Adaptive Control Approach for Cylinder Balancing in a Hydraulic Linear Engine

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Abstract—This paper considers cylinder balancing of a novel hydraulic linear engine developed by the United States Environmental Protection Agency as an evolution of previous free-piston engine concepts. We employ an adaption algorithm that adjusts individual fuel injector commands to ensure the estimated kinetic energy of the crank tracks a set point. A control-oriented model and a high-fidelity model demonstrate the adaptive controller performance. The algorithm successfully reduces perturbations by 99% at a target speed of 1000 RPM. Additionally, the algorithm enables the engine to operate with different loads and conditions in each cylinder.

I. INTRODUCTION

A prototype engine has been developed by the United States Environmental Protection Agency (EPA) as an evolution of Free Piston Engine (FPE) concepts. Unlike a conventional internal combustion engine, an FPE has no crank. Consequently, piston travel and compression ratio are unconstrained and can be optimized for a variety of fuels and combustion modes. As an additional benefit, an FPE has fewer moving parts translating to lower friction and cost. Further, the absence of slider-crank dynamics reduces normal forces contributing to friction, alleviates wear, and allows the use of low-friction piston rings.

In conventional automotive applications, engine power is transmitted through a rotating crankshaft to the wheels. For a hybrid vehicle, the crank can be used to drive an electric generator or hydraulic pump. By contrast, an FPE utilizes the linear motion of the pistons. Some concepts, such as the FPE being developed at Sandia National Laboratories [1], use a linear alternator for electrical power generation. Preliminary studies performed at Sandia use a rapid compression and expansion machine to simulate FPE conditions and report thermal efficiencies up to 56% [1].

A hydraulic FPE uses the oscillating piston as a linear pump. Fig. 1 illustrates a dual piston, hydraulic FPE. Because both pistons are rigidly connected, the force from combustion in one cylinder compresses the charge contained within the other. Energy is stored in a high pressure accumulator or used to power a hydraulic motor. The CHIRON engine, developed by Innas BV, is a single-cylinder, two-stroke, hydraulic FPE reported to reach indicated efficiencies as high as 50% [2]. Similarly, Hibi and Ito have presented an opposed piston, two-stroke design capable of 31% hydraulic conversion efficiency [3].

As part of a Hydraulic Hybrid Vehicle (HHV) development program, the EPA designed and developed a hydraulic FPE [4]. The prototype achieved hydraulic conversion efficiencies around mid-30%. Shown in Fig. 2, the EPA FPE was a direct injected diesel engine consisting of three dual piston assemblies coupled by rack and pinion mechanisms. Unlike other FPE designs, the six-cylinder configuration enables four-stroke operation, which is characterized by lower emissions and higher efficiency compared to the two-stroke cycle.

Changing engine speed to modify power is not an option because the operating frequency of an FPE is nearly constant regardless of load [5], [6]. Instead, the EPA FPE used a bypass valve, presented in Fig. 1, to adjust engine power output. The bypass valve circumvents the low pressure check valve for a portion of each stroke. Opening the bypass valve for the entire stroke is analogous to idle, while keeping it completely closed results in the maximum hydraulic output.

Fig. 1. FPE with hydraulic bypass schematic. HP denotes the high pressure accumulator and LP is the low pressure reservoir

Fig. 2. EPA six-cylinder prototype Hydraulic FPE [4].

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FPE. Without a crank to constrain the piston motion, variations in stroke length can cause combustion instabilities or even lead to mechanical failure. If, for instance, too much fuel is added, the piston can travel further than prescribed and impact the cylinder head.

To address controllability and robustness concerns, the EPA has modified their FPE design. The dual piston assembly, consisting of cylinders 3b and 4b, has been removed and replaced with a crank device. Fig. 3 displays the new configuration, referred to as a Hydraulic Linear Engine (HLE). Unlike a conventional engine, the crank does not transmit power to a load. Instead, the apparatus functions as a safety mechanism to limit piston travel by exerting a force through the rack and pinion. The crank also actuates the intake and exhaust valves. Work, however, is still extracted hydraulically from the linear piston motion. Like a true FPE, side loads and friction are reduced because the pistons do not interact directly with the crank, but rather with the rack and pinion. A prototype of the HLE has been constructed and is currently being tested at EPA facilities.

