Abstract—The main objective of this work is to design a coordinated distributed model predictive control (CDMPC) architecture for nonlinear systems using the price-driven coordination method. One fundamental assumption of this work is that a centralized model predictive control (MPC) scheme can be designed based on successive linearization of the nonlinear system at the origin and the coordination schemes strives to get to the very same performance via adding a coordinator level to the existing decentralized structure. In other words, the coordinator and the distributed MPCs exchange information and calculate the optimal future input trajectories iteratively. In order to deploy the price-driven coordination algorithm developed by Marcos [1], which is applicable to linear or linearized systems, the alkylation process of benzene used as a case study was successively linearized around the operating points. The simulation results demonstrate that the algorithm can be successfully applied to nonlinear systems using a successive linearization strategy.

I. INTRODUCTION

Generally speaking, there are three choices for large-scale MPC design: decentralized, distributed and centralized. Decentralized schemes are traditionally preferred, since they are easy to implement. In fact, decentralized schemes completely ignore the interactions between subsystems, which may result in comparatively poor plant-wide performance in the sense that they cannot capture the whole plant behaviour. On the contrary, centralized schemes are based on designing a monolithic controller for the entire plant; however, they may become too complicated to implement as the size of the problem grows large, require substantial maintenance effort and show poor fault tolerant behaviour [2]–[4].

One of the techniques used in distributed control is the coordination method. This technique includes several approaches, such as the price-driven, prediction-driven and primal decomposition [5].

The coordinated approach aims to achieve centralized optimal performance by adding a coordination level to an already existing decentralized system, which requires some minor modifications to the existing decentralized system [1], [6]–[17].

Based on information flow between the subsystems in a network, there exists several coordinated control schemes in the literature. Coordinated distributed control schemes can be classified into four methods. First, the price-driven approach in which the coordinator provides pricing information for the local controllers in order to allocate the shared resources [1], [5], [8], [14]–[17]. Second, the primal decomposition approach in which the coordinator allocates the shared resources to the controllers [5], [8]. Third, the prediction-driven approach in which each operating unit optimizes its own objective function based on the gathered information from predicted resource consumption computed in the other units [1], [5]–[8], [18], [19].

In this work, we will address the problem of designing the coordination level for distributed MPC structures via the price-driven coordination algorithm. With minimal modifications to an existing decentralized system, the CDMPC method tries to match the centralized performance as close as possible. The CDMPC strategy is shown in Fig. 1. Note in Fig. 1 that there exists a coordinator that manipulates the local MPCs in order to obtain the centralized performance.

The price-driven coordination method proposed by Marcos [1] is used as the coordination technique in this work, and it comprises two parts. In a lower level, overall process is decomposed into a certain number of sub-systems. And in an upper level, the coordinator adjusts a price that it is sent to the sub-systems, in an iterative procedure, in order to obtain the optimal performance of the system. The adjustment of the price is done by Newton’s method.

The first section of this paper is dedicated to the distributed MPC formulation and the corresponding successive linearization procedure; accordingly, we define the price-driven coordination formulation. In the second section, we will focus on the coordination level design for the networked MPC problem. Next, the stability of the closed-loop system under the CDMPC algorithm is discussed. Finally, the applicability and effectiveness of the proposed approach is illustrated via its application to a large-scale process, namely the alkylation process of benzene.
II. DISTRIBUTED MPC FORMULATION

A. Successive Linearization of the Centralized and the Decentralized Nonlinear Models

Consider the continuous plant model defined by the following equations:

\begin{align*}
\dot{x} &= f(x,u) \\
y &= h(x,u)
\end{align*}

where $x \in \mathbb{R}^m$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are defined as the vector of states, manipulated variables and control outputs.

