Delay Distribution Dependent Stability Criteria for Discrete-Time Systems with Interval Time-Varying Delay*

Nan Xiao 1, Yingmin Jia 2 and Fumitoshi Matsuno 3

Abstract—This paper studies the stability problem for discrete-time systems with interval time-varying delay. By dividing delay interval into two subintervals, a delay-dependent exponential stability criterion is obtained based on Lyapunov stability theory and reciprocally convex lemma. Furthermore, by assuming that the distribution of time-varying delay is known, the difference of Lyapunov functional is allowed to have positive upper bound for the value of time-varying delay in one subinterval, and a new delay distribution dependent stability criterion is obtained. The obtained result is also extended to cope with the robust delay distribution dependent stability problem for uncertain time-varying delay systems. All the obtained criteria are presented in terms of Linear Matrix Inequalities (LMIs). Finally one numerical example is given to show the effectiveness of the proposed method.

I. INTRODUCTION

Time delay is a common phenomenon in many industrial and engineering systems. Since the existence of time delay may lead to system instability and poor performance, the problem of stability analysis for time-delay systems has attracted many researcher’s attention in recent years [1-5]. For the discrete-time case, systems with interval time-varying delay have strong background in engineering applications such as network-based control systems, and much work have been done to the stability analysis problem for discrete-time systems with interval time-varying delay [6-8].

For getting less conservative results, various methods have been utilized, such as the use of Park and Moon’s inequalities [1-2], descriptor model transformation method [3], discretized Lyapunov functional method [4], free-weighting matrices method [5,14], the use of Jensen’s inequality [6], Augmented Lyapunov functional and delay-partitioning method [8-10], convex analysis method [7,11], reciprocally convex approach [12], etc. In the case that the distribution of time-varying delay is known a priori, delay-distribution dependent stability criteria are obtained for time-delay systems both in continuous form [15-16] and discrete form [17], which can guarantee system’s exponential stability in mean square sense.

When researching this problem, we find that the difference of Lyapunov functional is restricted to be negative for time delay in all the subintervals in the literature that used delay-partitioning method. While by assuming that the distribution of time delay is known, we can relax the condition and allow that the difference of Lyapunov functional has some positive upper bound in some case. Motivated by the above ideals, we investigate the stability problem for discrete-time systems with interval time-varying delay in this paper. We divide the delay interval into two subintervals and construct the corresponding Lyapunov functional. Based on Lyapunov stability theory and reciprocally convex lemma, delay-dependent exponential stability criterion is obtained. Then by allowing the difference of Lyapunov functional have positive upper bound for the value of time-varying delay in one subinterval, delay distribution dependent stability criterion and robust delay distribution dependent stability criterion are obtained. All the derived criteria are expressed in terms of LMIs that can be solved by using Matlab Toolbox. The method used in this paper is different from the existing ones [15-17]. For a given distribution of time delay, we can choose different values of the exponential convergence and divergence rates in solving our obtained LMIs, and get the corresponding admissible upper delay bounds. Then less conservative results can be chosen from these upper delay bounds. Finally we give one numerical example to verify the effectiveness of the obtained criteria.

Notations: In this paper, the notation $P > 0 \ (\geq 0)$ means that $P$ is real symmetric and positive definite (semi-definite). The superscripts $T$ and $-1$ denote matrix transposition and matrix inverse. $\| \cdot \|$ stands for the Euclidean vector norm. A symmetric term in a symmetric matrix is denoted by $*$, $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. $I_n$ and $0_{n \times m}$ denote the $n \times n$ dimensional identity matrix and $n \times m$ dimensional zero matrix, respectively.

II. PRELIMINARIES

Consider the following discrete-time system with time-varying delay:

$$x(k+1) = Ax(k) + A_d x(k-h(k)), \ k \geq k_0$$

$$x(k) = \phi(k), \ k \in [k_0, h_2, k_0]$$

(1)

where $x(k) \in \mathbb{R}^n$ is the state vector, $\phi(k)$ is an initial value at $k$, $A$ and $A_d$ are constant system matrices of appropriate dimensions, integer $h(k)$ is a time-varying function satisfying

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\begin{align}
0 < h_1 &\leq h(k) \leq h_2, \quad (2)
\text{where } h_1 < h_2 \text{ are integers.}
\end{align}

In this paper, we will research the stability problem for system (1) with interval time-varying delay satisfying (2). Before deriving our main results, the following definition and lemmas are given.

