Adaptive Torque Ripple Compensation Technique Based on the Variable Structure Control and its Applications to Gear Driven Motion Systems

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Abstract—In this paper, a robust adaptive control scheme is proposed to reduce the effect of harmonic disturbances in gear driven feed drive systems. Harmonic forces are one of the most common disturbances in feed drives utilized in wide variety of applications such as in robotic systems or in various machine tools. They are generated, for instance, due to the external disturbances or internally by the servo structure itself such as torque ripples of the motor and gear mechanism. An adaptive torque ripple compensation technique is developed in this research, which can track the amplitude and phase of the sinusoidal disturbance forces. A suppression signal is then generated to reject the specific frequency of the disturbances. In conjunction with the ripple compensator, a robust feedback controller is designed employing the sliding mode variable structure control framework. Both, controller stability and the adaptation convergence are proven using the Lyapunov theory. The controller is then transformed into a PID-like structure and implemented in various gear driven servo systems where its performance is verified experimentally.

I. INTRODUCTION

Servo drives such as rotary or linear permanent magnet (PM) motors are heavily used in positioning applications such as in robotic systems or in the computer numerical controlled (CNC) machine tools. In PM motor drives, ‘torque ripples’ are one of the common error sources. Torque ripples or ‘bumps’ lead to speed oscillations, which deteriorate the positioning performance by limiting the application of PM motors in high-performance motion control.

In servo systems torque ripples generally originate from two main sources. As reported in the past literature, firstly they are inherent by the design of the permanent magnet motor, where the ripples occur due to the cogging torque, reluctance torque, mutual torque, and the DC offset depending on the motor position. Practically, in an optimally designed permanent PM drive the cogging, reluctance and mutual torque ripples can be neglected [1]. However, the DC current offset ripple is dominant and difficult to eliminate [2]. It occurs due to the unreliable measurement of the stator currents and its conversion into voltage signals. Particularly, presence of any unbalanced DC supply voltage in the current sensors are inherent, and offsets in the analog electronic devices give rise to those ripple forces [3],[4].

Secondly, when the PM motor is actually attached to the rest of the mechanical system, torque ripples arise due to the load variations [3]. For instance, “worm” or “spur” gears may be utilized for transferring the motor torque from motor to the components [5]. As the gears engage each other, the contact force creates high frequency load variations. These types of position dependent motion errors are also called ‘torque ripples’, and as the harmonics of those ripples are well above the bandwidth of the feedback controller, their compensation by classical control theory becomes a difficult task. Similarly, external forces also cause high frequency harmonic disturbances. Those are mostly experienced in manufacturing/machining applications where sinusoidal process forces are generated at the fundamental frequency of the cutter’s rotational frequency at its various harmonics [6].

There have been numerous solutions to tackle torque ripples and compensate them within the control system. Most of the approaches base on the Internal model principle [7] (IMP) where a complex pole pair is placed at the ripple frequency to generate infinite gain. In this design approach, it is generally assumed that the ripple frequency is fixed and does not vary by time. Wang [8] addressed this problem by designing a gain-scheduled controller that accommodates only for the slowly time-varying frequencies. Repetitive controllers [6][9] also base on the IMP where they can be employed to suppress the fundamental ripple frequency and its high frequency harmonics at once. Recently repetitive controllers with improved robustness have become attractive in eliminating multi frequency harmonic disturbance cancellation. In contrary, real-time implementation of repetitive controllers is cumbersome as the sampling frequency needs to be adjusted so that it is an integer multiple of the disturbance frequency. Iterative learning controllers (ILC) have also been exploited to overcome the ripple disturbances [10] where the learning can be performed in both time or the in the frequency domain [11].

In order to tackle the shortcomings of feedback controllers, feed-forward controller have been used to cancel ripple forces [5]. The fundamental idea is to identify ripple amplitude and phase in advance and cancel the harmonic disturbance in the feed-forward path [12][13]. It was shown that this gives satisfactory results in precision systems. However, due to the nature of the ripples, the amplitude of disturbance varies with load and velocity. In particular, load dependent ripples cannot be successfully compensated with this approach. The ripple phase is contingent on the reference position, which needs to be memorized leading to problems in practical implementation. In these cases adaptive feed-forward compensation (AFC) becomes attractive [14]. AFC can be used to adaptively estimate the magnitude and
phase of the “equivalent” disturbance at the ripple frequency, and the cancellation signal can be injected accordingly. Hosseinkhani has shown that the AFC controller can be used in conjunction with the feedback controller to suppress the harmonic disturbances in feed drive systems [15].