Using the first-order Taylor expansion of (1) and (2), we obtain equations (3) and (4); equations (3) and (4) are the linearized versions of equations (1) and (2) around the current nominal states ($x_0$) and inputs ($u_0$) values.

\begin{align*}
\dot{x}(t) &= A(t)\hat{x}(t) + B(t)\hat{u}(t) \\
y(t) &= C(t)\hat{x}(t) + D(t)\hat{u}(t)
\end{align*}

where $\hat{x}(t)$ and $\hat{u}(t)$ are deviations from the nominal corresponding values. Additionally, the values of the matrices $A$, $B$, $C$ and $D$ are shown in equations (5)-(8)

\begin{align*}
A(t) := \frac{\partial f(x,u)}{\partial x} |_{x=x_0,u=u_0} \\
B(t) := \frac{\partial f(x,u)}{\partial u} |_{x=x_0,u=u_0} \\
C(t) := \frac{\partial h(x,u)}{\partial x} |_{x=x_0,u=u_0} \\
D(t) := \frac{\partial h(x,u)}{\partial u} |_{x=x_0,u=u_0}.
\end{align*}

Accordingly, we can discretize the state-space model of the system, defined in equation (3), as shown in equation (9)

\begin{equation}
\Delta x(k+1) = \Psi(k)\Delta x(k) + \Gamma(k)\Delta u(k)
\end{equation}

where $\Psi$ and $\Gamma$ are the discrete-time version of matrices $A$ and $B$.

We suppose that there are predefined set-points ($Y_{sp}$) that are obtained by an upper level optimizer, so the MPC formulation of the centralized problem, within each sampling time can be represented in equation (10)

\begin{align*}
J = \min_{Y_{sp},\Delta U_k} \left( \frac{1}{2}((Y_{sp} - Y(k))^TQ(Y_{sp} - Y(k))
+ \Delta U_k^T R \Delta U_k) \right)
\end{align*}

subject to

\begin{align*}
\hat{x}(k+1) &= \Psi(k)\hat{x}(k) + \Gamma(k)\hat{u}(k) + \Delta x(k), \\
\hat{u}(k+1) &= \sum_{n=0}^{l} \Delta \hat{u}(k+n) + u(k), \\
\Delta \hat{u}(k+1) &= 0, H_u \leq l \leq H_p - 1, \\
\hat{y}(k+1) &= C\hat{x}(k) + l + 1|k, \\
y_{\min} \leq \hat{y}(k+1) \leq y_{\max}, \\
\Delta u_{\min} \leq \Delta \hat{u}(k+m) \leq \Delta u_{\max}
\end{align*}

for $l = 0, \cdots, H_p - 1$ and $m = 0, \cdots, H_u - 1$, where $Y(k) = [\hat{y}(k+1) \cdots, \hat{y}(k+1)]^T$, $\Delta \hat{u}(k+1) = [\Delta \hat{u}(k+1) \cdots, \Delta \hat{u}(k+1)]^T$. The vector of the predicted outputs, $\Delta U_k = [\Delta \hat{u}(k) \cdots, \Delta \hat{u}(k+1)]^T$ is the vector of the calculated manipulated variables, $Q = diag[Q(l+1)]$ is the weighting matrix for the states of the system, and $R = diag(R(m))$ is the weighting matrix for the manipulated variables; in fact, $Q$ and $R$ are positive semi-definite matrices. In addition, $H_p$ and $H_u$ are the numbers of prediction and control horizons, respectively.

The decentralized problem for $N$ subsystems is shown in equation (11)

\begin{align*}
J_k = \min_{Y_{sp},\Delta U_k, V_k} \left( \frac{1}{2}((Y_{sp} - Y(k))^T Q_{ii} (Y_{sp} - Y(k))
+ \Delta U_k^T R_{ii} \Delta U_k) \right)
\end{align*}