\textbf{Definition 1.} The discrete time-delay system (1) is said to be globally exponentially stable if there exist positive constants $M > 0$ and $0 < \theta < 1$ such that

$$\|x(k)\| \leq M\theta^{k-k_0}\|x_{k_0}\|, \quad \forall k \geq k_0,$$

where $\|x_{k_0}\| = \sup_{h_0 \leq s \leq h_0}\|x(s)\|$.

\textbf{Lemma 1.} For any constant matrix $P = P^T > 0$, integers $0 \leq h_1 < h_2$, constant $\rho < 1$ and vector function $\omega(i) : (h_1, h_1 + 1, \ldots, h_2) \to \mathbb{R}^n$, the following inequality holds:

$$\sum_{i=h_1}^{h_2} (1-\rho)^i \omega(i)^T P \omega(i) \geq \frac{\rho(1-\rho)^{h_2}}{1-(1-\rho)^{h_2}} \omega(i)^T P \omega(i),$$

where $h_{12} = h_2 - h_1$.

\textbf{Proof.} By Schur Complement [13], we have

$$\begin{bmatrix}
(1-\rho)^i \omega(i)^T P \omega(i) & \omega(i)^T P T - (1-\rho)^i P^{-1}
\end{bmatrix} \geq 0,$$

we sum it from $h_1$ to $h_2$, then by Schur Complement again we can get Lemma 1.

\textbf{Lemma 2.} [12] For functions $k_1(t), k_2(t) \in [0,1], k_1(t) + k_2(t) = 1$, and vectors $\eta_1, \eta_2$ which satisfy $\eta_1 = 0$ with $k_1(t) = 0$ and $\eta_2 = 0$ with $k_2(t) = 0$, matrices $P > 0$, there exist matrices $T$, satisfies

$$\begin{bmatrix}
P & T
\end{bmatrix} \geq 0,$$

such that the following inequality holds

$$\frac{1}{k_1(t)} \eta_1^T P \eta_1 + \frac{1}{k_2(t)} \eta_2^T Q \eta_2 \geq \begin{bmatrix}
\eta_1 & \eta_2
\end{bmatrix} \begin{bmatrix}
P & T
\end{bmatrix} \begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix}.$$

\textbf{III. MAIN RESULTS}

\textbf{A. Exponential Stability Criterion}

In this subsection, we investigate the exponential stability problem for system (1) with interval time-varying delay satisfying (2). We divide the time interval $[h_1, h_2]$ into two subintervals: $[h_1, h_a]$ and $[h_a + 1, h_2]$, where $h_a$ is an integer between $h_1$ and $h_2$, denote $\delta_1 = h_a - h_1$, $\delta_2 = h_2 - h_a$, and $\bar{\xi}(k) = \begin{bmatrix} x^T(k) & x^T(k-h_1) & x^T(k-h_a) & x^T(k-h_2) \end{bmatrix}^T$, and $e_i$ are block entry matrices such that $e_i \xi(k) = \xi(k) (i = 1, \ldots, 5)$, for example, $e_i = \begin{bmatrix} 0_{n \times (i-1) n} & I_n & 0_{n \times (5-i) n} \end{bmatrix}$.

Then corresponding to the divisions we construct the Lyapunov-Krasovskii functional as follows:

$$\begin{align}
V(x(k)) &= V_1(x(k)) + V_2(x(k)) + V_3(x(k)) + V_4(x(k)), \quad (4)
V_1(x(k)) &= x^T(k) P x(k),
V_2(x(k)) &= \sum_{i=h_1-1}^{h_a-1} \sum_{j=i+1}^{h_2-1} (1-\rho)^{k-i-1} x^T(i) Q x(i),
V_3(x(k)) &= \sum_{i=h_a-1}^{h_2-1} (1-\rho)^{k-i-1} x^T(i) Q x(i),
V_4(x(k)) &= \sum_{i=h_1-1}^{h_a-1} \sum_{j=i+1}^{h_2-1} (1-\rho)^{k-i-1} x^T(i) Q x(i),
\end{align}$$
Proof. Taking the difference of Lyapunov functional (4) along the trajectory of system (1), we have
\[ \Delta V_1 = x^T(k + 1)Px(k + 1) - (1 - \rho)x^T(k)Px(k) - \rho V_1, \]
\[ \Delta V_2 \leq (h_{12} + 1)x^T(k)Qx(k) - \rho V_2 \]
\[ - (1 - \rho)h^2_1 x^T(k-h(k))Qx(k-h(k)), \]
\[ \Delta V_3 = x^T(k)Qx(k) - (1 - \rho)h^1_1 x^T(k-h_1)Qx(k-h_1) \]
\[ + (1 - \rho)h^1_1 x^T(k-h_1)Qx(k-h_1) - \rho V_3 \]
\[ - (1 - \rho)h^2_1 x^T(k-h_2)Qx(k-h_2) \]
\[ + (1 - \rho)h^2_1 x^T(k-h_2)Qx(k-h_2), \]
\[ \Delta V_4 = \eta^T(k)(h_1 \delta_1 + h_2 \delta_2) \eta(k) - \rho V_4 \]
\[ - \sum_{i=1}^{h_1}(1 - \rho)\eta^T(k-i)R_0 \eta(k-i) \]
\[ - \sum_{i=1}^{h_2}(1 - \rho)\eta^T(k-i)R_1 \eta(k-i) \]
\[ - \sum_{i=1}^{h_2}(1 - \rho)\eta^T(k-i)R_2 \eta(k-i). \]

When \( h(k) \in [h_1, h_2] \), by using Lemma 1, we have
\[ - \sum_{i=1}^{h_1}(1 - \rho)\eta^T(k-i)R_0 \eta(k-i) \leq \lambda_0(1 - \rho)\sum_{i=1}^{h_1}(1 - \rho)\eta^T(k-i)R_0 \eta(k-i) \]
\[ = \eta^T(k)(h_1 \delta_1 + h_2 \delta_2) \eta(k-i) \]
\[ - \sum_{i=1}^{h_2}(1 - \rho)\eta^T(k-i)R_1 \eta(k-i) \]
\[ - \sum_{i=1}^{h_2}(1 - \rho)\eta^T(k-i)R_2 \eta(k-i). \]

Then by using Lemma 2, there exists matrix \( T_1 \), such that
\[ \begin{bmatrix} R_1 & T_1 \\ \ast & R_1 \end{bmatrix} \geq 0, \]
(7)
and
\[ -\lambda_1(1 - \rho)\left[ \begin{bmatrix} \eta_1^T R_1 \eta_1 + \eta_2^T R_1 \eta_2 \\ \eta_1^T R_1 \eta_1 \end{bmatrix} \right] \leq 0. \]

According to above inequalities we can finally get
\[ \Delta V(x(k)) + \rho V(x(k)) \leq \xi^T(k)(\Pi_0(\rho) + \Pi_1(\rho))\xi(k). \]

When \( h(k) \in [h_1, h_2] \), similar to the above procedure we can get
\[ \Delta V(x(k)) + \rho V(x(k)) \leq \xi^T(k)(\Pi_0(\rho) + \Pi_2(\rho))\xi(k). \]

Then we can conclude that if (5-6) are satisfied, the following inequality (10) can be guaranteed for \( h(k) \in [h_1, h_2] \):
\[ \Delta V(x(k)) < -\rho V(x(k)), \]
(10)
thus we have
\[ V(x(k)) < (1 - \rho)^{k-h_0}V(x(k_0)). \]

It is easy to see from (4) that
\[ V(x(k)) \geq n\|x(k)\|^2, V(x(k_0)) \leq m\|x(k_0)\|^2, \]
from these inequalities we can finally conclude that
\[ \|x(k)\| \leq \sqrt{\frac{n}{m}}(1 - \rho)^{k-h_0}\|x(k_0)\| \]
This completes the proof.

Remark 1. We apply Lemma 1 to the cross terms that appeared in the difference of Lyapunov functional, and enlarge these terms by using reciprocally convex lemma, which can lead less conservative results as well as avoid the use of many free-weighting matrices.

