Abstract— This paper is concerned with the problem of finite-time $H_\infty$ control for a class of discrete-time Markovian jump nonlinear systems with time delays represented by Takagi-Sugeno (T-S) model. First, by using fuzzy stochastic Lyapunov-Krasovskii functional approach, sufficient conditions are derived such that the resulting close-loop system is stochastic finite-time bounded and satisfies a prescribed $H_\infty$ disturbance attenuation level in a given finite-time interval. Second, sufficient criteria on stochastic finite-time $H_\infty$ stabilization via fuzzy state feedback are provided, and the fuzzy state feedback controller is designed by solving an optimization problem in terms of linear matrix inequalities. Finally, a numerical example is given to show the validity of the proposed designed techniques.

I. INTRODUCTION

In the past few decades, the fuzzy logic control has been utilized as an alternative approach to conventional control for complex nonlinear systems. As one of most important form of fuzzy systems, Takagi-Sugeno (T-S) fuzzy model [1] was introduced and has been recognized as a popular and powerful tool in approximating and described complex nonlinear systems. The main reason of this attention for T-S fuzzy model is due to the fact that it can combine the merits of both fuzzy logic theory and linear systems, and the stability analysis and controller design of the overall fuzzy systems can be carried out in the Lyapunov function framework. Therefore, many problems have been tackled and some appealing results for T-S fuzzy systems have been reported in the literature. For example, the stability and stabilization for T-S fuzzy systems have been studied in [2]. The output feedback control has been discussed in [3]. The robust and $H_\infty$ control has been considered in [4, 5]. The $H_\infty$ filtering problem has been addressed in [6, 7]. Some other important results like the guaranteed cost control, variable structure control and reliable control have also been provided for the class of T-S fuzzy systems in [8-10]. More results on fuzzy systems could be found in [11-14] and the references therein.

On the other hand, considerable attention has been paid to Markovian jump systems (MJSs) in the control community since the class of stochastic systems have been extensively applied to modeling various practical processes that can experience abrupt changes in their structure and parameters, possibly caused by the phenomena such as component failures, sudden environmental disturbances, and changing subsystem interconnections, and so on. Therefore, many attracting results and a large variety of control problems have been reported in the literature. For example, the authors considered the stability analysis and state feedback stabilization problems for MJSs in [15, 16]. The references [17, 18] studied the robust and $H_\infty$ control problem, and the $H_\infty$ filtering problem has been investigated in [19-22]. The sliding mode control and passivity analysis for the class of stochastic systems have also been presented in [23-25]. More detailed results on the topic could be found in [26] and the references therein. Recently, fuzzy MJSs, as a special form of MJSs, have also received many researches. Some results on fuzzy MJSs have been investigated and studied, such as the stability analysis and state feedback stabilization problem [27, 28], the output feedback stabilization [29] and the $H_\infty$ control [30].

In many practical applications, however, many concerned problems are that described system state does not exceed a certain bound in some finite time interval. Compared with classical Lyapunov asymptotical stability, finite-time stability or short-time stability was investigated to deal with the transient performances of system trajectories in a finite-time interval. Finite-time stability or short-time stability was first introduced in the 1960s [31, 32], and then the definition of finite-time stability was extended to finite-time boundedness in [33]. Further, using the linear matrix inequality (LMI) technique and Lyapunov approach, many results on finite-time stability and stabilization have been investigated for linear systems, nonlinear systems, stochastic systems, switching systems, fuzzy systems and singular systems. For instance, the authors [34] studied the state feedback finite-time stabilization for discrete-time linear systems. The problem of finite-time stability and stabilization was tackled for nonlinear stochastic hybrid systems in [35]. The results on robust finite-time stabilization were provided for uncertain continuous-time fuzzy MJSs in [36]. For more details of the literature related to finite-time stability, finite-time stabilization and finite-time $H_\infty$ control, the reader is referred to [37-40].