subject to

\begin{align*}
\hat{x}_i(k+1 + 1|k) &= \Psi_{ii}(k)\hat{x}_i(k+1 + 1|k) + \Gamma_{ii}(k)\hat{u}_i(k+1 + 1|k), \\
\hat{u}_i(k+1 + 1|k) &= \sum_{n=0}^{l} \Delta \hat{u}_i(k+n) + u_i(k), \\
\Delta \hat{u}_i(k+1 + 1|k) &= 0, H_u \leq l \leq H_p - 1, \\
\hat{y}_i(k+1 + 1|k) &= C_i\hat{x}_i(k+1 + 1|k), \\
y_{i,\min} \leq \hat{y}_i(k+1 + 1|k) \leq y_{i,\max}, \\
\Delta u_{i,\min} \leq \Delta \hat{u}_i(k+m) \leq \Delta u_{i,\max}
\end{align*}

for $l = 0, \cdots, H_p - 1$ and $m = 0, \cdots, H_u - 1$.

B. Price-Driven Coordinated MPC Formulation

The overall problem defined in equation (10) can be formulated as $N$ subproblems shown in equations (12a)-(12i)

\begin{align*}
J_k = \min_{Y_{sp},\Delta U_k, V_k} \left( \frac{1}{2}((Y_{sp} - Y(k))^T Q_{ii} (Y_{sp} - Y(k))
+ \Delta U_k^T R_{ii} \Delta U_k) \right)
\end{align*}

subject to

\begin{align*}
\hat{x}_i(k+1 + 1|k) &= \Psi_{ij}(k)\hat{x}_i(k+1 + 1|k) + \Gamma_{ij}(k)\hat{u}_i(k+1 + 1|k), \\
\hat{u}_i(k+1 + 1|k) &= \sum_{n=0}^{l} \Delta \hat{u}_i(k+n) + u_i(k), \\
\Delta \hat{u}_i(k+1 + 1|k) &= 0, H_u \leq l \leq H_p - 1, \\
\hat{y}_i(k+1 + 1|k) &= C_i\hat{x}_i(k+1 + 1|k), \\
y_{i,\min} \leq \hat{y}_i(k+1 + 1|k) \leq y_{i,\max}, \\
\Delta u_{i,\min} \leq \Delta \hat{u}_i(k+m) \leq \Delta u_{i,\max}
\end{align*}

and

\begin{align*}
\hat{u}_i(k+1 + 1|k) &= \sum_{n=0}^{l} \Delta \hat{u}_i(k+n) + u_i(k), \\
\Delta \hat{u}_i(k+1 + 1|k) &= 0, H_u \leq l \leq H_p - 1, \\
\hat{y}_i(k+1 + 1|k) &= C_i\hat{x}_i(k+1 + 1|k), \\
y_{i,\min} \leq \hat{y}_i(k+1 + 1|k) \leq y_{i,\max}, \\
\Delta u_{i,\min} \leq \Delta \hat{u}_i(k+m) \leq \Delta u_{i,\max}
\end{align*}

for $l = 0, \cdots, H_p - 1$ and $m = 0, \cdots, H_u - 1$, where $\hat{v}_i$ is defined as the interacting or linking variable among the different sub-units in the plant and $V(k) = [\hat{v}(k +
1[k]^T, \cdots, \hat{v}(k + H_p[k]^T]^T is the vector of the predicted linking variables; the matrix \( \Theta_i \) is the coefficient matrix for the linking constraints; and \( p \) is the price vector coming from the coordinator.

Finally, the problem formulation of (12) can be further simplified as equation (13)

\[
Z_i^* = \arg \min \left( \frac{1}{2} ((Z_i(k))^T \Theta_i (Z_i(k)) + \Phi_i^T Z_i(k) + p^T \Theta_i Z_i(k)) \right) \tag{13a}
\]

subject to

\[
\begin{align*}
B_i^{eq} Z_i(k) &= b_i^{eq} \\
B_i^{ineq} Z_i(k) &\leq b_i^{ineq} \tag{13b}
\end{align*}
\]

where \( Z_i(k) \) represents the vector of decision variables, namely, \( Z_i(k) = [Y_i(k)^T, \Delta U_i(k)^T, V_i(k)^T]^T \).

