Multi-link Mechanical Locomotors in Natural Gaits - Controller Design and Experiments

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Abstract—Recently, a so-called natural oscillation gait was studied for multi-link systems and a class of biologically inspired controllers was designed for the achievement of the gait. In this paper, the theoretical design is applied on a mechanical multi-link testbed of two posture configurations in rayfish-like flapping-wing motion and snake-like serpentine motion. The effectiveness of the design is cross examined by theoretical analysis, numerical simulation, and experiments.

Index Terms—Robotics, biologically inspired control, locomotion, central pattern generator (CPG)

I. INTRODUCTION

The research on limbless snake-like multi-link robots is of constant interest for biologists and engineers. Analytical studies of snake locomotion as a multi-segmental system can be traced back to Gray’s work [1]. A variety of snake-like gaits such as undulation, side-winding, concertina, rectilinear and side-pushing were studied in [2]. In [3], the author studied biological snakes and developed mathematical relationships characterizing their motion. Specifically, the locomotion mode was characterized by wave-like movement or lateral bending patterns being propagated along the body from head to tail and acting to propel an animal forward. A class of wheeled snake robots was modelled in [4] and an asymptotically stable controller was proposed in [5] which aimed at minimizing the lateral constraint forces on the wheels during locomotion. Another snake-like robot with elegant mechanical design was presented in [6]. All those designs with active or passive wheels have the advantage of continuing forward propulsion. However, they are not effective on rugged environments such as rough and/or muddy terrains. A planar snake robot without wheels was designed in [7] and the optimal motion was studied in computer simulation. The further work on wheel free robots can be found in more literature including [8]–[10], etc.

In this paper, we consider a class of multi-link mechanical systems under continual interactions with the surrounding environment, encompassing robotic locomotors inspired by snakes crawling on the ground and fishes swimming in water [8], [11]. For this class of systems, various locomotion gaits have been studied, e.g., the optimal gait in [12] and the natural oscillation gait in [13], [14]. In particular, natural oscillation is an inherent characteristic of a mechanical rectifier interacting with environment, which is defined as the free response in persistent oscillation of the system with its damping properly compensated. Once the natural oscillation is defined for a mechanical rectifier, the following question is how to design a controller such that the locomotion with a gait in natural oscillation is achieved. This question has been answered in [13], [14] based on a central pattern generator (CPG) inspired design. Actually, a CPG is a group of neurons interconnected in a specific manner, which can endogenously (i.e. without rhythmic sensory or central input) produce rhythmic outputs to activate muscle contractions, resulting in coordinated rhythmic body motions [15].

To verify the theoretical framework of natural oscillation and its CPG based control design proposed in [13], [14], a simple testbed, called prototype mechanical rectifier (PMR), has been established in our recent work [16]. However, because of the simple structure of PMR model, complicated locomotion behaviors cannot be demonstrated. In this paper, we focus on a more sophisticated structure, which is used as the experimental platform to examine more practical locomotion behaviors. The platform consists of multiple links jointed by actuating motors which may demonstrate more locomotion patterns than the PMR system. Specifically, the natural oscillation concept and the CPG based control design framework are tested on this multi-link platform. The tests are conducted in terms of two locomotion posture configurations in rayfish-like flapping-wing motion and snake-like serpentine motion.

II. STRUCTURES AND MODELS

The mechanical locomotors studied in this paper consist of multiple rigid links, which are connected in series with flexible joints. They are pictured in Figs. 1 and 2 in two configurations. In each configuration, it consists of multiple rigid links, wCK modules from Robobuilder (see www.robobuilder.net), which are connected in series with flexible joints. Each link is driven by a DC servo motor through a gear box. In Fig. 1, the middle motor is dumb and the other four motors supply joint toques; in Fig. 2, the first motor from the right is dumb and the other four motors supply joint toques. These symmetric structure are designed for two different gaits and they have two different nominal postures. The ventral contact of the links is equipped with directional friction material, e.g. lint cloth, for better grip on wooden surface. This is analogous to the frictional anisotropy of snake scales to slither on flat surface.