In this research a variable structure controller [16] (VSC) is designed to tackle those high frequency torque ripples and improve the positioning accuracy of the motion system. The objective is to accurately generate required compensation torque to cancel out high frequency ripple forces by adapting to ripple amplitude and phase during the motion of the servo. The proposed controller utilizes adaptive sliding mode control (ASMC) technique to widen the control bandwidth and at the same time generate accurate ripple compensation signal to cancel out the ripple forces. The Lyapunov energy function is used to prove the stability of the controller with the ripple adaptation law. It was experimentally shown that the proposed controller could successfully suppress harmonic disturbances in gear systems.

II. MODELING OF THE SERVO SYSTEM

The controller is designed for suppressing ripple disturbances in gear driven servo systems (See Fig. 1). The simplified model of the feed drive dynamics can be presented by considering only the rigid body motion of the servo system, and the governing differential equation is:

\[ u(t) - d(t) - d_e(t) = B_e \dot{x}(t) + J_e \ddot{x}(t) \]  

(1)

where \( \dot{x} \) is the drive velocity and \( \ddot{x} \) is acceleration. \( J_e \) is the control equivalent reflected inertia, \( B_e \) is the reflected friction in the system. \( u(t) \) is the control input, which corresponds to the torque command to the motor, \( d(t) \) is control equivalent disturbances and \( d_e(t) \) represents control equivalent ripple disturbances. The ripple disturbances in gear system are defined as position dependent forces and can be given for a single harmonic as:

\[ d_e(t) = A \cos(\omega x(t) + \phi) = a_1 \sin(\omega x(t)) + a_2 \cos(\omega x(t)) \]

(2)

where \( \omega = N/L \) represents the fundamental ripple frequency. \( N \) is the number of ripples, e.g. gear teeth. \( L \) is the displacement interval, e.g. the motor revolution. Hence, \( a_1 \) and \( a_2 \) define the ripple amplitude and phase with respect to the reference position. It should be noted that although contact in an actual gear system is rather complex and ought to be represented with a number of frequency harmonics, \( \omega = N/L \) being the known fundamental one. Consequently, the tracking error dynamics \( (e = x_r - x) \) of the servo system can be written from (1) and (2) as:

\[ \dot{e} = \ddot{x} - \frac{1}{J_e} \left[ u - d - B_e \dot{x} - a_1 \sin(\omega x) - a_2 \cos(\omega x) \right] \]

(3)

where \( x_r \) is the interpolated reference position command. In (3), the drive parameters such as the reflected inertia, friction as well as the ripple frequency \( \omega \) can be identified by simple frequency or time domain identification methods [17][4]. However, the identification of the ripple amplitude \( A = \sqrt{a_1^2 + a_2^2} \) is cumbersome. Since it is caused by the gear contact, it depends on the load attached to the system. Furthermore, the ripple frequency shifts depending on the speed of the motion, and thus the amplitude of the ripple cancellation control signal needs to be adjusted. In addition, the ripple phase \( \phi = \tan^{-1}(a_2/a_1) \) varies depending on the initial position of the motion as well as the speed of the motion. Considering a gear system, it particularly bases on when the gears start engaging each other and delayed by the phase of the closed loop feedback system. Hence, the objective of the proposed controller is to adaptively trace the high frequency ripple amplitude and phase, generate and inject a disturbance cancellation signal accordingly.

III. DESIGN OF THE CONTROL SYSTEM

A variable structure controller is designed to minimize the tracking errors of the system \( e \) and at the same time adaptively reject the ripple disturbances. Both objectives can be achieved by designing an adaptive sliding mode control as follows. The first step of designing the sliding mode controller is to select the sliding surface. A 1st order sliding surface is selected as:

\[ S = \lambda(x_r - x) + (\dot{x}_r - \dot{x}) = \lambda e + \dot{e} \]

(4)

where \( \lambda \) is the desired bandwidth of the control system. In other words the errors will tend to zero \( e, \dot{e} \to 0 \), i.e. \( x \to x_r \), and \( \dot{x} \to \dot{x}_r \), with a decay of \( \lambda \) on the sliding surface. In order to pull the errors on the sliding surface and adapt for the unknown ripple disturbance parameters, the following Lyapunov energy function is postulated as:

\[ V = \frac{1}{2} \left[ J_e S^2 + \frac{(d - \hat{d})^2}{\rho_1} + \frac{(a_1 - \hat{a}_1)^2}{\rho_2} + \frac{(a_2 - \hat{a}_2)^2}{\rho_3} \right] \]

(5)