III. COORDINATION OF DISTRIBUTED MPC

The price-driven coordination algorithm was used to solve the algebraic optimization problems [20], [21] and [14]; also, this algorithm was used for dynamic optimization problems [17].

Using an efficient price-updating method, such as Newton’s method, we can achieve the plant-wide optimum performance for the coordination problem defined in (13a)-(13c) for \( i = 1, \cdots, N \). Using Newton’s method, the price vector can be updated as in equation (14) [14]:

\[
p(k+1) = p(k) - \alpha J^{-1} \Delta(k), \tag{14}
\]

where \( \alpha \) is the step size in Newton’s method, \( \Delta(k) = \Theta_i Z_i(k) \), and the Jacobian matrix \( J \) can be calculated as

\[
J = \frac{d\Delta(k)}{dp} = \Sigma_i \Theta_i \frac{dZ_i(k)}{dp(k)} \tag{15}
\]

The Jacobian matrix, equation (15), requires information of the sensitivity matrix \( \frac{dZ_i(k)}{dp(k)} \). In fact, during the coordination procedure, the coordinator should be provided with the decision variables \( Z_i(k) \) in every iteration.

The sensitivity analysis approach, used in equations (16) and (17) was developed in several publications [14], [16], [17], [22] and [1], can be used to solve problem (15). By performing a sensitivity analysis, the matrix \( \frac{dZ_i(k)}{dp(k)} \) can be calculated. This requires solving the following system of linear equations:

\[
\begin{bmatrix}
\nabla_p Z_i(k) \\
\nabla_p \lambda_i \\
\n\nabla_p \Lambda_i \\
\n\nabla_p \sigma_i
\end{bmatrix} = -\begin{bmatrix}
\Theta_i^T \\
0 \\
0 \\
0
\end{bmatrix} \tag{16}
\]

where

\[
\Gamma_i = \begin{bmatrix}
\Upsilon_i & B_i^{eqT} & B_i^{ineqT} & 0 \\
B_i^{eqT} & 0 & 0 & 0 \\
B_i^{ineqT} & 0 & 0 & 0 \\
 inactive B_i^{ineqT} & 0 & 0 & I
\end{bmatrix} \tag{17}
\]

A detailed derivation of equations (16) and (17) can be found in [14], [16] and [1]. The procedure for implementing the coordinated distributed MPC is described in [17] and [1].

IV. STABILITY OF THE CLOSED-LOOP SYSTEM UNDER THE CDMPC ALGORITHM

In this section, we discuss the nominal stability of the CDMPC algorithm. This analysis, without the loss of generality, is based on the regulation control problem. The stability of the closed-loop system is analyzed at each operating point obtained on the basis of the successive linearization of the plant. Here, the analysis is based on the convergence of the coordination algorithm to the centralized solution.

Proposition 1: Consider the successively linearized process system (1)-(2) is controlled by the CDMPC algorithm (13a)-(13c). Under the conditions that all of the sub-systems are stable, matrices \( Q_{ii} \) and \( R_{ii} \) are positive definite and fixed, there are control actions obtained by the centralized controller (10) that can stabilize the nonlinear model, the binding set of constraints are known, the convergence of first element of the control sequence of the prediction horizon to the first element of control action sequence in the prediction horizon trajectory of the centralized optimal at each operating point [1] is guaranteed, and the sampling interval is sufficiently long to permit convergence of the CDMPC algorithm, then the entire closed-loop system under CDMPC is said to be asymptotically stable.