In this paper, we deal with the problem of finite-time $H_\infty$ control for discrete-time Markovian jump T-S fuzzy systems.
with time delays. The concepts of stochastic finite-time boundedness and stochastic $H_\infty$ finite-time stabilization of stochastic systems are first given. Then, sufficient conditions of stochastic finite-time boundedness or stochastic $H_\infty$ finite-time stabilization via fuzzy state feedback are obtained for the family of fuzzy stochastic systems. The main aim of this paper is to design a finite-time $H_\infty$ controller which can ensure stochastic finite-time boundedness of the time-delay fuzzy MJS and a prescribed $H_\infty$ performance level can be achieved in the given finite-time interval. Sufficient criteria on stochastic finite-time boundedness or stochastic $H_\infty$ finite-time stabilization can be tackled by a feasibility problem in terms of LMIs with a fixed parameter. Finally, a numerical example is provided to illustrate the validity of the proposed methods.

The rest of this paper is structured as follows. Section 2 is devoted to the problem statement and preliminaries. The main results for finite-time $H_\infty$ control analysis and synthesis are in Section 3 provided. Section 4 presented some conclusions are drawn on stochastic finite-time boundedness or stochastic $H_\infty$ finite-time stabilization.

II. PROBLEM STATEMENT AND PRELIMINARIES

The following notations will be used in this paper. $\mathbf{R}^n$, $\mathbf{R}^{n \times m}$ and $\mathbf{Z}_{k \geq 0}$ is used to denote the sets of $n$ component real vectors, $n \times m$ real matrices, and the set of nonnegative integers, respectively. $\sigma_{\min}(M)$ and $\sigma_{\max}(M)$ denote the smallest and the largest eigenvalue of matrix $P$, respectively. $M^T$ and $M^{-1}$ stand for the matrix transpose and matrix inverse, respectively. The symbol $*$ is used to denote a matrix which can be inferred by symmetry and diag{$\cdots$} stands for a block-diagonal matrix. $(\Omega, \mathcal{F}, \mathcal{P})$ is probability space, $\Omega$ is the sample space, $\mathcal{F}$ is the $\sigma$-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation with some probability measure $\mathcal{P}$.

Consider the following discrete-time Markovian jump system (DMJS) with time delay which could be represented by a T-S fuzzy model over the probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

**Plant Rules**: IF $\theta_1$ is $\mu_{i1}$, $\theta_2$ is $\mu_{i2}$, $\cdots$, $\theta_g$ is $\mu_{ig}$, THEN

$$
x(k+1) = A_i(r_k)x(k) + A_{di}(r_k)x(k-d)$$
$$+ B_i(r_k)u(k) + G_i(r_k)w(k),$$
$$z(k) = C_i(r_k)x(k) + C_{di}(r_k)x(k-d)$$
$$+ D_{i1}(r_k)u(k) + D_{i2}(r_k)w(k),$$
$$x(k) = \phi(k), k \in \{-d, \cdots, 0\},$$

where $x(k) \in \mathbf{R}^{n_1}$, $u(k) \in \mathbf{R}^{n_2}$ and $z(k) \in \mathbf{R}^{p_1}$ are the system state, the control input, and the control output, $d$ is a positive integer denoting the constant delay time of the state in the system, $\phi(k), k \in \{-d, \cdots, 0\}$ is a vector-valued initial discrete sequence. The stochastic jump process $\{r_k, k \geq 0\}$ is a discrete-time, discrete-state Markov chain taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, s\}$ with transition probabilities $\pi_{lm}$, $\pi_{lm} > 0$ and $\sum_{m=1}^{s} \pi_{lm} = 1$ for all $l \in \mathcal{S}$. $A_i(r_k), A_{di}(r_k), B_i(r_k), G_i(r_k), C_i(r_k), D_{i2}(r_k)$, $C_{di}(r_k)$, $D_{i1}(r_k)$ and $D_{i2}(r_k)$ are known constant matrices of appropriate dimensions. $\theta_j$ and $\mu_{ij}$ ($i = 1, \cdots, f, j = 1, \cdots, g$) are respectively the premise variables and the fuzzy sets, $f$ is the number of IF-THEN rules. The fuzzy basis functions are given by

$$h_i(\theta(k)) = \prod_{j=1}^{f} \mu_{ij}(\theta_j(k)), \quad i = 1, \cdots, f,$$

in which $\mu_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in $\mu_{ij}$. It is obvious that $\sum_{i=1}^{f} h_i(\theta(k)) = 1$ with $h_i(\theta(k)) > 0$. Moreover, the noise signal $w(k) \in \mathbf{R}_p^{2}$ satisfies