Suppose the mechanical system comprises a chain of \( n \) identical rigid links connected by \( n \) rotational joints \(( n = 4 \) in the present case.) For this model, undulatory...
Fig. 1. The experimental multi-link system in flapping-wing configuration. The middle motor is dumb and the other four motors supply joint torques.

body movements are supposed to occur within the \((x, y)\) plane, where the \(x\)-axis is taken to be the direction of locomotion. Let \(\theta_i\) be the absolute angle between the \(i\)-th link and the negative \(x\)-axis, and \((x_i, y_i)\) be the coordinates of the center of mass of the \(i\)-th link. Each joint is flexible with torsional stiffness \(k\), and the \(i\)-th joint is driven by an actuator that generates torque \(\tau_i\). Each link is subject to the environmental force (e.g., ground friction for crawling and fluid drag for swimming), which is approximately modelled as linear functions of link velocities. In particular, the force components tangential and normal to the \(i\)-th link are given by \(f_{ti} = -c_t v_{ti}\) and \(f_{ni} = -c_n v_{ni}\), respectively, where \(c_t\) and \(c_n\) are constant coefficients, and \(v_{ti}\) and \(v_{ni}\) are the components of the link velocity in the respective directions. The actual forces may be more complex, but the linear approximation is chosen to qualitatively capture the resistive property in the simplest manner. Directional preference \((c_n \gg c_t, \text{i.e., tendency of the link to slide much more easily in the tangential direction than in the normal direction})\) is known to be important for thrust generation [8]. We define the coordinates of the center of mass of the whole body \((x_c, y_c)\) with

\[
x_c = \sum_{i=1}^{n+1} \frac{x_i}{n+1}, \quad y_c = \sum_{i=1}^{n+1} \frac{y_i}{n+1}.
\]

Hence, we define the forward velocity \(v := \dot{x}_c\), side velocity \(w := \dot{y}_c\), and their vector \(\varpi := [v \ w]^T \in \mathbb{R}^2\).

The equations of motion for the multi-link system have been derived using the Newton’s law [8], [13] and the Euler-Lagrange equation [11]. The fully nonlinear equations of motion derived in [8], [11] are briefly summarized as follows. When the links of the multi-link system are identical with mass \(m_o\), length \(2\ell_o\), and moment of inertia \(m_o\ell_o^2/3\), the system is described by

\[
\begin{align*}
J(\theta)\ddot{\theta} + D(\theta, \dot{\theta})\dot{\theta} + K\dot{\theta} + \Lambda(\theta)\varpi &= B\tau, \\
m\ddot{\varpi} + \zeta(\theta, \dot{\theta}) + \Psi(\varpi)\varpi &= 0,
\end{align*}
\]

(1)

The quantities \(J(\theta), D(\theta, \dot{\theta}), K, B, \Lambda(\theta),\) and \(Q(\varpi)\), depend on \(m_o, \ell_o\). For a given constant vector \(\theta^0 \in \mathbb{R}^{n+1}\), the system presents a nominal posture when \(\theta = \theta^0\). Then, \(\dot{\vartheta} := \theta - \theta^0\) represents the angle deviation of the links from the nominal posture. An approximate model in the neighborhood of \(\theta = \theta^0, \dot{\theta} = 0\), and \(\varpi = \varpi^0 := [v^0, w^0]^T\) is given as follows

\[
\begin{align*}
J\ddot{\vartheta} + D\dot{\vartheta} + K(\varpi^0)\vartheta + \Phi(\varpi^0)\varpi &= B\tau, \\
m\ddot{\varpi} + \zeta(\vartheta, \dot{\vartheta}) + \Phi(\varpi)\varpi &= 0,
\end{align*}
\]

(2)

The details of the quantities \(J, D, K(\varpi^0), E(\varpi), \zeta(\vartheta, \dot{\vartheta}),\) and \(\Phi(\varpi)\) used in the equations are omitted.

In what follows, we will study two specific locomotion postures, i.e., flapping-wing locomotion and serpentine locomotion. Then, the model (2) will take more specific forms in these two cases.