Proof: As defined in section II-B, the overall objective function used in the CDMPC algorithm consists of all sub-systems as stated in equation (18)

\[
J_{CDMPC} = \sum_{i=1}^{N} J_i \tag{18}
\]

where \( J_i \) is the value of objective function for sub-system \( i \). Let the optimum value of the CDMPC at each operating point \( k \) is denoted by \( J_{CDMPC}^*(k) \). Considering a finite prediction horizon, \( J_{CDMPC}^*(k) \) can be described in equation (19)

\[
J_{CDMPC}^*(k) = \sum_{i=1}^{N} (Z_i^T(k)\Upsilon_i Z_i^*(k) + Z_i^T(k + H_p)\Upsilon_i(k)Z_i^*(k + H_p)) \tag{19}
\]

where \( \Upsilon_i(k) = diag(\hat{Q}_i, \lambda_i) \) and \( \hat{Q}_i(k) \) is the terminal penalty for each subsystem and it is obtained by solving the Lyapunov equation (20)

\[
\hat{Q}_i = C_i^T Q_i C_i + \Phi_i^T(k) \hat{Q}_i(k) \Phi_i(k) \tag{20}
\]

for \( i = 1, \cdots, N \).

Knowing the active set of constraints and assuming feasibility of the solution and convergence of the CDMPC algorithm at time \( k \) [1], feasibility of the control sequence \( \{u(k + 1|k), \cdots, u(k + H - 1|k)\} \) as well as convergence of the control trajectory to the centralized optimal solution is guaranteed. Therefore, in the presence of inequality constraints the closed-loop system is asymptotically stable [23].

V. NUMERICAL IMPLEMENTATION

The simulation studies were performed on a 2.3 GHz Intel® Core-i7 3610QM processor with 8 GB of RAM. For the numerical calculations, we used MATLAB® 2012a. The quadprog command was used to solve the MPC optimization problems, using interior-point option. The illustrative
example of this work is the application of the proposed coordination technique to control the process of alkylation of benzene with ethylene to produce ethylbenzene.

More specifically, the process consists of four CSTRs and a flash tank separator, as shown in Fig. 2. Pure benzene is fed from stream \( F_1 \) and pure ethylene is fed from streams \( F_2, F_4 \) and \( F_6 \). Two catalytic reactions take place in CSTR-1, CSTR-2 and CSTR-3. In reaction 1, benzene \((C)\) reacts with ethylene \((B)\) and produces the desired product ethylbenzene \((D)\). Further, in reaction 2, ethylbenzene reacts with ethylene to form a byproduct, namely 1,3-diethylbenzene \((E)\). In the flash tank separator, most of benzene is separated overhead by vaporization and condensation techniques and recycled back to the plant. A portion of the recycle stream \( F_{r1} \) is fed to CSTR-4 and the other portion \( F_{r2} \) is fed back to CSTR-1 together with an additional feed stream \( F_{10} \) which contains 1,3-diethylbenzene. Moreover, reaction 2 and reaction 3 in which 1,3-diethylbenzene reacts with benzene to produce ethylbenzene take place in CSTR-4. Finally, we assume that throughout the process all the materials are in liquid phase and their molar volumes are constant [24].

![Process flow diagram of alkylation of benzene](image)

The nonlinear state-space model of the process is described in equations (21)-(25)

**CSTR-1:**

\[
\frac{dC_{A1}}{dt} = \frac{F_1 C_{A0} + F_2 C_{Ar} - F_3 C_{A1}}{V_1} - \frac{r_1(T_1, C_{A1}, C_{B1})}{V_1} \quad \text{(21a)}
\]

\[
\frac{dC_{B1}}{dt} = \frac{F_2 C_{B0} + F_2 C_{Br} - F_3 C_{B1}}{V_1} - \frac{r_1(T_1, C_{A1}, C_{B1})}{V_1} \quad \text{(21b)}
\]

\[
\frac{dC_{C1}}{dt} = \frac{F_2 C_{CBr} - F_3 C_{C1}}{V_1} + \frac{r_1(T_1, C_{A1}, C_{B1})}{V_1} \quad \text{(21c)}
\]

\[
\frac{dC_{D1}}{dt} = \frac{F_2 C_{Dr} - F_3 C_{D1}}{V_1} + \frac{r_1(T_1, C_{B1}, C_{C1})}{V_1} \quad \text{(21d)}
\]