![Fig. 3. EPA hydraulic linear engine.](image)

The HLE has low rotational inertia, facilitating minimal transients and fast startups. However, the engine is also more sensitive to cylinder imbalance, which results in oscillations that reduce lifespan. Unfortunately, the prototype HLE is inherently unbalanced. Cylinder displacements are mismatched because two cylinders have been removed from the six-cylinder design. In Fig. 2, cylinders 3a and 3b are considered a pair because they compress and burn simultaneously. Likewise, cylinders 4a and 4b are a pair. In order to balance the original FPE design, the combined displacement of cylinder pair 3a and 3b or pair 4a and 4b should match the displacement of either cylinder 1 or 2. For this reason, cylinders 3a, 3b, 4a, and 4b have a smaller bore than cylinders 1 and 2.

Current approaches to cylinder balancing take advantage of various torque estimation techniques. Once reconstructed, torque imbalances can be compared on a cycle-to-cycle basis and attributed to specific combustion events [7], [8]. Alternatively, a periodic engine description can be ‘lifted’ to create a time-invariant representation of the plant dynamics. Grizzle et al. [9] used this procedure to correct air-to-fuel ratio discrepancies in a spark-ignited engine with a single EGO sensor in the exhaust stream.

In this paper we use adaptive control techniques to balance an estimate of engine rotational kinetic energy stroke-to-stroke in the presence of an unknown, periodic disturbance. We design the controller using a simplified representation and validate with a high-fidelity, physics-based simulation.

II. CONTROL-ORIENTED MODEL

Typically, an automotive injector control unit calculates a fuel command using open-loop maps that depend on the torque demand and the current operating conditions. At steady state, each injector receives the same instructions whether or not the system is balanced. A new control structure is proposed to balance each cylinder individually using rotational kinetic energy estimates.

Consider a single cylinder engine near dead center: the piston is momentarily stationary. At this location, all mechanical kinetic energy is stored within the rotating components. Performing a simple energy balance between two dead center positions yields,

\[
\text{KE}(\Theta_2) - \text{KE}(\Theta_1) = \frac{1}{2} J (\dot{\Theta}_2^2|\Theta = \Theta_2 - \dot{\Theta}_1^2|\Theta = \Theta_1) = W_2, \quad (1)
\]

where KE is the kinetic energy, \(J\) is the rotational inertia, \(\Theta\) is the crank angle, and \(W_2\) is the work done on the engine. Equation (1) is a convenient expression because \(W_2\) is strictly a function of rotational velocity. Work is attributable to three components: frictional forces, hydraulic pressure forces, and in-cylinder pressure forces. Sampling every 180 Crank Angle Degrees (CAD), at every dead center position, each of the four cylinders experiences a different process of the four-stroke cycle. Expanding the work term produces

\[
W_2 = W_{\text{fric}} + W_{\text{comp}} + W_{\text{exp}} + W_{\text{exh}} + W_{\text{int}} + W_{\text{hyd}}, \quad (2)
\]

where \(W_{\text{fric}}\) and \(W_{\text{hyd}}\) are the frictional and hydraulic contributions. Work terms \(W_{\text{int}}, W_{\text{comp}}, W_{\text{exp}},\) and \(W_{\text{exh}}\) result from the in-cylinder processes intake, compression, expansion, and exhaust. Since each term refers to a different cylinder with varying geometry, the sum of the work components changes from stroke-to-stroke.

Control input is introduced to the system through the hydraulic and expansion terms. Performing a first law balance on the cylinder undergoing the expansion stroke generates,

\[
\Delta E_{\text{exp}} = Q_{\text{comb}} - Q_{\text{ht}} - W_{\text{exp}} = V_f \times \rho_f \times \text{LHV} - Q_{\text{ht}} - W_{\text{exp}}. \quad (3)
\]

Heat loss, \(Q_{\text{ht}}\), and the work done by the control volume, \(W_{\text{exp}}\), both decrease the change in internal energy during expansion, \(\Delta E_{\text{exp}}\). The heat addition from combustion, \(Q_{\text{comb}}\), is the product of injected fuel volume, \(V_f\), fuel density, \(\rho_f\), and the fuel lower heating value, \(\text{LHV}\). Fuel volume is the controllable parameter. Rearranging Equation (3):

\[
W_{\text{exp}} = V_f \times \rho_f \times \text{LHV} - Q_{\text{ht}} - \Delta E_{\text{exp}}. \quad (4)
\]

