Wheel Slip Estimation Based on Real-Time Identification of Tire-Road Friction Conditions

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Abstract—This paper is to develop efficient nonlinear observers for wheel slip with real-time friction parameter estimation only using wheel angular velocity. The model-based wheel slip control approach is adopted, which models the friction forces between the tires of the vehicle and the road surface, and an adaptive algorithm is designed for each wheel. It is shown that the adaptive observer is uniformly globally asymptotically stable under the condition of uniform δ-persistence of excitation. In addition, a stronger local stability result is obtained, where uniform local exponential stability is proven. Simulation results are provided to demonstrate the performance of the adaptive observer, even on the low friction road surface.

I. INTRODUCTION

Automotive antilock brake system (ABS) controls the wheel slip of a vehicle to prevent it from locking such that high friction forces are achieved and steerability is maintained. ABS brakes are characterized by robust adaptive behavior with respect to highly uncertain tire characteristics, mainly the wheel slip characteristics, and fast changing road surface conditions, and they have been commercially available in vehicles for more than 20 years [1].

The wheel slip, which is the difference between the vehicle velocity and the rotational wheel velocity, is regarded as one of the most important process parameters affecting the quality of vehicle control. In general, the wheel slip is highly dependent on particular road characteristics, such as whether the road is dry or wet. Real-time tire-road friction coefficient estimation is also extremely valuable for active safety applications, such as anti-lock braking systems (ABS), electronic stability program (ESP), etc. In particular, knowledge about the current road surface conditions is important for the observer to work properly [2], [3].

Many control schemes have been proposed to control wheel slip, such as sliding mode control, fuzzy logic control, adaptive control, etc. Contributions to model-based wheel slip control for ABS can be found in the literature. An adaptive control Lyapunov approach is investigated in [4], [5]. Feedback linearization in combination with gain scheduling is suggested in [6]. The observer-based adaptive methods for obtaining information about road surface conditions have been studied in [7], [8]. In [9], the wheel slip is used to estimate tire-road friction, which appears appealing as it needs only standard ABS sensor equipment. However, such methods are based on velocity measurement.

In this paper, we assume that the vehicle velocity measurement is not available, and develop efficient nonlinear observers for wheel slip with real-time friction parameter estimation. The approach is to use tire-road friction model together with a nonlinear observer to estimate a parameter that reflects changes in the tire-road characteristics. Such a parametrization can describe situations where the surface is different below each wheel. The stability analysis presented in this paper relies crucially on the condition of persistent excitation. It is shown that the adaptive observer is uniformly globally asymptotically stable under the excitation condition and a set of technical assumptions, using results related to nested Matrosov theorem [10].

The paper is organized as follows. In Section II, the wheel slip dynamics and friction model parametrization is introduced. In Section III, the nonlinear wheel slip observer with adaptation to different road conditions, and the stability properties are presented. In Section IV, the robustness is analyzed, where the uniform local exponential stability is proven. The adaptive algorithm is extended to yaw rate motion in Section V, and the simulations results are given in Section VI. Finally, the paper concludes in Section VII.

II. SYSTEM MODELING

A. Vehicle System

As shown in Fig. 1, we consider a vehicle system consisting of 7 degrees of freedom (DOF), which include longitudinal, lateral and yaw motions of the vehicle as well as the rotational dynamics of the four wheels. The dynamic equations can be summarized as follows

\[ M \dot{\psi} + C(\psi)\dot{\psi} = \tau \]  

where the vehicle dynamics are described by the longitudinal velocity \( v_x \), lateral velocity \( v_y \), and yaw rate \( r \), for which

\[ M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_z \end{bmatrix}, \quad C(\psi) = \begin{bmatrix} 0 & -mr & 0 \\ mr & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]  

\( v = [v_x, v_y, r] \) is a vector containing the body generalized velocities, and the vector \( \tau = [F_x, F_y, F_z] \) consists of forces and
moments acting on the vehicle, where $F_r$ can be formulated as
\[
F_r = [-l_d \ l_f] \left[ F_{x1} \ \ F_{y1} \right] + [l_d \ -l_f] \left[ F_{x2} \ \ F_{y2} \right] + [l_d \ -l_f] \left[ F_{x3} \ \ F_{y3} \right] + [l_d \ -l_f] \left[ F_{x4} \ \ F_{y4} \right]
\]
where $l_f$ and $l_r$ and $l_d$ represent the distances from CG to the front and rear axles, and to the wheel side. $m$ and $J_z$ are the mass of vehicle and the inertia about $z$ axis, respectively.