**CSTR-2:**

\[
\frac{dC_{A2}}{dt} = \frac{F_3 C_{A1} + F_2 C_{A2}}{V_2} - \frac{r_1(T_2, C_{A2}, C_{B2})}{V_2} \quad \text{(22a)}
\]

\[
\frac{dC_{B2}}{dt} = \frac{F_3 C_{B1} + F_4 C_{B0} - F_3 C_{B2}}{V_2} - \frac{r_1(T_2, C_{A2}, C_{B2})}{V_2} \quad \text{(22b)}
\]

\[
\frac{dC_{C2}}{dt} = \frac{F_3 C_{C1} - F_3 C_{C2}}{V_2} + \frac{r_1(T_2, C_{A2}, C_{B2})}{V_2} \quad \text{(22c)}
\]

\[
\frac{dC_{D2}}{dt} = \frac{F_3 C_{D1} - F_3 C_{D2}}{V_2} + \frac{r_1(T_2, C_{A2}, C_{B2})}{V_2} \quad \text{(22d)}
\]

**CSTR-3:**

\[
\frac{dC_{A3}}{dt} = \frac{F_3 C_{A2} + F_3 C_{A3}}{V_3} - \frac{r_1(T_3, C_{A3}, C_{B3})}{V_3} \quad \text{(23a)}
\]

\[
\frac{dC_{B3}}{dt} = \frac{F_3 C_{B2} + F_3 C_{B0} - F_7 C_{B3}}{V_3} - \frac{r_1(T_3, C_{A3}, C_{B3})}{V_3} \quad \text{(23b)}
\]

\[
\frac{dC_{C3}}{dt} = \frac{F_3 C_{C2} - F_7 C_{C3}}{V_3} + \frac{r_1(T_3, C_{A3}, C_{B3})}{V_3} \quad \text{(23c)}
\]

\[
\frac{dC_{D3}}{dt} = \frac{F_3 C_{D2} - F_7 C_{D3}}{V_3} + \frac{r_1(T_3, C_{A3}, C_{B3})}{V_3} \quad \text{(23d)}
\]

**Separator:**

\[
\frac{dC_{A4}}{dt} = \frac{F_7 C_{A3} + F_9 C_{A5} - F_7 C_{Ar} - F_6 C_{A4}}{V_4} \quad \text{(24a)}
\]

\[
\frac{dC_{B4}}{dt} = \frac{F_7 C_{B3} + F_3 C_{B5} - F_7 C_{Br} - F_8 C_{B4}}{V_4} \quad \text{(24b)}
\]
\[ \frac{dC_{C4}}{dt} = F_r C_{C3} - F_r C_{C5} - F_r C_{Cr} - F_r C_{C4} \quad (24c) \]
\[ \frac{dC_{D4}}{dt} = F_r C_{D3} - F_r C_{D5} - F_r C_{Dr} - F_r C_{D4} \quad (24d) \]
\[ \frac{dT_4}{dt} = Q_4 + \sum_{i=A}^{D} (F_r C_{i3} H_i(T_3) + F_r C_{i5} H_i(T_5)) \quad (24e) \]

**CSTR-4:**
\[ \frac{dC_{A5}}{dt} = F_n C_{Ar} + F_r C_{A5} - r_3(T_5, C_{A5}, C_{D5}) \quad (25a) \]
\[ \frac{dC_{B5}}{dt} = F_n C_{Br} + F_r C_{B5} - r_2(T_5, C_{B5}, C_{C5}) \quad (25b) \]
\[ \frac{dC_{C5}}{dt} = F_n C_{Cr} + F_r C_{C5} - r_2(T_5, C_{B5}, C_{C5}) \quad (25c) \]
\[ \frac{dC_{D5}}{dt} = F_n C_{Dr} + F_r C_{D5} - F_r C_{D5} \quad (25d) \]
\[ \frac{dT_5}{dt} = Q_5 + F_n C_{D5} \cdot H(T_5) \quad (25e) \]