1) Flapping-wing locomotion: The flapping-wing posture configure runs in a symmetric mechanical manner. In particular, the nominal posture is set symmetrically satisfying

\[
\theta_i^0 + \theta_o^0 = \theta_2^0 + \theta_4^0 = 2\theta_o^0 = 2\pi.
\]

With the initial posture at the nominal posture, and applying exactly symmetric torques \(\tau_1(t) = \tau_2(t)\) and \(\tau_3(t) = \tau_4(t)\). The system is always in a symmetric position with

\[
\vartheta_1(t) = -\vartheta_3(t), \quad \vartheta_2(t) = -\vartheta_4(t), \quad \vartheta_3(t) = 0.
\]

Since there is no outstanding force in the \(y\)-axis, the velocity vanishes with \(w(t) = 0\). The original model (2) is generally underactuated as \(\vartheta \in \mathbb{R}^5\) but \(\tau \in \mathbb{R}^4\). However, in the symmetric flapping-wing locomotion, the degrees of freedom of \(\vartheta\) and \(\tau\) reduce to two, resulting in a fully actuated system. More specifically, we define

\[
\bar{\vartheta} = T\vartheta = [\vartheta_4, \vartheta_5]^T, \quad \bar{\tau} = T\tau, \\
\bar{\varpi} = S\varpi = [\tau_3, \tau_4]^T, \quad \bar{\tau} = \bar{S}\bar{\tau},
\]

for

\[
T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
\]

\[
\bar{T} = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \bar{S} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}.
\]

As a result, the model (2) reduces to a three dimensional system governing \(\bar{\vartheta}\) and \(\bar{\varpi}\). Furthermore, let

\[
\phi = \bar{B}^T\bar{\vartheta} = [\bar{\vartheta}_1, \bar{\vartheta}_2 - \bar{\vartheta}_1], \quad \bar{B} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.
\]

The system governing \(\phi\) and \(\varpi\) becomes

\[
\dot{\phi} + M\phi = u, \quad m\dot{\varpi} + \zeta(\phi, \dot{\phi}) + \Psi(\phi)\varpi = 0,
\]

(3)
where
\[ M = B^T J^{-1} K (v^o, 0) T B^{-T}, \]
\[ \xi (\phi, \dot{\phi}) = \zeta_1 (T B^{-T} \phi, T B^{-T} \dot{\phi}), \]
\[ \Psi (\phi) = \Phi_{11} (T B^{-T} \phi) \]
and
\[ u = B^T J^{-1} B \tilde{S} \tau - B^T J^{-1} E (v, 0) + B^T J^{-1} D T B^{-T} \dot{\phi}. \]

2) Serpentine locomotion: In the serpentine posture configuration, the nominal posture is set as
\[ \theta^o = [0^o, 0^o, 0^o, 0^o, 0^o]^T. \]
Also, we consider a trivial nominal \( y \)-directional velocity \( w^o = 0 \). In this specific scenario, the quantities in (2) take the following form:
\[ E (\varpi) = 0, \quad D = \mu J, \quad \mu = c_n / m_o \]
which will be used to simplify the system model later. Since \( B \) is not a full rank matrix, the dynamics in (2) are underactuated. To facilitate the analysis, we introduce the coordinate transformation
\[ \phi = B^T \vartheta, \quad \varphi = e^T \vartheta, \quad e = [1, \cdots, 1]^T / (n + 1), \]
where \( \varphi \in \mathbb{R} \) is the average link angle representing the body orientation with respect to the inertial frame, and \( \phi \in \mathbb{R}^n \) is the vector of relative angles between two adjacent links, representing the body shape. Define
\[ \begin{bmatrix} B^T \\ e^T \end{bmatrix} = \begin{bmatrix} B^T \\ e^T \end{bmatrix}^{-1} \]
with which, \( \vartheta = B^T \phi + e^T \varphi \). Moreover, when \( c_n \gg c_t \), we have \( K (v^o, 0) \vartheta \approx K (v^o, 0) B^T \phi \). As a result, the system (2) is approximately equivalent to
\[ \dot{\phi} + \mu \dot{\phi} + M \phi = u, \quad \dot{\varphi} + \mu \dot{\varphi} + p^T \phi = q^T u, \]
\[ m \ddot{w} + \xi (\phi, \dot{\phi}, \dot{\varphi}) + \Psi (\phi, \varphi) \varpi = 0, \]
where
\[ M = B^T J^{-1} K (v^o, 0) B^\dagger, \quad p^T = e^T J^{-1} K (v^o, 0) B^\dagger, \quad q^T = e^T J^{-1} B (B^T J^{-1} B)^{-1}, \quad \Psi (\phi, \varphi) = \Phi (B^T \phi + e^T \varphi), \]
\[ \xi (\phi, \dot{\phi}, \dot{\varphi}) = \begin{bmatrix} \zeta_1 (B^T \phi, B^T \dot{\phi} + e^T \dot{\varphi}) \\ 0 \end{bmatrix}, \]
and \( u = B^T J^{-1} B \tau \).