$$\sum_{k=0}^{\infty} w^T(k)w(k) \leq \omega^2, \quad \omega \geq 0. \quad (3)$$

To simplify the notation, in the sequel, for each possible $r_k = l, l \in \mathcal{S}$, matrix $M_i(r_k)$ will be denoted by $M_{i,l}$; for instance, $A_i(r_k)$ will be denoted by $A_{i,l}$, $A_{di}(r_k)$ by $A_{di,l}$, $A_{dl}(r_k)$ by $A_{di,l}$, and so on. In addition, $h_i(\theta(k))$ denotes $h_i(\theta_1(k))$, the set $\Lambda$ denotes $\{1, \cdots, f\}$, and $\tilde{P}_i$ denotes $\sum_{m=1}^{s} \pi_{lm} P_{m}$. By using the fuzzy blending method, the overall fuzzy DMJS could be inferred as follows:

$$x(k+1) = \sum_{i=1}^{f} h_i(\theta(k))[A_{i,l}x(k) + A_{di,l}x(k-d) + B_{i,l}u(k) + G_{i,l}w(k)],$$
$$z(k) = \sum_{i=1}^{f} h_i(\theta(k))[C_{i,l}x(k) + C_{di,l}x(k-d) + D_{i1,l}u(k) + D_{i2,l}w(k)],$$
$$x(k) = \phi(k), k \in \{-d, \cdots, 0\}. \quad (4)$$

The design of controllers in this paper is performed through the parallel distributed compensation and the overall controller is thus inferred as

$$u(k) = \sum_{i=1}^{f} h_i(\theta(k))K_{i,l}x(k), \quad (5)$$

where $K_{i,l}$ is state feedback gain to be designed. Then, the resulting closed-loop fuzzy DMJS can be written in the following form:

$$x(k+1) = A_i(h)x(k) + A_{di}(h)x(k-d) + G_i(h)w(k),$$
$$z(k) = C_i(h)x(k) + A_{di}(h)x(k-d) + D_i(h)w(k), \quad (6)$$

where

$$A_i(h) = \sum_{i=1}^{f} \sum_{j=1}^{s} h_i(\theta(k))J_{i,j,l}[A_{i,l} + B_{i,l}K_{j,l}],$$
$$A_{di}(h) = \sum_{i=1}^{f} \sum_{j=1}^{s} h_i(\theta(k))J_{i,j,l}[A_{di,l}],$$
$$C_i(h) = \sum_{i=1}^{f} \sum_{j=1}^{s} h_i(\theta(k))J_{i,j,l}[C_{i,l} + D_{i1,l}K_{j,l}],$$
$$C_{di}(h) = \sum_{i=1}^{f} \sum_{j=1}^{s} h_i(\theta(k))J_{i,j,l}D_{i2,l}. \quad (4893)$$
Definition 2.1 (stochastic finite-time stable (SFTS)). The time-delay fuzzy DMJS (6) with \( w(k) = 0 \) and \( u(k) = 0 \) is said to be SFTS with respect to \((\delta_x, \epsilon, R_l, N)\), where \( 0 < \delta_x < \epsilon, R_l > 0 \) and \( N \in \mathbb{Z}_{k \geq 0} \), if

\[
E\{x^T(k_1)R_l x(k_1)\} \leq \delta_x^2 \Rightarrow E\{x^T(k_2)R_l x(k_2)\} < \epsilon^2,
\]

\( k_1 \in \{-d, \cdots 0\}, \) \( k_2 \in \{1, 2, \cdots N\} \).

(7)

Definition 2.2 (stochastic finite-time bounded (SFTB)). The time-delay fuzzy DMJS (6) is said to be SFTB with respect to \((\delta_x, \epsilon, R_l, N, \gamma, \varpi)\), where \( 0 < \delta_x < \epsilon, R_l > 0 \) and \( N \in \mathbb{Z}_{k \geq 0} \), if the constraint relation (7) holds.