where \( r_1, r_2 \) and \( r_3 \) are reaction rates defined as
\[ r_1(T, C_A, C_B) = 0.084 \times 10^{-5} \frac{C_A^{0.32} C_B^{1.5}}{s} \quad (26a) \]
\[ r_2(T, C_B, C_C) = 0.085 \times 10^{-5} \frac{C_B^{0.5} C_C^{0.5}}{s} \quad (26b) \]
\[ r_3(T, C_A, C_D) = 66.1 \times 10^{-5} \frac{C_A^{1.018} C_D}{s} \quad (26c) \]

The definition of process variables used in the nonlinear model above is given in Table I. In addition, all other system parameters, such as steady state values of the state variables and manipulated variables, can be found in [24].

In order to improve solver performance, the process model variables were scaled between 0.1 and 1. The control problem for this example is explained in equation (27).

\[ J = \min_{y, \Delta u} \left\{ \frac{1}{2} (Y(k)^T Q Y(k)) + \Delta U(k)^T R \Delta U(k) \right\} + F(x(k + H_p - 1)) \quad (27a) \]

subject to
\[ \hat{x}(k + l + 1 | k) = \Phi \hat{x}(k + l | k) + \Gamma \hat{u}(k + l | k), \quad (27b) \]
\[ \hat{u}(k + l | k) = \sum_{n=0}^{l} \Delta \hat{u}(k + n | k) + u(k - 1), \quad (27c) \]

\[ \Delta \hat{u}(k + l | k) = 0, \quad H_u \leq l \leq H_p - 1, \quad (27d) \]
\[ \hat{y}(k + l + 1 | k) = \hat{x}(k + l + 1 | k), \quad (27e) \]
\[ \Delta u_{\min} \leq \Delta \hat{u}(k + m | k) \leq \Delta u_{\max} \quad (27f) \]

for \( l = 0, \ldots, H_p - 1 \) and \( m = 0, \ldots, H_u - 1 \). The following subsystems are defined based on physical decomposition in the overall process model:

\[ X = (X_1 | X_2)^T \quad (28a) \]
\[ U = (U_1 | U_2)^T \quad (28b) \]
\[ \Delta U = (\Delta U_1 | \Delta U_2)^T \quad (28c) \]

where
\[ X_1 = [C_{A1}, C_{B1}, C_{C1}, C_{D1}, T_1, C_{A2}, C_{B2}, C_{C2}, C_{D2}, T_2] \quad (29a) \]
\[ X_2 = [C_{A3}, C_{B3}, C_{C3}, C_{D3}, T_3, C_{A4}, C_{B4}, C_{C4}, C_{D4}, T_4, C_{A5}, C_{B5}, C_{C5}, C_{D5}, T_5] \quad (29b) \]
\[ U_1 = [Q_1, Q_2, F_4] \quad (29c) \]
\[ U_2 = [Q_3, Q_4, F_5] \quad (29d) \]

The operating range of the manipulated variables around their corresponding steady state values is defined in Table II.