Wheel rotational dynamics are described by
\[
I_{w,j} \ddot{\delta}_j = T_j - r_{ej} \left[ \cos \delta_j \ \sin \delta_j \right] \left[ F_{x,j} \ \ F_{y,j} \right]
\]
where $I_{w,j}$ and $r_{ej}$ represent the moment of inertia and the effective radius of wheel $j$, respectively. $T_j$ and $\delta_j$ are the wheel torque and wheel steering angle. Frictional forces $F_{x,j}$ and $F_{y,j}, j = 1, \ldots, 4,$ defined in the body coordinate system $(x,y)$ can be expressed as
\[
\left[ F_{x,j} \ \ F_{y,j} \right] = F_{x,j} \frac{\mu_{Res}([S_j]|,k_H)}{[S_j]} \left[ \cos \beta_j \ \sin \beta_j \right] \left[ 1 \ 0 \right] \left[ S_{L,j} \ S_{S,j} \right]
\]
where $k_H$ is an attenuation factor used for the presence of tire tread profile. The resultant friction coefficient $\mu_{Res}([S_j]|,k_H)$ is directed in the same direction as the resultant slip $[S_j]$. In general, $k_H$ is the parameter which characterizes the shape of the function $\mu_{Res}([S_j]|,k_H)$; namely, it defines the tire-road friction characteristics. $S_{L,j}$ is the longitudinal slip, defined in the direction of the wheel ground contact point velocity $v_{wj}, j = 1, \ldots, 4,$ and the lateral slip $S_{S,j}$ at right angles to this. The combined wheel slip $S_j$ of wheel $j$ is characterized by
\[
S_j = \frac{S_{L,j}}{S_{S,j}} = \frac{1}{\max(v_{r,j} \cos \alpha_j, ||v_{wj}||)} \left[ v_{r,j} \cos \alpha_j - ||v_{wj}|| \right] v_{r,j} \sin \alpha_j
\]
where the longitudinal slip must be always between -1 and 1, and $\alpha_j$ is the tire side slip angle, which can be expressed by the longitudinal and lateral velocities $v_{r,j}$ and $v_{r,j}$ of the wheel center in the body-fixed coordinate system
\[
\alpha_j = \delta_j - \beta_j, \ \ \beta_j = \arctan(v_{r,j}/v_{y,j})
\]
The normal loads $F_{z,j}$ on the four wheels can be obtained as
\[
F_{z,j} = \frac{1}{2} \left( \frac{l_j mg}{l_f + l_r} - \frac{h \alpha_j}{l_f + l_r} \right), \ \ j = 1, 2
\]
\[
F_{z,j} = \frac{1}{2} \left( \frac{l_j mg}{l_f + l_r} + \frac{h \alpha_j}{l_f + l_r} \right), \ \ j = 3, 4
\]