The positive definite Lyapunov function in (5) penalizes deviation of errors from the sliding surface and the discrepancy between the actual and adapted disturbances as well as the ripple parameters. In this representation, \( \hat{d} \) is the adapted disturbance, \( \hat{a}_1 \) and \( \hat{a}_2 \) are the adapted ripple parameters. \( \rho_1 \) is the adaptation gain for slowly varying disturbances, and \( \rho_2, \rho_3 \) is the adaptation gain for the torque ripple parameters assuming that they are also slowly varying. For asymptotic stability of non-linear systems, the derivative of the Lyapunov function must be negative definite, meaning that the rate of change in the energy and prediction error must decrease in a stable system. An important aspect of this
design is to select appropriate adaptation laws for the disturbances and the ripple parameters so that the derivative of the Lyapunov function is strictly negative to generate a stable control law. The following disturbance observer is utilized [18] for adapting arbitrary forces, which integrates the sliding surface as:

\[ \hat{d} = \rho \kappa S \]  
(6)

where \( \kappa \) is used to impose limits on the integral action,

\[ \kappa = \begin{cases} 
0 & \text{if } \hat{d} \leq d^- \text{ and } S \leq 0 \\
0 & \text{if } \hat{d} \geq d^+ \text{ and } S \geq 0 \\
1 & \text{otherwise}
\end{cases} \]  
(7)

so that the estimated disturbance is always kept within the boundaries of the adaptation \( d \in [d^-, d^+] \). The adaptation law for the ripple parameters is also chosen to be a direct function of the sliding surface and the corresponding trigonometric function pairs. Thus, ripple adaptation laws are postulated as:

\[ \hat{a}_1 = \rho \kappa_{r,3} S \sin(\omega x) \text{ and } \hat{a}_2 = \rho \kappa_{r,2} S \cos(\omega x) \]  
(8)

where \( \kappa_{r,3} \) and \( \kappa_{r,2} \) are used to impose limits on the adapted ripple gain and the phase parameters by

\[ \kappa_{r,3} = \begin{cases} 
0 & \text{if } \hat{a}_1 \leq a_1^- \text{ and } S \sin(\omega x) \leq 0 \\
0 & \text{if } \hat{a}_1 \geq a_1^+ \text{ and } S \sin(\omega x) \geq 0 \\
1 & \text{otherwise}
\end{cases} \]  
(9a)

\[ \kappa_{r,2} = \begin{cases} 
0 & \text{if } \hat{a}_2 \leq a_2^- \text{ and } S \sin(\omega x) \leq 0 \\
0 & \text{if } \hat{a}_2 \geq a_2^+ \text{ and } S \sin(\omega x) \geq 0 \\
1 & \text{otherwise}
\end{cases} \]  
(9b)

so that \( \hat{a}_1 \in [\hat{a}_1^-, \hat{a}_1^+] \) and \( \hat{a}_2 \in [\hat{a}_2^-, \hat{a}_2^+] \) are satisfied. Once the adaptation laws and the parameter bounds are determined, the derivative of the Lyapunov function is computed from (5) as:

\[ \dot{V} = J S \left[ \lambda (\dot{x}_r - \dot{x}) + \dot{x} \right] - Su + SB + S\dot{d} - S\dot{\kappa}(d - \hat{d}) + S \hat{a}_1 \sin(\omega x) + \hat{a}_2 \cos(\omega x) \]  
(10)

Some simplifications can be undertaken to force the derivative of the Lyapunov function to be strictly negative. Firstly, the bounds on the disturbance adaptation are utilized from (7) and the following can be concluded:

\[ S d - S \kappa (d - \hat{d}) = \frac{S(d - d')(1 - \kappa) + S \hat{d}}{\leq 0} \]  
(11)

Similar conclusion can be drawn due to the limits imposed in (9) as:

\[ S a_1 \sin(\omega x) - S \kappa_1 (a_1 - \hat{a}_1) \sin(\omega x) \]
\[ = S \sin(\omega x)(a_1 - \hat{a}_1)(1 - \kappa_1) + S \hat{a}_2 \kappa_1 \sin(\omega x) \]  
\[ \leq 0 \]

\[ S a_2 \sin(\omega x) - S \kappa_2 (a_2 - \hat{a}_2) \sin(\omega x) \]
\[ = S \sin(\omega x)(a_2 - \hat{a}_2)(1 - \kappa_2) + S \hat{a}_2 \kappa_2 \sin(\omega x) \]  
\[ \leq 0 \]

Employing the simplifications from (11) and (12), the derivative of Lyapunov function in (10) can be constrained to be negative by only considering the remaining terms:

\[ \dot{V} \leq 0 \rightarrow J S \left[ \lambda (\dot{x}_r - \dot{x}) + \dot{x} \right] - Su + SB + S\dot{d} + S \hat{a}_1 \sin(\omega x) + \hat{a}_2 \cos(\omega x) + K_S S \]  
\[ \leq 0 \]  
(13)