**Table I:** Process parameters of alkylation of benzene [24]

<table>
<thead>
<tr>
<th>Process Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{A1}, C_{B1}, C_{C1}, C_{D1} )</td>
<td>Concentration of species in CSTR-1</td>
</tr>
<tr>
<td>( C_{A2}, C_{B2}, C_{C2}, C_{D2} )</td>
<td>Concentration of species in CSTR-2</td>
</tr>
<tr>
<td>( C_{A3}, C_{B3}, C_{C3}, C_{D3} )</td>
<td>Concentration of species in CSTR-3</td>
</tr>
<tr>
<td>( C_{A4}, C_{B4}, C_{C4}, C_{D4} )</td>
<td>Concentration of species in Separator</td>
</tr>
<tr>
<td>( C_{A5}, C_{B5}, C_{C5}, C_{D5} )</td>
<td>Concentration of species in ( F_r, F_{r1}, F_{r2} )</td>
</tr>
<tr>
<td>( T_1, T_2, T_3, T_4, T_5 )</td>
<td>Temperature in each vessel</td>
</tr>
<tr>
<td>( T_{ref} )</td>
<td>Reference Temperature</td>
</tr>
<tr>
<td>( F_3, F_3, F_4, F_4, F_4 )</td>
<td>Effluent flow rates from each vessel</td>
</tr>
<tr>
<td>( F_3, F_3, F_3, F_3, F_3 )</td>
<td>Feed flow rates to each vessel</td>
</tr>
<tr>
<td>( F_{r1}, F_{r2} )</td>
<td>Recycle flow rates</td>
</tr>
<tr>
<td>( H_{vap}, H_{vap}, H_{vap}, H_{vap}, H_{vap} )</td>
<td>Enthalpies of vaporization</td>
</tr>
<tr>
<td>( H_{vap, 1}, H_{vap, 1}, H_{vap, 1}, H_{vap, 1}, H_{vap, 1} )</td>
<td>Enthalpies of vaporization</td>
</tr>
<tr>
<td>( \Delta H_{H_1}, \Delta H_{H_2}, \Delta H_{H_3} )</td>
<td>Heat of reactions</td>
</tr>
<tr>
<td>( V_1, V_2, V_3, V_4, V_5 )</td>
<td>Volume of each vessel</td>
</tr>
<tr>
<td>( Q_3, Q_3, Q_3, Q_3, Q_3 )</td>
<td>External heat/coolant input to each vessel</td>
</tr>
<tr>
<td>( C_{A1}, C_{B1}, C_{C1}, C_{D1} )</td>
<td>Feed temperature of pure species</td>
</tr>
<tr>
<td>( \rho A_0, \rho B_0, \rho C_0, \rho D_0 )</td>
<td>Feed temperature of pure species</td>
</tr>
<tr>
<td>( T_{H_3}, T_{H_2}, T_{H_1} )</td>
<td>Fraction of overhead flow recycled to the reactors</td>
</tr>
</tbody>
</table>

In addition, the prediction horizon \( H_p \) and the control horizon \( H_u \) were selected 3 and 2, respectively; and the sampling time was set to 30 s. The penalty term \( F(x_{k+H_p-1}) \) in equation (27) can be ignored because the real parts of all eigenvalues of the matrix of overall system \( \Psi \) are within the unit circle throughout the simulations performed, and \( H_p \) was selected long enough in order to have a good control performance. The weighting matrices \( Q \) and \( R \) are selected.
$10^3 \times I_{25}$ and $10^{-1} \times I_7$, respectively. Furthermore, the initial conditions of this system are shown in [24].

The trends of the overall objective functions $J = \sum_{i=1, \ldots, N} J_i$ are depicted in Fig. 3. As can be seen, CDMPC can perfectly track the centralized optimum performance. The result obtained by decentralized MPC shows that the decentralized MPC is not able to achieve the centralized optimum performance.

VI. CONCLUSION

The main scope of this work is to investigate the use of the price-driven coordination method which is applicable to linear or linearized models, for nonlinear systems. In order to verify this objective, a number of simulations were performed using an alkylation process of benzene model as a benchmark, which is successively linearized around its operational points. The results achieved for the centralized and coordinated distributed control techniques completely match; indeed, the price-driven coordination approach applied to the nonlinear case study can ideally track the centralized performance. The achieved results show the coordination method used in this work can also be applied to nonlinear systems. Here we have shown that for some nonlinear systems, successive linearization can provide a reliable and simple algorithm approach based on Marcos [1] to control networked nonlinear systems and to achieve the centralized optimum performance.

REFERENCES