B. Wheel Slip Dynamics

The horizontal quarter car model is described by
\[
m_{w,j} \ddot{v}_{x,j} = F_{x,j}
\]
\[
m_{w,j} \ddot{v}_{y,j} = F_{y,j}
\]
\[
I_{w,j} \ddot{\beta}_j = T_j - r_{ej} \left[ \cos \delta_j \ \sin \delta_j \right] \left[ F_{x,j} \ \ F_{y,j} \right]
\]
To avoid singularities, we assume that control input no longer applies when velocity reaches a small neighborhood of zero. Differentiating (6), we obtain the longitudinal wheel slip dynamics as follows (only braking)
\[
\dot{S}_{L,j} = -\frac{r_{ej} \cos \alpha_j}{I_{w,j} v_{wj}} \left( \phi_j(S_{L,j}, S_{S,j}, k_H) - T_{ej} \right)
\]
where
\[
f_1(S_{L,j}, S_{S,j}, v_{wj}) = (1 + S_{L,j}) \frac{v_{x,j}}{v_{w,j}} + S_{S,j} \frac{v_{y,j}}{v_{w,j}}
\]
\[
f_2(S_{L,j}, S_{S,j}, v_{wj}) = (1 + S_{L,j}) \frac{v_{y,j}}{v_{w,j}} - S_{S,j} \frac{v_{x,j}}{v_{w,j}}
\]
\[
\phi_j(S_{L,j}, S_{S,j}, k_H, v_{wj}) = \left( r_{ej} \cos \delta_j + \frac{I_{w,j}}{r_{ej} \cos \alpha_j m_{w,j}} \right) F_{x,j}
\]
\[
\left( r_{ej} \sin \delta_j + \frac{I_{w,j}}{r_{ej} \cos \alpha_j m_{w,j}} \right) F_{y,j}
\]
For the sake of simplicity, in the rest of the paper the dependence of $\phi_j$ on $S_{S,j}$ will be neglected. Maybe such simplicity is an imprecise description of the true physical system. However, for the wheel slip estimation in this paper, it is enough to identify the effect of surface changes on the resultant tire forces, without regard to how this manifests itself physically.

C. Friction Model Parametrization

In the following, we denote by $z$ a vector containing all time-varying signals used in the friction model, except the longitudinal wheel slip $S_{L,j}$ and the coefficient $k_H$. The contents of $z$ may be different depending on the friction model used, but we assume that certain measurements to be included in $z$ are available
1) the longitudinal acceleration $a_z$;
2) the lateral acceleration $a_y$;
3) the yaw rate $\gamma$;
4) the wheel angular velocities $\omega_j, j = 1, \ldots, 4$;
5) the wheel steering angles $\delta_j, j = 1, \ldots, 4$. 

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Because estimation of the yaw rate $r$ is not considered until Section V, it is at this point assumed available as a measurement and included in $z$.

In this paper, the friction model is valid for different road surface. A closely related parameter $\theta$ is chosen for adaptation, and we may write

$$\phi_j = \theta \phi_j^*(z, S_{Lj})$$  \hfill (11)

The function $\phi_j^*$ is defined as

$$\phi_j^* = \phi_j^*(z, S_{Lj}, k_H)$$  \hfill (12)

where $k_H$ is some fixed nominal value of $k_H$. Compared with using $k_H$ for adaptation, choosing $\theta$ as described above offers certain benefits, because the parameter enters linearly into (11), and it will be easier to deal with and result in better performance. Moreover, Such a parametrisation can describe situations where the surface is different below each wheel. And the longitudinal acceleration is used to design nonlinear observer with adaptation to different road surface.

III. OBSERVER DESIGN AND STABILITY ANALYSIS

From (10), using small angle approximation, the simplified longitudinal wheel slip dynamics is rewritten as

$$\dot{S}_{Lj} = -\frac{r_{ej}}{F_{vyw}} (\theta \phi_j^*(z, S_{Lj}) - T_{bj})$$  \hfill (13)

In observer design, it is often required that the states to be observed must exist for all future time. Therefore, we include a result on forward completeness for (13), although this may be obvious from physics. We define the vector $x = [S_{Lj}, \theta]^T$ of states to be estimated.

Assumption 1: There exist compact sets $D_z \subset \mathbb{R}^m$, $D_{sl} \subset \mathbb{R}$ such that

a) $(z, S_{Lj}) \in D_z \times D_{sl}$;

b) Assuming that $z, S_{Lj}$ are uniformly continuous in $t$ on $\mathbb{R}$;

c) $\phi_j^*(z, S_{Lj}, k_H)$ and its derivative with respect to $S_{Lj}$ are continuous on $D_z \times D_{sl} \times \{k_H\}$.