Assuming the accumulator pressure is constant over the duration of the stroke, the hydraulic work can be defined as

\[
W_{\text{hyd}} = \int (p_{hp} - p_{lp}) dV = -\Delta p_{hc} A_{\text{hyd}} D_s \alpha_{pz}. \quad (5)
\]
The controllable parameter, \( \alpha_{px} \), is the ratio of the power extraction distance to the stroke length, \( D_s \). We define \( A_{hyd} \) as the cross sectional area of the hydraulic pistons and \( \Delta p_{hc} \) as the difference in pressure between the high and low pressure accumulators, \( p_{hp} \) and \( p_{lp} \). Substituting (4) and (5) into (1), and sampling every 180 CAD yields,

\[
KE_{k+1} = KE_k + V_f \times \rho_f \times LHV - \Delta p_{hc}A_{hyd}D_s\alpha_{px} + g_k,
\]

where

\[
g_k = W_{comp} + W_{exh} + W_{int} + W_{fric} - Q_{ht} - \Delta E_{exp},
\]

As defined, \( g_k \) is unknown and varies periodically with time step, \( k \). We consider \( g_k \) a disturbance that is a function of cylinder geometry, speed, and load.

Using the aforementioned definitions, the state space representation of (6) is

\[
x_{k+1} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} \rho_f LHV & 0 \\ -\Delta p_{hc} A_{hyd} D_s & 0 \end{bmatrix} u_k + \begin{bmatrix} g_k \\ 0 \end{bmatrix}
\]

\[
y_k = [0 \ 1] x_k,
\]

with states

\[
x_{1,k} = KE_k, \ x_{2,k} = KE_{k-1}, \ u_{1,k} = V_f, \text{ and } u_{2,k} = \alpha_{px}.
\]

Because the injection corresponding to \( u_k \) occurs just before engine speed is sampled at TDC, the control update must be calculated using the information from \( KE_{k-1} \). State \( x_2 \) accounts for the delay in the output.

### III. CONTROLS FORMULATION

Fig. 4 illustrates the adaptive control structure used to adjust for cylinder imbalance.

[Diagram of adaptive control scheme]

We break the disturbance, \( g_k \), into four unknown elements, each corresponding to the combustion event of a different cylinder. The individual disturbances, \( \theta_1 - \theta_4 \), interact with the system through a known regressor, \( \phi_k \). Let \( \phi_k \theta = g_k \frac{\rho_f LHV}{1 + LHV} \). The regressor is a collection of four step functions such that \( \phi_k = [\phi_1, \phi_2, \phi_3, \phi_4] \) and

\[
\phi_k \theta = \begin{cases} \theta_1 & k = 1, 5, 9, \ldots \\ \theta_2 & k = 2, 6, 10, \ldots \\ \theta_3 & k = 3, 7, 11, \ldots \\ \theta_4 & k = 4, 8, 12, \ldots \end{cases}
\]

Assuming zero power extraction (\( u_2 = 0 \), engine at idle), (8) is written as

\[
x_{k+1} = Ax_k + Bu_k + \phi_k \theta,
\]

where

\[
A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \rho_f LHV \\ 0 \end{bmatrix}, \quad \text{and} \quad C = [0 \ 1],
\]

The reference is the kinetic energy at the desired speed, \( \dot{\Theta}_r \), such that \( r = \frac{1}{2} J \dot{\Theta}_r^2 \). The periodic disturbance is canceled using an estimation, \( \hat{\theta} \), based on a recursive least squares algorithm. The control law has the following form:

\[
u_k = -[K, K_I] \begin{bmatrix} \dot{x}_k \\ w_k \end{bmatrix} - \phi_k \theta,
\]

where \( K \) is a stabilizing state feedback gain and \( K_I \) is an integrator gain. Gains \( K \) and \( K_I \) are chosen such that the eigenvalues of the closed-loop state matrix lie within the unit disc.