\( K_s \) is a positive feedback to be tuned. This yields the asymptotically stable control law of

\[ u = J \lambda (\dot{x}_r - \dot{x}) + J \dot{x} + B \dot{x} + K_S S \]  
\[ + \frac{S \hat{a}_1 \sin(\omega x) + \hat{a}_2 \cos(\omega x)}{\text{Disturbance Observer}} + K_S S \]  
(14)

The final control law can then be obtained by inserting the disturbance observer from (6) and the ripple parameter adaptation laws from (8) in to (14) as presented below:

\[ u = J \lambda (\dot{x}_r - \dot{x}) + J \dot{x} + B \dot{x} + K_S S + \frac{\rho \int_0^T S d \tau}{\text{Disturbance Observer}} \]
\[ + \frac{\rho \sin(\omega x) \int_0^T S \cos(\omega x) d \tau}{\text{Adaptive Ripple Compensator(K_s)}} \]  
(15)

IV. CONTROLLER SYNTHESIS

The control law in (15) is linear in the parameters and resembles a proportional integral derivative (PID)-like structure with the addition of feed-forward dynamics cancellation as well as the ripple compensation control. The tuning parameters of the controller can be summarized as follows:

- Firstly, \( \lambda \) determines the decay of the errors on the sliding surface. Hence, it can be used to tune the bandwidth of the controller, or so called the desired speed of the tracking response.
- \( \rho \) is the gain of the disturbance observer, which is derived by the integration of the sliding surface. It is used to counteract slowly changing disturbances against the motion of the system and introduced robustness against the un-modeled dynamics.
- \( K_S \) is the feedback gain of the controller. It should be selected as high as possible to push the states onto the sliding surface while care needs to be given not to inject excess noise into the system.
• At last, \( \rho \) is the adaptation gain for the ripple parameters. Higher gains allow faster convergence of the ripple parameters. However, may jeopardize the stability of the control system by tightening the phase margin and causing actuator saturation.

\[
\begin{align*}
\rho_r &= \int F(t)dt \\
\rho &= \frac{\sin(\omega x)}{\cos(\omega x)} \\
&= \frac{S}{e} \\
&= \rho_r \\
&= \rho \\
&= \rho_i \\
&= \rho_f \\
&= \rho_c
\end{align*}
\]

Figure 2. Proposed AFC Element

For practical implementation, the controller parameters in the proposed adaptive ripple compensating sliding mode control law from (15) can be grouped and mapped into a PID-like form, and the PID gains can be computed as:

- \( J \hat{x} + B \dot{x} \) are the inertia and viscous friction feed-forward terms,
- \( \lambda K_s + \rho \) is the proportional (P) gain,
- \( \rho \lambda \) is the integral gain (I) and
- \( \lambda J_s + K_r \) is the derivative (D) gain of the controller.

At last, the ripple compensation feed-forward control term is extracted from (15).

\[
\begin{align*}
\rho_r &= \rho \left[ \sin(\omega x) \int_0^r S \sin(\omega x) d\tau + \cos(\omega x) \int_0^r S \cos(\omega x) d\tau \right] \\
&= \rho_r
\end{align*}
\]

It should be noted \( u_r \) resembles the commonly used detachable AFC [19] structure with an exception that it does not operate on the tracking error state \( (e) \), but the proposed one operates directly on the sliding surface, \( S = \lambda e + \dot{e} \). The block diagram of the proposed ripple compensating AFC element is given in Fig. 2. It is detachable, which allows the ripple compensation signal to be switched on or off depending on the application. Furthermore, analogous to the well-known AFC structure, the dynamics of the proposed ripple compensating term can be inspected in a linear time invariant (LTI) system structure. Following the LTI transformation methods introduced by Bayard [14] and utilizing the modulation property of the Laplace transform, transfer function form of (16) can be written as:

\[
\frac{e}{u_r} = \rho_r \frac{s(s + \lambda)}{s^2 + \omega^2}
\]

Hence, the proposed adaptive torque ripple compensating sliding mode controller can be implemented in a PID form with the detachable AFC element for torque ripple compensation and the block diagram of the control system is presented in Fig. 3. This allows us to analyze the stability of the closed loop system conveniently.

V. EXPERIMENTAL RESULTS

In order to validate the performance of the proposed adaptive ripple compensating sliding mode control system, experiments are performed on two different servo setup. In both setups rotary motors is attached to different gear systems for power transmission.