III. Controller Design

In this section, we aim to develop an effective control algorithm to achieve a desired behavior in terms of three actions: oscillation \( \phi \), orientation \( \varphi \) (for the serpentine posture only), and locomotion \( \varpi \). The desired behavior is to be realized as a stable autonomous motion through feedback control, rather than as a forced response to a fixed trajectory command. In this way, the design would be more robust against disturbances, noises, and neglected higher order nonlinear dynamics. The specific objective is to find a dynamic state feedback controller for the locomotion system described by (3) or (4), such that the closed-loop system achieves the following three properties:
(i) The multi-link body shape \( \phi \) displays the “natural oscillation” of the locomotion system.
(ii) The body orientation is regulated to match its forward direction, i.e. \( \varphi \) is periodic with zero average.
(iii) A desired forward velocity \( v = v^o \) is generated while the side velocity \( w \) is zero on average.
The general theoretical problem was studied in [13], [14]. The main design approach is summarized in this section.

A. Oscillation

A precise definition of the natural oscillation for the locomotor system is given as follows.

Definition 3.1: Consider the \( \phi \)-system in (3) or (4) where all the eigenvalues of \( M \) are simple and have positive real parts. Let the damping parameter \( \mu \) be adjusted by \( \epsilon \in \mathbb{R} \) so that \( \epsilon (\omega) = \rho \). Then, the state feedback controller for the locomotion system described by (3) or (4), such that the closed-loop system achieves the desired behavior is to stabilize the modal oscillation \( \omega \) of the locomotion system.

Let \( \varsigma \) by (3) or (4), such that the closed-loop system achieves the desired behavior is to stabilize the modal oscillation \( \omega \) of the locomotion system.

Let \( \varsigma \) be defined by (6), and \( (\omega, z) \) be the corresponding natural oscillation (5). Let \( \varsigma \) be defined by (6), and \( (\omega, z) \) be the corresponding natural oscillation (5). Let \( \varsigma \) be defined by (6), and \( (\omega, z) \) be the corresponding natural oscillation (5). Let
\[ u = \zeta (\phi) := \epsilon (\| R^T \phi \|) \dot{\phi} \]
with \(R^\dagger := 2 \begin{bmatrix} \Im(\ell) & \Re(\ell) \end{bmatrix}^T \in \mathbb{R}^{2 \times n}\) achieves the entrainment to the natural oscillation, i.e., for each initial state \((\phi(0), \dot{\psi}(0))\) that is sufficiently close to the orbit \(\mathcal{O}\), there exists \(t_\circ \in \mathbb{R}\) such that
\[
\lim_{t \to \infty} \|\phi(t) - \psi(t + t_\circ)\| = 0,
\]
where \(\ell\) is the left eigenvector of \(M\) associated with the eigenvalue \(\zeta\), normalized to satisfy \(\ell^T z = 1\).