At time step \( k \), variable \( z_k \) is constructed using known inputs and measurements,

\[
z_k = \frac{1}{\rho_f LHV} [y_k - y_{k-1}] - u_{k-2} + \phi_k \theta = \psi_k \theta,
\]

where \( \psi_k = \phi_{k-2} \). Equation (17) allows for the construction of a priori error, \( \epsilon_k \), or

\[
\epsilon_k = z_k - \psi_k \hat{\theta}_k.
\]

A Recursive Least Squares (RLS) algorithm with exponential forgetting yields the adaptive disturbance estimation [10],

\[
\hat{\theta}_k = \hat{\theta}_{k-1} + \Gamma_k \epsilon_k,
\]

\[
\Gamma_k = \frac{\lambda^{-1} P_{k-1}}{1 + \lambda^{-1} \psi_k P_{k-1} \psi_k^T},
\]

\[
P_k = \lambda^{-1} P_{k-1} - \lambda^{-1} \Gamma_k \psi_k P_{k-1} \psi_k^T.
\]

Persistence of excitation is a sufficient condition for parameter convergence [10], requiring that

\[
0 < c_1 I < \sum_{n=k}^{k+T_p} \psi_n^T \psi_k < c_2 I < \infty,
\]
where $c_1$ and $c_2$ are constants such that $0 < c_1 < c_2 < \infty$. Because $\psi_k$ (and by extension $\psi'_k$, $\psi''_k$) periodically repeats every four iterations, consider $T_p = 4$: $\sum_{n=k}^{k+4} \psi'_k \psi_k = I$. It follows that for any $T_p > 4$, condition (22) is satisfied and parameter convergence is guaranteed. Further, because the plant is considered linear and closed-loop stable, the 'key technical lemma' in Goodwin and Sin [10] can be applied to show that the system, including the RLS algorithm, is closed-loop stable and that the output tracking error approaches zero.

Fig. 5 shows the system response to an arbitrary disturbance of $\theta = [-3, -2, -5, -1]^T$, commanded to a reference of $r = 6$. Before time step zero, the closed-loop system is allowed to converge without the RLS algorithm. A periodic disturbance creates the periodic error. At time step 10, the adaptive controller is activated with an initial estimate of $\hat{\theta}_0 = [0, 0, 0, 0]^T$. There is a perturbation while the updated disturbance estimate is abruptly added to the control input. As the disturbance estimation begins to converge to the correct values, the oscillations approach zero and the integrator brings the system to the desired reference. The perturbation can be significantly decreased by using a better initial guess for the disturbance estimate, $\hat{\theta}_0$.


A. Dynamics

A model was constructed by Filipi and Assanis [12] to simulate the speed fluctuations of a single cylinder conventional engine. Much like a single-cylinder engine, the HLE has low rotational inertia and is therefore subject to analogous dynamics. Our model for slider-crank dynamics follows a similar structure.

A torque balance describes the rotation of the crank. Cylinder-pressures and piston inertia contributions to net torque are a function of crank angle and act through a geometric projection. Because the movement of the slider, the pistons, and the pinions are linearly coupled, they are lumped together as an equivalent mass in a single inertial force balance.

B. Thermodynamics

The in-cylinder control volume is considered. A mass balance tracks individual species across the boundary during intake, exhaust, and injection. We assume injection rate as constant during the injection pulse, calculated using calibrations from the prototype HLE. Valve flow is modeled as compressible flow through an orifice [13], and is driven by up and downstream pressures.

The combustion model tracks specific species generation and consumption based on a generalized chemical reaction. Common diesel fuel consists of a wide variety of compounds and additives. For simplicity we use dodecane, $C_{12}H_{26}$, as a diesel surrogate. The rate of change in mass fraction of a particular species follows the Watson correlation to capture both the premixed and diffusion processes of a two staged diesel combustion [14]. We model ignition delay using an autoignition ignition integral based on the Arrhenius equation as demonstrated by Assanis and Heywood [15].

The first law of thermodynamics describes temperature and pressure changes in the control volume. We assume a convective heat loss model, governed by a Hohenberg correlation [16].

C. Hydraulics

We model hydraulic pumping as turbulent flow through an orifice, similar to Yuan and Wu [17]. Flow is driven by the pressure differential between the pumping chamber and the accumulator. We treat the accumulator as a constant pressure reservoir with infinite volume while the change in chamber pressure is a function of the bulk modulus multiplied by the flow and divided by volume.