Definition 2.3 (stochastic \( H_{\infty} \) finite-time stabilizable). The time-delay fuzzy DMJS (6) is said to be stochastic \( H_{\infty} \) finite-time stabilizable with respect to \((\delta_x, \epsilon, R_l, N, \gamma, \varpi)\), where \( 0 < \delta_x < \epsilon, R_l > 0, \gamma > 0 \) and \( N \in \mathbb{Z}_{k \geq 0} \), if the time-delay fuzzy DMJS (6) is SFTB with respect to \((\delta_x, \epsilon, R_l, N, \gamma, \varpi)\) and under the zero-initial condition the output \( z(k) \) satisfies

\[
E\{\sum_{k=0}^{N} z^T(k)z(k)\} < \gamma^2 E\{\sum_{k=0}^{N} w^T(k)w(k)\}
\]

(8)

for any nonzero \( w(k) \) which satisfies (3), where \( \gamma \) is a prescribed positive scalar. Moreover, the control law (5) is called as finite-time \( H_{\infty} \) controller of the time-delay fuzzy DMJS (1).

The main aim of this paper is to design a fuzzy state feedback controller of the form (5) which can ensure stochastic \( H_{\infty} \) finite-time stabilization of the time-delay fuzzy DMJS (6).

III. MAIN RESULTS

In this section, we consider stochastic \( H_{\infty} \) finite-time stabilization of the time-delay fuzzy DMJS described by (1). LMI conditions will be established to show that the fuzzy DMJS (6) is SFTB and the output \( z(k) \) and disturbance \( w(k) \) satisfies the constraint relation (8).

Theorem 3.1. The time-delay fuzzy DMJS (6) is SFTB with respect to \((\delta_x, \epsilon, R_l, N, \varpi)\), if there exist scalars \( \lambda \geq 1 \) and \( \epsilon > 0 \), a symmetric positive-definite matrix \( Q \), sets of symmetric positive-definite matrices \( \{P_l, l \in \mathcal{S}\} \) and \( \{Q_l, l \in \mathcal{S}\} \), for all \( l \in \mathcal{S} \) and \( i, j \in \Lambda \), such that

\[
\Pi(h) \triangleq \begin{bmatrix}
\Pi_{11}(h) & * & * \\
\Pi_{21}(h) & \Pi_{22}(h) & * \\
\Pi_{31}(h) & \Pi_{32}(h) & \Pi_{33}(h)
\end{bmatrix} < 0,
\]

(9a)

\[
|\bar{\sigma} P + \sigma Q d| \delta_z^2 + \sigma Q \varpi^2 < 2 \sigma \lambda^{-N} \epsilon^2,
\]

(9b)

where

\[
\Pi_{11}(h) = A_l(h)P_lA_l(h) - \lambda P_l + Q,
\]

\( \Pi_{21}(h) = A_l(h)P_lA_l(h) \), \( \Pi_{22}(h) = A_l(h)P_lA_l(h) - Q \), \( \Pi_{31}(h) = G_l(h)P_lA_l(h), \) \( \Pi_{32}(h) = G_l(h)P_lA_l(h), \), \( \Pi_{33}(h) = G_l(h)P_lG_l(h) - Q_l, \)

\[
\bar{\sigma} P = \max_{l \in \mathcal{S}} \{\sigma_{\max}(\bar{P}_l)\}, \sigma P = \min_{l \in \mathcal{S}} \{\sigma_{\min}(\bar{P}_l)\}, \\
\sigma Q = \max_{l \in \mathcal{S}} \{\sigma_{\max}(Q_l)\}, \sigma Q = \min_{l \in \mathcal{S}} \{\sigma_{\min}(Q_l)\}, \\
\bar{P}_l = R_l^{-1/2} P_l R_l^{-1/2}, \bar{Q}_l = R_l^{-1/2} Q_l R_l^{-1/2}.
\]

Proof. We first choose the following fuzzy Lyapunov-Krasovskii functional:

\[
V(k) = x^T(k)P_l x(k) + \sum_{n=k-d}^{k-1} x^T(n)Q x(n).
\]

(10)

Then, simple computation yields

\[
E\{V(k+1)\} - V(k) = E\{ \sum_{n=1}^{\infty} \gamma \cdot \max_{l \in \mathcal{S}} \{\sigma_{\max}(\bar{P}_l)\} \cdot \max_{l \in \mathcal{S}} \{\sigma_{\max}(Q_l)\} \cdot |\bar{\sigma} P + \sigma Q d| \delta_z^2 + \sigma Q \varpi^2 \} \leq \lambda \cdot E\{V(k)\} + \lambda^k \sigma Q \varpi^2.
\]

(11)

(12)

(13)

(14)

(15)

(16)

(17)

Therefore, we have

\[
E\{V(k)\} < \lambda E\{V(0)\} + \lambda^k \sigma Q \varpi^2.
\]

(13)

(14)

(15)

(16)

(17)

Therefore, it follows from (9a) that \( E\{x^T(k)R_l x(k)\} < \epsilon^2 \) for all \( k \in \{1, 2, \cdots, N\} \). The proof of this theorem is completed.