Assumption 2: There exist known functions $\xi_j : D_z \times \mathbb{R} \to \mathbb{R}$, $j = 1, \ldots, 4$, such that $\xi_j(z, S_{Lj})$ and $\partial \xi_j / \partial S_{Lj}$ are continuous on $D_z \times \mathbb{R}$ and

$$\phi_j^*(z, S_{Lj}) - \phi_j^*(z, S_{Lj}) = \xi_j(z, S_{Lj})(S_{Lj} - S_{Lj})$$  \hfill (14)

According to the mean value theorem, there always exist values such that (14) holds. In fact, $\xi_j(z, S_{Lj})$ is just some partial derivative function. Using Newton’s second law, we may write $ma_x = \sum_{j=1}^{4} F_{ij}(z, S_{Lj}, k_H)$, using the expression $\phi_j(z, S_{Lj}, k_H) = r_{ej} F_{ij}(z, S_{Lj}, k_H)$, then the error dynamics can be approximated by

$$\ddot{a}_x = a_x - \frac{4}{mr_{ej}} \dot{\theta} \phi_j^*(z, S_{Lj})$$  \hfill (15)

This represents the difference between the actual acceleration and the estimate obtained using the friction model. The notation $\ddot{\phi}_j = \dot{\phi}_j^*(z, S_{Lj}), \ddot{\xi}_j = \dot{\xi}_j(z, S_{Lj})$, $j = 1, \ldots, 4$, and $\ddot{a}_x = \ddot{a}_x(t, \bar{x})$ are used for the sake of brevity.

For the vehicle system (13), the following observer is proposed

$$\dot{S}_{Lj} = -\frac{r_{ej}}{F_{vyw}} (\dot{\theta} \phi_j^*(z, S_{Lj}) - T_{bj})$$

$$\dot{\theta} = \frac{mr_{ej}}{4} \dot{\phi}_j^*(z, S_{Lj})$$  \hfill (16)

where $K_d, K_s$ and $\Gamma$ are positive gains and $\Lambda_4 = \Lambda_4(z, S_{Lj})$, $j = 1, \ldots, 4$ refers to some choice function such that $\Lambda_4$ and $\partial \Lambda_4 / \partial S_{Lj}$ are continuous on $D_z \times \mathbb{R}$. These functions are used to scale the observer equation for numerical simulations, and their exact shape are not important for the stability analysis.

Define the estimation error variables as $\tilde{S}_{Lj} = S_{Lj} - S_{Lj}$, and $\tilde{\theta} = \theta - \hat{\theta}$, and we obtain the observer error dynamics by subtracting (16) from (13)

$$\dot{\tilde{S}}_{Lj} = -\left(\frac{r_{ej}}{F_{vyw}} + K_d \Lambda_4 \tilde{\theta}ight) \frac{mr_{ej}}{4} \dot{\theta}$$

$$\dot{\tilde{\theta}} = -\Gamma \tilde{K}_d \Lambda_4 \theta \frac{mr_{ej}}{4}$$  \hfill (17)

where $\tilde{K}_d$ satisfies the expression $\tilde{K}_d \Lambda_4 \tilde{\theta} = \frac{r_{ej}}{F_{vyw}} + K_d \Lambda_4 \tilde{\theta}$. For the vehicle system (13), the following observer is proposed

One may write

$$\frac{mr_{ej}}{4} \dot{\theta} \phi_j^*(z, S_{Lj}) = \theta (\phi_j^*(z, S_{Lj}) - \phi_j^*(z, \bar{S}_{Lj})) + \dot{\theta} \phi_j^*(z, S_{Lj})$$  \hfill (19)

Using this for substitution yields

$$\dot{V}(t, \bar{x}) = -\Gamma \tilde{K}_d \Lambda_4 \bar{\theta}^2 / 16$$  \hfill (20)

It follows that the origin of (17) is UGS.