The first setup contains a spur gear for power transmission as illustrated in Fig. 4. The motor is attached to a gearbox, where the power is transmitted to a set of rollers for pulling a wire system. The power amplifier is set to operate in torque mode and the direct torque signal is send to the amplifier. The rotary encoder at the back of the motor is used for closed loop control. The gearbox has a gear ratio of \( \frac{1}{2} \) where the smaller gear on the motor side has 8 teeth. The identified system parameters as well as the controller gains are given in Table I.

<table>
<thead>
<tr>
<th>TABLE I. CONTROLLER PARAMETERS</th>
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<tbody>
<tr>
<td>Drive Parameters</td>
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<tr>
<td>( J_e = 0.00078458 ) [V/rad/sec³]</td>
</tr>
<tr>
<td>( B_e = 0.0014112 ) [V/rad/sec]</td>
</tr>
</tbody>
</table>

The controller is implemented at a sampling period of 0.1[msec]. Trapezoidal reference position trajectory is send to the motors where the maximum acceleration is set to 150[rads/sec²] and cruise velocity of 20[rads/sec] is achieved within 0.4[sec] from the start of the motion. The total travel distance is 250[rad]. At first, the proposed adaptive ripple compensating sliding mode...
controller parameters are mapped to P, I, D gains and the controller is implemented in PID form without the ripple compensation signal, $u_r = 0$ to inspect effect of the ripple disturbances on tracking errors. Tracking errors and its frequency spectrum are presented in Fig. 5. The second peak corresponds the first lower harmonic of the gear frequency, i.e. $N_f=8$ is set to cancel out the highest ripple frequency where $\omega = N_f/L = 8/2\pi$. Results are depicted in Fig 6. As shown, the adaptation parameters converge to their steady state values less than $1[\text{sec}]$. The intended frequency component of the ripple disturbances is canceled out completely, leading to reduction in the overall tracking errors. The improvement on the tracking performance is summarized in Table II.

### TABLE II. TRACKING ERRORS

<table>
<thead>
<tr>
<th>Ripple Compensation</th>
<th>Tracking Errors at $20[\text{rad/sec}]$</th>
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<tbody>
<tr>
<td></td>
<td>RMS[rad] x$10^3$</td>
</tr>
<tr>
<td>No Compensation</td>
<td>1.6337</td>
</tr>
<tr>
<td>$N_f=8$</td>
<td>0.93363</td>
</tr>
<tr>
<td>$N_f=8$ and $N_r=4$</td>
<td>0.67669</td>
</tr>
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</table>

As given in Table II, the RMS and maximum tracking errors are reduced by utilizing the proposed controller. By canceling the fundamental gear excitation frequency, $N_f=8$ errors are reduced around $35\%$. Furthermore, the first harmonic of the gear frequency, i.e. $N_f=4$ can also be canceled by including one more ripple compensating element into the controller structure. Its stability can be proven by the presented technique given in Section III. This introduces further improvement into the tracking performance of the system. The experimental tracking response is presented in Fig. 7, and the overall RMS and maximum errors could be reduced around $50\%$.

The controller performance was also validated on a worm gear driven servo system. Similar to the first setup, the equivalent drive parameters are obtained from identification tests, and the controller parameters are tuned for moderate performance and stability. Due to the nature of the worm gear, contact occurs fundamentally once in every rotation of
the motor. Hence \( N_r = 1 \) is considered to be the fundamental frequency of ripples forces. In addition, \( N_r = 2 \) can be added and double ripple cancellation is undertaken for better performance. In these experiments, reference trajectory is a back-and-forth motion with cruise speed of 25[rad/sec]. Results of the tracking experiment are shown in Fig 8. Main ripple errors are eliminated and the power spectrum of errors indicate that the intended frequencies are successfully canceled. Hence, the RMS errors are reduced around 40% from 6.2106e-4[rad] down to 3.9546e-4[rad] on this setup.

Figure 7. Compensation of the torque ripples by the proposed control (fundamental frequency and its harmonic \( N_r = 4 \) and \( N_r = 8 \))

Figure 8. Compensation of the torque ripples in worm gear setup including fundamental frequency and one harmonic \( N_r = 1 \) and 2

VI. CONCLUSION

This paper presented design of a novel ripple compensating adaptive sliding mode controller. The controller has effectively eliminated the ripples that occur due to load variations in gear driven servo system. The controller can easily be implemented in a PID-like form and the ripple compensating adaptive element can be attached in the form of an AFC. The effectiveness of the controller is validated in basic spur as well as the worm gear systems. The controller design presented in this paper can also be utilized to compensate for different high frequency repetitive errors in precision motion systems.

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