Theorem 3.2. The time-delay fuzzy DMJS (6) is stochastic \( H_{\infty} \) finite-time stabilizable with respect to \((\delta_x, \epsilon, R_l, N, \gamma, \varpi)\), if there exist scalars \( \lambda \geq 1, \epsilon > 0, \gamma > 0 \), a symmetric positive-definite matrix \( Q \), a set of symmetric...
Denote $Q_l = \lambda^{-N}\gamma^2 I$ and notice the form of $P_l, L_{ii}(h), L_{2i}(h), L_{3i}(h)$ and $P$, it is obvious that $(21)$ is equivalent to condition $(9a)$. Thus, conditions $(19)$ and $(18b)$ can guarantee that the time-delay DMJS $(6)$ is SFTB with respect to $(\delta_x, \epsilon, R_l, N, \omega)$ according to Theorem $3.1$. On the other hand, consider the change of fuzzy Lyapunov-Krasovskii functional as Theorem $3.1$. Taking into account that condition $(19)$ and $Q_l = \lambda^{-N}\gamma^2 I$, one can derive from $(19)$ that the following inequality

$$E\{V(k+1)\} < \lambda V(k) - z^T(k)z(k) + \gamma^2 \lambda^{-N} w^T(k)w(k)$$

holds for all $l \in S$. According to $(20)$, we can derive that condition $(8)$ holds. This completes the proof of the theorem.

In order to solve Theorem $3.2$, the following theorem gives LMI conditions to ensure stochastic $H_{\infty}$ finite-time stabilization via fuzzy state feedback of the fuzzy DMJS $(1)$.

**Theorem 3.3.** Consider the time-delay fuzzy DMJS $(6)$, there exists a state feedback controller $K_{i,l} = Y_{i,l}X_{i,l}^{-1}$ such that the time-delay fuzzy DMJS $(6)$ is $H_{\infty}$ finite-time stabilizable with respect to $(\delta_x, \epsilon, R_l, N, \omega)$, if there exist scalars $\lambda \geq 1, \sigma > 0, \epsilon > 0, \gamma > 0, \eta_1 > 0$ and $\eta_2 > 0$, a symmetric positive-definite matrix $M$, a set of mode-dependent symmetric positive-definite matrices $\{X_l, l \in S\}$, a set of matrices $\{Y_{i,l}, i \in \Lambda, l \in S\}$, for all $l \in S$ and $i, j \in \Lambda$, such that

$$\Xi_{i,l} < 0, \quad \Xi_{ij,l} + \Xi_{ji,l} < 0, \quad i < j,$$

$$\sigma R_l^{-1} < X_l < R_l^{-1}, \quad \eta_1 R_l^{-1} < M < \eta_2 R_l^{-1},$$

$$(\omega^2\gamma^2 - \epsilon^2)\lambda^{-N} \sigma^2 \lambda - \delta_x \sigma^2 \lambda - \delta_x \eta_1 < 0,$$

where $\Xi_{ij,l} = \begin{bmatrix} \Xi_{i,j,l} & 0 \\ 0 & -\Xi_{i,j,l} \end{bmatrix}$ with

$$\Xi_{i,l} = \begin{bmatrix} -\lambda X_l & 0 \\ 0 & -\lambda^{-N} \gamma^2 I \end{bmatrix}, \quad \Xi_{ij,l} = \begin{bmatrix} -I & 0 \\ 0 & -I \end{bmatrix}, \quad \Xi_{ji,l} = \begin{bmatrix} -I & 0 \\ 0 & -I \end{bmatrix},$$

$$L_{ij,l} = \begin{bmatrix} X_{i,j,l} & 0 \\ 0 & -X_{i,j,l} \end{bmatrix}, \quad L_{ji,l} = \begin{bmatrix} X_{j,i,l}^{-1} & 0 \\ 0 & -X_{j,i,l}^{-1} \end{bmatrix},$$

$$\Xi_{2i,l} = \begin{bmatrix} -\lambda^{-N} \gamma^2 I & 0 \\ 0 & -\lambda^{-N} \gamma^2 I \end{bmatrix}.$$

Proof. The proof is omitted due to the space limitation.