Since the uniform stability has been confirmed, the excitation condition is used to establish the uniform global asymptotic stability (UGAS) of the origin of the error dynamics (17).
We define the following functions

\[ \eta_{sl} = \begin{cases} \frac{4 \theta^*}{m_r e_j} \phi^*_j(z, S_{Lj}^s) - \phi^*_j(z, S_{Lj}^\tilde{t}) & S_{Lj}^s \neq 0 \\ \frac{4 \theta^*}{m_r e_j} \phi^*_j(z, S_{Lj}^\tilde{t}) & S_{Lj}^s = 0 \end{cases} \]

for some \( c > 0 \). According to the result of Khalil [11], the origin of (17) is ULES.

Using these functions, we rewrite (19) as

\[ \tilde{a}_x = \eta_{sl} \tilde{S}_{Lj} + \eta_{\bar{\theta}} \tilde{\theta} \quad (21) \]

**Theorem 1:** Suppose that for each \( S_{Lj} \in \mathbb{R} \), there exist \( T > 0 \) and \( \bar{\zeta} > 0 \) such that for all \( t \in \mathbb{R} \),

\[ \int_{t}^{t+T} \eta_{\bar{\theta}}^2(\tau, \tilde{x}) d\tau \leq \bar{\zeta} \quad (22) \]

If Assumption 1-2 hold, then the origin of the error dynamics (17) is uniformly globally asymptotically stable (UGAS).

**Proof:** From Lemma 1, we know that the origin of the error dynamics is UGS. We may use nested Matrosov theorem to the establish the uniform global asymptotic stability. The function \( \tilde{a}_s \) is locally Lipschitz continuous in \( \tilde{x} \), uniformly in \( t \). Hence, for each \( \Delta > 0 \), there exists a constant \( \rho > 0 \) such that for all \( (t, \tilde{x}) \in \mathbb{R} \times B(\Delta) \),

\[ \max \left\{ \| V(\tilde{x}) \|, |a_t| \right\} \leq \rho, \quad \text{where} \quad V(\tilde{x}) = \text{the Lyapunov function in Lemma 1}. \]

We define \( Y : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R} \) as

\[ Y(\tilde{x}, \psi) = -\Gamma \tilde{K}_d \bar{\Delta} \tilde{a}_s^2 / 16 \]

We have that \( \dot{V}(t, \tilde{x}) \leq Y(z, \psi), \quad Y(z, \psi) \leq 0 \), and \( Y(z, \psi) = 0 \Rightarrow \psi = 0 \). We may write

\[ \int_{t}^{t+T} \dot{a}_s(\tau, \tilde{x}) d\tau = \dot{x}^T \bar{M}(t, \tilde{x}) \]

where

\[ \bar{M}(t, \tilde{x}) = \begin{bmatrix} \int_t^{t+T} \eta_{\bar{\theta}}^2(\tau, \tilde{x}) d\tau & \int_t^{t+T} \eta_{\bar{\theta}} \eta_{\bar{\theta}}(\tau, \tilde{x}) d\tau \\ \int_t^{t+T} \eta_{\bar{\theta}} \eta_{\bar{\theta}}(\tau, \tilde{x}) d\tau & \int_t^{t+T} \eta_{\bar{\theta}}^2(\tau, \tilde{x}) d\tau \end{bmatrix} \]