D. Model Behavior

Fig. 6 compares the instantaneous crank dynamics of the preliminary results from the prototype HLE to those simulated by the high fidelity model. Both cases received the same fuel command and were allowed to converge to a steady state. The maximum variation between the modeled and measured engine speed is approximately 5% of full scale.
The experimental behavior is partially the result of unintended compliance in rigidly designed components. Further engine testing and repairs will reduce modeling errors by facilitating the identification and calibration of dynamic and thermodynamic expressions.

![Graph](image)

**Fig. 6.** Comparison of experimental and modeled engine dynamics at steady state with same fueling.

V. RESULTS

The adaptive controller described in sections II and III has been applied to the high-fidelity model outlined in section IV. Fig. 7 shows the response of the HLE model with the adaptive RLS control scheme. The engine is at idle with zero power take-off, commanded to a speed of 1000 RPM, which corresponds to a rotational kinetic energy of approximately 1.6 kJ. Prior to iteration zero an integral controller has converged but is unable to reject the periodic disturbance resulting from the imbalanced cylinder geometry. At iteration 50 the adaptive controller is activated with an initial guess of \( \hat{\theta} = [0, 0, 0, 0] \). Similar to the low fidelity response in Fig. 5, the engine speed fluctuates as the disturbance estimate is abruptly added. Subsequently, the disturbance estimate converges and engine speed settles to the set-point. The adaptive controller successfully decreases the amplitude of oscillations around the target by 99%.

The fuel command, \( u_k \), is measured as volume in cubic millimeters. It follows from the definition in equation (12), that the disturbance, \( \theta \), has the same units as the fuel command. Also note that \( \theta \) is a scalar multiple of \( g_k \), which is roughly a measurement of losses over a single stroke. The disturbance estimate can be interpreted as the amount of fuel, in cubic millimeters, that does not contribute to useful work over the duration of a single stroke. In fact, the data presented in Fig. 7 is consistent with the idea of engine idle. The disturbance estimate converges to the same value but opposite sign as the fuel command, suggesting that none of the energy in the fuel is extracted as useful work. Instead, it is all used to overcome friction and heat transfer losses.

Injector calibrations convert a fuel command into a pulse width. An error in calibration would result in the injection of an incorrect fuel quantity for a given command. The adaptive control scheme described in section III automatically adjusts for any calibration errors. However, a particular fuel command would no longer directly correspond to the actual volume of fuel injected into the cylinder. As a result, the physical interpretation of the disturbance estimate loses its meaning.

Fig. 7 displays the rotational velocity as sampled by the controller every 180 CAD. The adaptive scheme proposed focuses specifically on reducing the differences in speed between each sample. This snapshot is sufficient to balance the cylinders but is not representative of instantaneous engine dynamics shown in Fig. 6. Large engine speed fluctuations are the result of low rotational inertia compounded by cylinder imbalance. All instantaneous variations cannot be completely eliminated because of the discrete nature of the input, yet they can be reduced through cylinder balancing. Fig. 8 superimposes the results of applying the adaptive controller at a reference speed of 900 rpm to the high fidelity model onto the fixed fueling data illustrated in Fig. 6. Balancing the engine reduces overall variation in engine speed at a given operating point.

Fig. 9 illustrates the controller’s capability to adjust for load discrepancies in each cylinder independently. A different load, \( \alpha_{\text{ref}} \), is applied to each cylinder at various iterations. After four load perturbations, the speed settles to the targeted 1000 RPM. Speed deviations can be minimized by including an open loop estimate of the load. Individual cylinder fuel control enables the engine to run in a number of attractive configurations. For instance, it may be more efficient at lower loads to run a number of cylinders near full load while
deactivating others. Alternatively, different cylinders could operate with different combustion modes, some with normal compression ignition and others with homogenous charge compression ignition (HCCI) type combustion.

An adaptive control scheme has been developed to correct cylinder imbalance for a novel hydraulic linear engine prototype developed by the EPA. The balancing relies on estimating engine speed near dead center every 180 CAD, computing the kinetic energy, and balancing the energy using RLS algorithm with exponential forgetting. A high fidelity physics-based model was developed and used for controller validation. We demonstrated that the controller minimizes speed variations stroke-to-stroke by adjusting the fuel command to each cylinder individually. This also enables the engine to operate with different loads for different cylinders.

We are currently investigating parameter learning techniques to intelligently update open-loop fuel maps using on-line results from the adaptive control algorithms presented in this paper.

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