**B. Orientation**

The second control objective is to keep the orientation angle aligned with the direction of locomotion on average; \(\bar{\varphi}(t) := \int_0^t \varphi(\tau) \, d\tau = 0\) for a period \(T\). This additional design is for the serpentine locomotion only. Let us first examine how this objective is addressed by the controller \(u = \zeta(\dot{\phi})\). Recall that this controller asymptotically achieves the entrainment to the natural oscillation; \(\bar{\phi}(t) \to \dot{\psi}(t)\). In the steady state, the orientation equation from (4) becomes
\[
\ddot{\phi} + \mu \dot{\phi} = \ddot{u},
\]
The equation has the general steady state solution \(\varphi(t) = \chi(t) + c_o\) with an arbitrary constant \(c_o \in \mathbb{R}\) and
\[
\chi(t) := \Re\left(\eta^T e^{j\omega_o t}\right), \quad \eta := \frac{j \omega_o q \sigma - p}{j \omega_o \mu - \omega^2} \in \mathbb{C}^n.\tag{9}
\]
Clearly, the solution has the average \(\bar{\varphi}(t) = c_o\) due to \(\bar{\chi}(t) = 0\) for \(T := 2\pi/\omega\) (\(\bar{\varphi}\) and \(\bar{\chi}\) are the average values of \(\varphi\) and \(\chi\) over one period, respectively.) Since the transfer function from \(\ddot{u}\) to \(\varphi\), i.e., \(1/(s^2 + \mu s)\), has a rigid body mode (the pole at \(s = 0\)), the average value \(\bar{c}_o\) depends on the initial condition. Thus, the controller \(u = \zeta(\dot{\phi})\) does not automatically lead to the desired orientation angle \(\bar{\varphi}(t) = 0\) in general. Therefore, we should modify the control law \(u = \zeta(\dot{\phi})\) so that the average value of the orientation angle is guaranteed to vanish asymptotically; \(\bar{\varphi}(t) \to 0\). Such a modified controller is given below
\[
\begin{align*}
\dot{u} &= \zeta(\dot{\phi}) - b \dot{\phi}, \\
\ddot{\phi} + \alpha_2 \dot{\phi} &= \varphi
\end{align*}\tag{10}
\]
for
\[
h := \alpha_1 M \beta/(b \dot{\beta}), \quad b = M^T q - p, \quad \beta = M^{-1} e
\]
and \(\alpha_1, \alpha_2 > 0\). For this controller, the first term \(\zeta(\dot{\phi})\) is exactly the one derived in the last section for achieving the natural oscillation, while the second term \(-h \dot{\phi}\) is the additional component for orientation regulation.

**C. Locomotion**

In the previous sections, the natural oscillation of the body shape and regulation of the body orientation have been achieved by the nonlinear feedback controller (7) or (10). Recall that the model has been developed under a nominal locomotion condition in which the locomotion velocity is nearly constant; \(\bar{v}(t) \cong v^o \neq 0 \|\|^T\). In particular, the coefficients \(M\) and \(p\) depend on the velocity \(v\) in general, but are chosen to take fixed values \(M(v^o)\) and \(p(v^o)\) for a selected nominal velocity \(v^o\). Hence, the control design would be valid only if the actual velocity \(\bar{v}\) is achieved by the controller (7) or (10) through the \(\bar{v}\)-dynamics, turns out to be equal to the nominal velocity \(v^o\). The objective of this section is to check and enforce the consistency. For a given constant velocity \(v^o\), let \((\omega^o, z^o)\) be the natural oscillation of (3) or (4) with \(\hat{M} = \hat{M}(v^o)\). When the corresponding natural oscillation \(\phi = \psi\) in (5) and the associated orientation \(\varphi = \chi\) in (9) are achieved, The force balance equation can be written as
\[
\bar{v} = a(\bar{v}^o) := \bar{\xi}_1(\psi, \dot{\psi}, \chi) \bar{\Psi}_1(\psi, \dot{\psi}) \tag{11}
\]
where \(\bar{\xi}\) and \(\bar{\Psi}\) are the average values of \(\xi\) and \(\Psi\) over one period (the arguments \(\chi\) and \(\bar{\chi}\) vanish in the flapping-wing case), and \(a\) is a function of \(v^o\) through the dependence of \((\omega^o, z^o)\) on \(v^o\). Recall that the definition of the natural oscillation does not specify the amplitude of oscillation \(\|z^o\|\). However, since \(a(v^o)\) is independent of \(\|z^o\|\), there is no ambiguity in defining the consistent velocity \(v^o\) by the condition \(\bar{v}^o = a(v^o)\). We call \(v^o\) satisfying (11) a natural velocity. Plotting the curve \(y = a(x)\) and the straight line \(y = x\) and searching for the intersection gives a numerical method to find a natural velocity as illustrated by the examples later.

When the controller (7) or (10) is designed for the natural oscillation associated with \(v^o\) satisfying (11), one can prove that controller (7) or (10) is able to achieve the natural oscillation \(\phi = \psi\) with the body orientation regulated at \(\varphi = \chi\), with small perturbations. Moreover, the same controller (7) or (10) also achieves regulation of the locomotion velocity at \(\bar{v} = v^o\) with small perturbations.