According to inequality (22), for each \( \tilde{x} \), \( M(t, \tilde{x}) \) is positive definite, then it follows that

\[ \int_t^{t+T} \dot{a}_s(\tau, \tilde{x}) d\tau \geq \bar{\zeta} \| \tilde{x} \| \quad (24) \]

for all \( t \in \mathbb{R} \). This means that \( \dot{a}_s \) is uniformly \( \delta \)-persistently exciting (U\( \delta \)-PE) with respect to \( \tilde{x} \), and it is also zero for \( \tilde{x} = 0 \). Define

\[ F_2(t, \tilde{x}) = \begin{bmatrix} -\tilde{K}_d \bar{L}_j \bar{\Delta} \tilde{a}_s / 4 \\ -\Gamma \tilde{K}_d \bar{L}_j \bar{\Delta} \tilde{a}_s / 4 \end{bmatrix} \]

then \( F_2(t, \tilde{x}) \) is bounded for all \( (t, \tilde{x}) \in \mathbb{R} \times B(\Delta) \). Hence, there exists a constant \( K_1(\Delta) \) such that for all \( (t, \tilde{x}) \in \mathbb{R} \times B(\Delta) \), \( F_2(t, \tilde{x}) \leq K_1(\Delta) |a_t| \).

**Remark 1:** Because \( \eta_{\bar{\theta}} \) and \( \eta_{\bar{\theta}} \) are measures of the influence of \( S_{Lj} \) and \( \bar{\theta} \) on \( \tilde{a}_s \). The condition (22) concerns the relationship of the signals \( \eta_{\bar{\theta}} \) and \( \eta_{\bar{\theta}} \). According to Cauchy-Schwarz inequality, the equalities hold if and only if any two continuous signals are linearly independent on \([t, t+T]\). Loosely speaking, the condition can be fulfilled by guaranteeing that the signals vary in a sufficiently independent manner within each time window. In practical, this amounts to requiring that the driving pattern is somewhat varied, not only long time period of steering, braking or acceleration.

**IV. ROBUSTNESS**

Based on theorem 1, the equilibrium points of the error dynamics is UGAS. Further, a stronger local stability result can be obtained, i.e., uniform local exponential stability (ULES).

**Theorem 2:** Suppose that Assumption 1-2 and the condition expressed by (22) hold, then the origin of the error dynamics (17) is uniformly globally exponentially stable (ULES).

**Proof:** We define the Lyapunov function

\[ V_{LE}(t, \tilde{x}) = V(\tilde{x}) - \gamma_1 \int_t^\infty e^{-\tau} \tilde{a}_s^2(\tau, \tilde{x}) d\tau \quad (26) \]

where \( \gamma \) is a constant and \( V(\tilde{x}) \) is the Lyapunov function from proof of Lemma 1. Because \( \tilde{a}_s \) is locally Lipschitz continuous in \( \tilde{x} \), uniformly in \( t \), and zero for \( \tilde{x} = 0 \), there exist constants \( K_2(\Delta) \) such that \(|\tilde{a}_s| \leq K_2(\Delta) \| \tilde{x} \| \). Choosing \( \gamma \) such that \( \gamma K_2^2(\Delta) < \min(\theta \Gamma, 1)/2m_r e_j \), we have

\[ \left( \frac{1}{2m_r e_j} \min(\theta \Gamma, 1) - \gamma K_2^2(\Delta) \right) \| \tilde{x} \|^2 \leq V_{LE}(t, \tilde{x}) \leq \frac{1}{2m_r^2} \min(\theta \Gamma, 1) \| \tilde{x} \|^2 \]

This means that \( \tilde{a}_s \) is bounded for all \( (t, \tilde{x}) \in \mathbb{R} \times B(\Delta) \). Hence, there exists a constant \( K_1(\Delta) \) such that for all \( (t, \tilde{x}) \in \mathbb{R} \times B(\Delta) \), \( F_2(t, \tilde{x}) \leq K_1(\Delta) |a_t| \).

\[ \int_t^{t+T} \tilde{a}_s(\tau, \tilde{x}) d\tau = \tilde{x}^T \bar{M}(t, \tilde{x}) \tilde{x} \geq K_1(\Delta) \| \tilde{x} \|^2 \quad (27) \]

From this, it follows that

\[ V_{LE}(t, \tilde{x}) \leq \frac{\Gamma \tilde{K}_d \bar{L}_j}{2m_r^2} \| \tilde{x} \|^2 \quad (28) \]

for some \( c > 0 \). According to the result of Khalil [11], the origin of (17) is ULES.
V. EXTENSION TO YAW MOTION

In the following, the yaw rate is included in the adaptive observer design. Similar as in Section III, we use the same type of parametrization for calculation of the yaw moment. Therefore, we may write $F_i(z, S_{Lj}, r, k_H) = \theta F_i^*(z, S_{Lj}, r)$.