**Corollary 3.1.** Consider the time-delay fuzzy DMJS $(6)$, there exists a state feedback control gain $K_{i,l} = Y_{i,l}X_{i,l}^{-1}$ such that the time-delay nominal fuzzy DMJS $(6)$ is SFTB with respect to $(\delta_x, \epsilon, R_l, N, \omega)$ if there exist scalars $\lambda \geq 1, \sigma > 0, \epsilon > 0, \delta_1 > 0, \delta_2 > 0, \eta_1 > 0$ and $\eta_2 > 0$, a symmetric positive-definite matrix $M$, sets of mode-dependent symmetric positive-definite matrices $\{X_l, l \in S\}$ and $\{Q_l, l \in S\}$, a set of matrices $\{Y_{i,l}, i \in \Lambda, l \in S\}$, for all $l \in S$ and $i, j \in \Lambda$, such that $(21c), (21d)$ and the following conditions hold:

$$\Xi_{i,l} < 0, \quad \Xi_{ij,l} + \Xi_{ji,l} < 0, \quad i < j,$$

$$\delta_1 I < Q_l < \delta_2 I,$$

$$(\omega^2\gamma^2 - \epsilon^2)\lambda^{-N} \sigma^2 \lambda - \delta_x \sigma^2 \lambda - \delta_x \eta_1 < 0,$$

where $\Xi_{ij,l} = \begin{bmatrix} -\lambda X_l & 0 \\ 0 & -Q_l \end{bmatrix}.$

**Remark 3.1.** It is significant to observe that conditions $(21a), (21b), (21e), (22a), (22b)$ and $(22d)$ are not strict LMI, however, once we fix the parameter $\lambda$, the conditions can be turned into LMI based feasibility problem. Thus, one can obtain that the feasibility of conditions stated in Theorem 3.1.

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3.3 and Corollary 3.1 can be turned into the following LMIs based feasibility problem with a fixed parameter $\lambda$, respectively:

$$\min_{X_i, Y_i, M, \eta_1, \eta_2, \sigma} (\varepsilon^2 + \gamma^2)$$

\text{s.t. LMIs (21a)-(21e).} 

$$\min_{X_i, Y_i, M, \eta_1, \eta_2, \theta_i, \sigma} (\varepsilon^2 + \gamma^2)$$

\text{s.t. LMIs (21c), (21d), (22a)-(22d).} 

Furthermore, by using the program $fminsearch$ in the unconstrained nonlinear optimization toolbox of Matlab, we can also find the locally convergent solution $\lambda$.

**Remark 3.2.** It should be pointed out that if we can find feasible solution when $\lambda = 1$, by the above discussion we can obtain that the designed stochastic finite-time $H_\infty$ controller can ensure SFTB and stochastic stabilization of the time-delay fuzzy DMJS with the disturbance attenuation $\gamma$.

**Remark 3.3.** Notice that the topic on robust control of T-S fuzzy systems has attracted considerable attention [3, 4, 9], [27]-[30]. We can also discuss the problem of robust finite-time $H_\infty$ control for uncertain discrete-time fuzzy jump nonlinear systems with time delays and obtain the corresponding similar results.

**IV. A NUMERICAL EXAMPLE**

In this section, we provide a simulation example to illustrate the proposed results.

**Example 4.1.** Consider the following time-delay fuzzy DMJS involving two modes with the following parameters:

$$A_{1,1} = \begin{bmatrix} 1.2 & 0.5 \\ 0.8 & 1 \end{bmatrix}, A_{d1,1} = \begin{bmatrix} 0.6 & 0 \\ 0 & -0.5 \end{bmatrix},$$

$$A_{1,2} = \begin{bmatrix} -0.6 & 0.4 \\ 1 & 1.5 \end{bmatrix}, A_{d1,2} = \begin{bmatrix} -0.6 & 0.1 \\ 0 & -0.4 \end{bmatrix},$$

$$A_{2,1} = \begin{bmatrix} -0.5 & 0.2 \\ 1 & 1 \end{bmatrix}, A_{d2,1} = \begin{bmatrix} 0.5 & -0.41 \\ 0 & -0.6 \end{bmatrix},$$