**IV. Simulation and Experiments**

In this section, we examine the effectiveness of the proposed controller (7) or (10) on a real testbed. We have the following system parameters: number of links, \(n = 5\); normal and tangential direction friction coefficients, \(\mu_n = 70\text{Ns/m}\) and \(\mu_t = 10\text{Ns/m}\) respectively; total system weight, \(m = \ldots\)
Fig. 4. Profiles of link oscillation $\phi$ and forward velocity $v$ in flapping-wing gait. Left: simulated results; right: experimental results.

Fig. 5. Profiles of link oscillation $\phi$, forward velocity $v$, and side velocity $w$ in serpentine gait. Left: simulated results; right: experimental results.

Fig. 6. Experimental snapshots of the locomotion in serpentine gait.
A. Flapping-wing locomotion

The flapping-wing posture studied in this section is with \( \theta = [160^\circ, 150^\circ, 90^\circ, 30^\circ, 20^\circ]^T \) and the controller (7). In Fig. 3, a desired forward velocity \( v^o = 0.045 \text{m/s} \) is solved from \( a(v^o) = v^o \) in flapping-wing gait. The observation \( a'(v^o) < 1 \) shows the stability of the locomotion as proved in [14]. With this velocity, \( \omega = 11.7 \) and \( \varrho = -22.7 \) are calculated. As a result, the controller (7) is designed with \( \varepsilon(x) = -23.1 + 5/(1 + x) \) satisfying \( \varepsilon(\omega) = \varrho \). Figure 4 represents the simulated and experimental profiles of link oscillation \( \phi \) and forward velocity \( v \). All the simulated and experimental results are summarized in Table I in cross check of theoretical analysis, simulation, and experiments. The phase of \( \phi_2 \) lags that of \( \phi_1 \) indicating the travelling waves propagating in the negative \( x \) direction. In the graph of experimental link oscillation, the profiles of \( \phi_1 \) and \( \phi_2 \) of one wing are plotted together with the corresponding oscillations of the other wing. The curves are well overlapped indicating the symmetric configuration of the two wings. Figure 7 shows the snapshots of the flapping-wing locomotion with experimental data which mimics rayfish-like movement.

B. Serpentine locomotion

The controller (10) is used in this posture. In Fig. 3, a desired forward velocity \( v^o = 0.053 \text{m/s} \) is solved from \( a(v^o) = v^o \) for this movement. It is also true that \( a'(v^o) < 1 \) that was proved in [14] for stability. With this velocity, \( \omega = 7.65 \) and \( \varrho = 1078 \) are calculated. As a result, the controller (7) is designed with \( \varepsilon(x) = 1077 + 8.65/(1 + x) \) satisfying \( \varepsilon(\omega) = \varrho \). The other parameters are \( \varepsilon_{x1} = 0.2 \) and \( \varepsilon_{x2} = 0.5 \). Figure 5 represents the simulated and experimental profiles of link oscillation \( \phi \), forward velocity \( v \), and side velocity \( w \). The results are also summarized in Tables II in cross check of theoretical analysis, simulation, and experiments. The phase of \( \phi_{x1+1} \) lags that of \( \phi_1 \) indicating the travelling waves propagating in the negative \( x \) direction. The side velocity in \( y \) direction is essentially invalid. Figure 6 shows the snapshots of the serpentine locomotion with experimental data which mimics snake-like movement.

V. conclusion

In this paper, we developed a multi-link mechanical system which works as an engineering analogous of multi-segmental limbless animals. Two typical limbless locomotion postures in natural gaits have been achieved by a biologically inspired controller. The control design was checked through theoretical analysis, numerical simulation and experiments.

REFERENCES


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The snapshots of the flapping-wing locomotion with experimental data which mimics rayfish-like movement.

V. conclusion

In this paper, we developed a multi-link mechanical system which works as an engineering analogous of multi-segmental limbless animals. Two typical limbless locomotion postures in natural gaits have been achieved by a biologically inspired controller. The control design was checked through theoretical analysis, numerical simulation and experiments.