**Assumption 3:** There exist compact sets $D_x \subset \mathbb{R}^m$, $D_y \subset \mathbb{R}$, and $D_z \subset \mathbb{R}$ such that

a) $(z, S_{Lj}, r) \in D_z \times D_y \times D_z$;

b) Assuming that $z, S_{Lj}$ and $r$ are uniformly continuous in $t$ on $\mathbb{R}$;

c) $\phi_j^*(z, S_{Lj}, r, k_H^j)$ and its derivatives with respect to $S_{Lj}$ and $r$ are continuous on $D_x \times D_y \times D_z \times \{k_H^j\}$.

**Assumption 4:** There exist known functions $\xi_j : D_z \times \mathbb{R}^2 \rightarrow \mathbb{R}$, $\zeta_j : D_z \times \mathbb{R}^2 \rightarrow \mathbb{R}$, such that

$\dot{\zeta}_j(z, \dot{S}_{Lj}, r) = \zeta_j(z, S_{Lj}, r)(S_{Lj} - \dot{S}_{Lj})$

(30)

For the sake of brevity, denote $F_i^* = F_i^*(z, S_{Lj}, r)$, $\xi_j = \xi_j(z, S_{Lj}, r)$, and $\zeta_j = \zeta_j(z, S_{Lj}, r)$, $\bar{a}_k = \bar{a}_k(t, \dot{x})$, where the vector $x = [S_{Lj}, \theta, r]$, and

$\bar{a}_k(t, \dot{x}) = a_k - \frac{4}{m r_{ej}} \hat{\theta} \phi_j^*(z, S_{Lj}, \dot{r})$

(31)

The observer is proposed as follows

$\dot{S}_{Lj} = -\frac{r_{ej}}{I_{w_{j}v_{w_{j}}}}(\hat{\theta} \phi_j^* - T_{bj}) + \Gamma_1^2 \bar{r}(r - \bar{r})$

$+ K_{sl} \bar{S}_{Lj} \left( \frac{m r_{ej} \bar{a}_k}{4} - \hat{\theta} \phi_j^* \right)$

$\dot{r} = \frac{1}{J_z} F_i^*(z, \dot{S}_{Lj}, \dot{r}) + \Gamma_1^2 \bar{r}(r - \bar{r})$

$\dot{\bar{r}} = \Gamma_1 K_{sl} \bar{S}_{Lj} \phi_j^* \left( \frac{m r_{ej} \bar{a}_k}{4} - \hat{\theta} \phi_j^* \right) + \Gamma_2 F_i^*(r - \bar{r})$

(32)

where $K_r$ and $\Gamma_2$ are positive gains.

**Theorem 3:** Suppose Assumption 3-4 and the condition (22) hold, then the origin of the error dynamics corresponding to the observer (32) is uniformly globally asymptotically stable (UGAS).

**Proof:** Select the Lyapunov function

$V(x) = \frac{1}{2} \left( \theta \Gamma_1 S_{Lj}^2 + \Gamma_2 J_z \bar{r}^2 + \bar{r}^2 \right)$

(33)

The derivative along the trajectories of (32) is

$\dot{V}(t, \bar{x}) = -\theta \Gamma_1 \bar{S}_{Lj} \left( \frac{r_{ej}}{I_{w_{j}v_{w_{j}}} + K_{sl} \phi_j^*} \right) \frac{m r_{ej} \bar{a}_k}{4}$

$+ \hat{\theta} \left( -\Gamma_1 K_{sl} \phi_j^* \frac{m r_{ej} \bar{a}_k}{4} - \Gamma_2 F_i^*(r - \bar{r}) \right)$

