zeta_{\Delta i} \zeta_{x_{iH}}(0) + \zeta_{\Delta i c} \frac{1}{1 - \pi_i \zeta_{\Delta i}} , i = 1, 2. 
\] (15)
From (14) and (15), one can see that \( \Delta i \) and \( x_{iH} (i = 1, 2) \) are bounded. Then from (18), it follows that \( \dot{q}_2 \) are bounded which means that \( \dot{x}_i (i = 1, 2) \) are bounded.
Moreover, from (11), one can obtain that
\[
| e_i(t) | \leq | C_i e^{A_i H} x_{iH}(0) | + \pi_i \| \Delta i \|_\infty , i = 1, 2. 
\] (16)
Therefore for given bounded initial conditions and given positive constants \( \varepsilon_i (i = 1, 2) \), one can find positive robust compensator parameters \( f_{ij} (i = 1, 2; j = 1, 2) \) with sufficiently large values and \( T_i \geq 0 (i = 1, 2) \) so that
\[
\sup_{t \geq T} | e_i(t) | < \varepsilon_i. 
\]
Remark 5: It should be noted that the robust compensator parameters \( f_{ij} (i = 1, 2; j = 1, 2) \) calculated by the robust approach can be conservative; that is, these parameters could be much smaller than the ones obtained by theoretical analysis, which can also guarantee the robust tracking properties.

V. EXPERIMENTAL RESULTS

The proposed robust controller is implemented in the processor of the dSPACE system. The elevation, travel, and pitch angles are measured by three encoders with effective resolutions of 0.0879 deg, 0.0439 deg, and 0.0879 deg, respectively [20]. The reference signals of the elevation and travel channels are obtained by \( r_i(p) = \bar{\omega}_i(p)/(\bar{\psi}_i(p) + 1) \) (i = 1, 2), where \( \bar{\omega}_i(p) (i = 1, 2) \) are corresponding reference input commands. The nominal helicopter parameters are shown in the first table, and the controller parameters in the second table respectively. Three experimental tests are carried out to examine the output tracking performances of the closed-loop system.

Case 1: In this case, reference input commands \( \bar{\omega}_1 \) and \( \bar{\omega}_2 \) are set to step signals. The corresponding transient responses and steady-state responses are shown in Fig. 4 and Fig. 5 respectively. Overshoots, 5%-zone settling times, and steady-state errors are 0.88% and 0.15%, 2.93s and 7.29s, and 0.0879 deg and 0.0439 deg for the elevation and travel channels respectively.

Case 2: The 3DOF helicopter is needed to track the large-angle references for the two angles simultaneously. Corresponding responses are depicted in Fig. 6. From this figure, one can see that the designed robust controller can achieve the desired performances under aggressive maneuvers. Note that in [13], the desired travel rate tracking control was dealt with whereas other angles can only be guaranteed to hold their position at 0 deg. Moreover, only one channel was considered in [13].

Case 3: In this experiment, the lab helicopter is aiming at tracking non-stationary sinusoidal signals for the two angles simultaneously, which describes a possible case in which a helicopter is required to land on the rocking deck of a ship [18]. The flight task includes hovering, landing, and synchronization with the motion of the oscillatory deck. The reference input commands \( \bar{\omega}_1(t) \) and \( \bar{\omega}_2(t) \) are given as

### TABLE I
**Nominal helicopter parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>( a_1 )</td>
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<td>( a_2 )</td>
<td>0</td>
<td>( a_3 )</td>
<td>0</td>
</tr>
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<td>( a_4 )</td>
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<td>( a_5 )</td>
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<td>( b_3 )</td>
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</table>

### TABLE II
**Controller parameters**

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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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<td>( \theta_2 )</td>
<td>1000</td>
<td>( \theta_1 )</td>
<td>0.67</td>
</tr>
<tr>
<td>( \psi_1 )</td>
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<td>( \psi_2 )</td>
<td>4</td>
<td>( f_{11} )</td>
<td>25</td>
</tr>
<tr>
<td>( f_{21} )</td>
<td>5</td>
<td>( f_{12} )</td>
<td>5</td>
<td>( f_{22} )</td>
<td>1</td>
</tr>
</tbody>
</table>
The tracking responses are presented in Fig. 7 and the corresponding tracking errors are shown in Fig. 8. One can see that the tracking errors of the elevation and travel channels are guaranteed to be less than 0.4 deg and 0.8 deg respectively. From these figures, the closed-loop system achieved good dynamical and steady-state tracking performances.

VI. CONCLUSIONS

A control approach combining the LQR control technique and robust compensation method was employed to achieve the automatic on a rocking deck for a 3DOF model helicopter. The designed linear time-invariant output feedback controller consists of two parts: a nominal controller and a robust compensator. Experimental results on the lab helicopter demonstrated the effectiveness of the control method.

APPENDIX

From (2), one can get positive constants $\zeta_{q1x}$ and $\zeta_{q1c}$ such that
\[
\|q_1\|_\infty \leq \zeta_{q1x} \|x_1 H\|_\infty + b_1 \rho_1 \|u_1\|_\infty + \zeta_{q1c}.
\]
From (4), (5), and (9), there exist positive constants $\zeta_{q1x}'$ and $\zeta_{q1c}'$ such that
\[
\|q_1\|_\infty \leq \zeta_{q1x}' \|x_1 H\|_\infty + \zeta_{q1c}'.
\]
Then, one can get positive constants $\zeta_{\Delta 1x}$ and $\zeta_{\Delta 1c}$ such that
\[
\|\Delta_1\|_\infty \leq \zeta_{\Delta 1x} \|x_1 H\|_\infty + \zeta_{\Delta 1c}.
\] (17)

Similarly, there exist positive constants $\zeta_{q2x}$, $\zeta_{q2c}$, $\zeta_{q2x1}$, and $\zeta_{q2c1}$ satisfying
\[
\|q_2\|_\infty \leq \zeta_{q2x} \|x_2 H\|_\infty + \zeta_{q2c},
\]
\[
\|\hat{q}_2\|_\infty \leq \zeta_{q2x1} \|x_2 H\|_\infty + \zeta_{q2c1}.
\] (18)

Furthermore, from (2), (4), (5), (9), and (18), there exist positive constants $\zeta_{u2x}$ and $\zeta_{u2c}$ such that
\[
\|u_2\|_\infty \leq \zeta_{u2x} \|x_2 H\|_\infty + \zeta_{u2c}.
\] (19)

Substituting (19) into (2), one can get two positive constants $\zeta_{\Delta 2x}$ and $\zeta_{\Delta 2c}$ such that
\[
\|\Delta_2\|_\infty \leq \zeta_{\Delta 2x} \|x_2 H\|_\infty + \zeta_{\Delta 2c}.
\] (20)

From (17) and (20), one can obtain the Lemma 1.
REFERENCES