$$A_{2,2} = \begin{bmatrix} -0.6 & 0.8 \\ 0.1 & 1.2 \end{bmatrix}, A_{d2,2} = \begin{bmatrix} 0.6 & -0.4 \\ 0 & -0.5 \end{bmatrix},$$

$$B_{1,1} = B_{2,1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B_{1,2} = B_{2,2} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$G_{1,1} = G_{2,1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G_{1,2} = G_{2,2} = \begin{bmatrix} 0 & 1 \\ 0 & 0.5 \end{bmatrix},$$

$$C_{1,1} = C_{2,1} = C_{1,2} = C_{2,2} = \begin{bmatrix} 0.5 & 0 \end{bmatrix},$$

$$C_{d1,1} = C_{d2,1} = C_{d1,2} = C_{d2,2} = \begin{bmatrix} 0.2 & 0 \end{bmatrix},$$

$$D_{11,1} = D_{12,1} = D_{11,2} = D_{12,2} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix},$$

$$D_{21,1} = D_{22,1} = D_{21,2} = D_{22,2} = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$ 

We assume that the transition rate matrix is given by

$$\begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}.$$ 

For simplicity, $h_1(k)$ and $h_2(k)$ are fuzzy basis functions defined, respectively, as $h_1(k) = h(x_1(k))$ and $h_2(k) = 1 - h(x_1(k))$ with

$$h(x_1(k)) = \begin{cases} \frac{1}{6} |3 - x_1(k)|, & |x_1(k)| < 3, \\
1, & |x_1(k)| \geq 3. \end{cases}$$ 

**Fig. 1.** The local optimal bound of $\epsilon$ and $\gamma$.

**Fig. 2.** The response of the system state $x(k)$.

Let $R_1 = R_2 = I_2$, $\delta_e = 1$, $N = 5$, $\omega = 2$ and $d = 1$, by Theorem 3.3, the optimal bound with minimum value of $\varepsilon^2 + \gamma^2$ relies on the parameter $\lambda$. One can find feasible solution when $1.90 \leq \lambda \leq 36.50$. Fig. 1 shows the optimal value with different value of $\lambda$. Then, by using the program $fminsearch$ in the optimization toolbox of Matlab starting at $\lambda = 2$, we can also find the locally convergent solution $\lambda = 2.1626$, $\gamma = 15.7421$, $\epsilon = 33.9467$, and the state feedback control gains can be derived as

$$K_{1,1} = \begin{bmatrix} 0.2804 & -0.4109 \\ -1.2848 & -0.4353 \end{bmatrix},$$

$$K_{2,1} = \begin{bmatrix} -1.4482 & -0.5412 \\ 0.4945 & -0.2787 \end{bmatrix},$$

$$K_{1,2} = \begin{bmatrix} -1.3937 & -0.0070 \\ 0.0348 & -0.8134 \end{bmatrix},$$

$$K_{2,2} = \begin{bmatrix} -0.7539 & 0.5136 \\ -0.4498 & -0.9513 \end{bmatrix}.$$ 

Now, we set the initial conditions are as $x(-1) = x(0) = \begin{bmatrix} 0.6 & -0.8 \end{bmatrix}^T$ and the initial mode $r_0 = 1$. We further assume that $w(k) = \begin{bmatrix} 0.9e^{-k} & 0.8e^{-k} \sin k \end{bmatrix}^T$, then, the simulation of the jumping modes and the state response of the closed-loop time-delay fuzzy DMJS are depicted in Fig. 2, which show the effectiveness of the proposed methods.
V. CONCLUSIONS

This paper studied the problem of finite-time $H_{\infty}$ control for a class of discrete-time Markovian jump T-S fuzzy systems with time delays. By applying the fuzzy Lyapunov-Krasovskii function method, a finite-time $H_{\infty}$ controller is designed such that the resulting closed-loop system is stochastic finite-time bounded and satisfies a prescribed $H_{\infty}$ performance level in the given finite-time interval. Sufficient criteria are provided for the solvability of the problem, which can be tackled by a feasibility problem in terms of LMIs. An example is presented which can show the validity of the proposed design approaches and more results related to stochastic finite-time control problem of time-delay fuzzy DMJSs will be investigated in the future.

REFERENCES