$+ \Gamma_2 J_z \bar{r} \left( \frac{1}{J_z} \theta \zeta_j \bar{S}_{Lj} + \hat{\theta} \phi_j^* \right) - K_r \bar{r}$

$- \theta \Gamma_1 \bar{S}_{Lj} \frac{\Gamma_2 \zeta_j}{\Gamma_1} (r - \bar{r})$

$\leq -m^2 r_{ej}^2 \Gamma_1 K_{sl} \phi_j^* \frac{\bar{a}_k^2}{16} - \Gamma_2 J_z K_r \bar{r}^2$

We define $Y : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$ as

$Y(z, \psi) = -m^2 r_{ej}^2 \Gamma_1 K_{sl} \phi_j^* \frac{\bar{a}_k^2}{16} - \Gamma_2 J_z K_r \bar{r}^2$

(34)

It follows that $V(t, \bar{x}) \leq Y(z, \psi), Y(z, \psi) \leq 0$ and $Y(z, \psi) = 0 \Rightarrow \psi = 0, \zeta_3 = 0$. Moreover, the functions $\bar{a}_k$ is U̅δ-PE with the rest of the states. We can take the same steps as the proof of Theorem 1, then the result follows invoking the nested Matrosov theorem.

**Theorem 4:** Suppose that Assumption 3-4 and the condition expressed by (22) hold, then the origin of the error dynamics corresponding to the observer (32) is ULES.

VI. SIMULATIONS

To examine the effectiveness of the control scheme, simulation tests are carried out as follows. The initial values of the friction parameters are chosen as $\theta_1(0) = 0, \theta_2(0) = 0.5, \theta_3(0) = 0.5$, and $\theta_4(0) = 0$. The observer is implemented using the gains $K_{sl} = 10^2/\omega_{w_{j}}, K_r = 20, \Gamma_1 = 4$, and $\Gamma_2 = 1/\omega_{w_{j}}$. In the simulations, some partial derivative functions are approximated as follows: $j = 1, \ldots, 4, \xi_j = \partial \phi_j^* / \partial S_{Lj}$, and the scaling $\Lambda_j = 1/\|\xi_j, \phi_j^*\|$. To be more realistic, white noise is considered because slow drift and bias in the accelerometer may have an adverse effect on performance of the observer.

In the first simulation, the vehicle is driven on asphalt. The nominal maximal tire-road friction coefficient that describes the asphalt surface is given by $k_H^z = 28$. Steering wheel angle and wheel torque is shown in Fig. 2, which are sinusoidal with different magnitudes. Fig. 3 depicts the longitudinal wheel slip of both the estimated and actual under the proposed observer in four wheels. It can be seen that the observed value approaches the actual value in a slightly swinging manner and eventually overlaps the actual one. And the noise has no obvious influence on the longitudinal wheel slip estimation.

In the second simulation, the vehicle is driven on ice with the same input signals as in the first simulation. The chosen value $k_H^z = 18$ gives the best average performance on this data set. From Fig. 5, it can be seen that the estimated longitudinal wheel slip converge to the actual value despite on low friction.
road. Although there exists some errors due to the effect of measurement errors, the performance of the observer is also acceptable.

Fig. 4 and Fig. 6 plot the results of the friction parameter estimation on asphalt and ice, respectively. The good estimation is due to the satisfaction of the U6-PE condition. This is because the persistent exciting signal is applied to the steering wheel angle and wheel torque, which activate the adaptive friction parameter estimation. And in the end the friction parameter performs asymptotically convergent to the actual values.

VII. CONCLUSIONS

A nonlinear observer is presented for estimation of the longitudinal wheel slip for different road surface without the vehicle velocity measurement. The tire-road friction model is parametrized with single friction parameter, and an adaptive algorithm is proposed for each wheel. The persistent excitation condition is essential for the stability results in this paper. Simulation results confirm the effectiveness of the performance the adaptive observer, even on the low friction road surface.

REFERENCES