In Theorem 1 we let \( \rho \neq 0 \) to avoid the infinite value of some terms, and get exponential stability criterion for system (1). Notice that Lemma 1 is equivalent to Jensen inequality by using L'Hospital rule for \( \rho \rightarrow 0 \), then by letting \( \rho = 0 \) in (4) and using the similar method we can get the following asymptotic stability criterion.

Corollary 1. For given integers \( 0 < h_1 < h_a < h_2 \), system (1) with interval time-varying delay satisfying (2) is asymptotically stable if there exist symmetric positive matrices \( P, Q, Q_0, R_0, Q_1 \) and \( R_1 \), matrices \( T_i(i=1,2) \), such that the following LMIs (11-12) hold:
\[ \Pi_0 + \Pi_1 < 0, \]
\[ \begin{bmatrix} R_1 & T_1 \\ \ast & R_1 \end{bmatrix} > 0, \]
(11)
\[ \Pi_0 + \Pi_2 < 0, \]
\[ \begin{bmatrix} R_2 & T_2 \\ \ast & R_2 \end{bmatrix} > 0, \]
(12)
where
\[ \Pi_0 = \tilde{A}_1^T P \tilde{A}_1 - e_1^T P e_1 + (h_{12} + 1)e_1^T Q e_1 - e_2^T Q e_2 \]
\[ + e_1^T Q_0 e_1 - e_2^T Q_0 e_2 + e_2^T Q_1 e_2 - e_3^T Q e_3 \]
\[ + e_3^T Q_2 e_3 - e_4^T Q_2 e_4 - \lambda_0(e_1 - e_2)^T R_0(e_1 - e_2) \]
\[ + \tilde{A}_2^T(h_1 R_0 + h_2 R_2) \tilde{A}_2, \]
\[ \Pi_1 = -\lambda_2(e_2 - e_4)^T R_2(e_2 - e_4) \]
\[ - \lambda_1 \begin{bmatrix} e_2 - e_4 \\ e_5 - e_3 \end{bmatrix}^T \begin{bmatrix} R_1 & T_1 \\ \ast & R_1 \end{bmatrix} \begin{bmatrix} e_2 - e_4 \\ e_5 - e_3 \end{bmatrix}, \]
\[ \Pi_2 = -\lambda_1(e_2 - e_4)^T R_2(e_2 - e_3) \]
\[ - \lambda_2 \begin{bmatrix} e_3 - e_5 \\ e_5 - e_4 \end{bmatrix}^T \begin{bmatrix} R_2 & T_2 \\ \ast & R_2 \end{bmatrix} \begin{bmatrix} e_3 - e_5 \\ e_5 - e_4 \end{bmatrix}, \]
\[ \lambda_0 = \frac{1}{h_1}, \lambda_1 = \frac{1}{h_0}, \lambda_2 = \frac{1}{h_2}. \]
B. Delay Distribution Dependent Stability Criteria

To obtain the exponential stability criterion and asymptotic stability criterion, we let \( \Delta V(x(k)) < 0 \) for the value of \( h(k) \) in all the subintervals. In the situation that the distribution of time-varying delay is unknown, Corollary 1 can get less conservative results compared to Theorem 1 since it allows the difference of Lyapunov function to have a bigger upper bound. However, when the distribution of time-varying delay is known, Corollary 1 will be less effective due to the fact that it can’t fully explore the distribute information of time delay. In this subsection, we will assume that the distribute information of time-varying delay is known, then by allowing the difference of Lyapunov function to have upper bound for the value of time delay in one subinterval, we can get the delay distribution dependent stability criterion for system (1). Define

\[
\begin{align*}
D_{[h_1,h_2]} &= \{ k | h(k) \in [h_1,h_2] \}, \\
D_{[h_a+1,h_2]} &= \{ k | h(k) \in [h_a+1,h_2] \}.
\end{align*}
\]

Let \( L_{[h_1,h_2]} \) and \( L_{[h_a+1,h_2]} \) denote the length of \( D_{[h_1,h_2]} \) and \( D_{[h_a+1,h_2]} \), respectively.

Fig. 1 shows an image of time-varying delay function \( h(k) \in [h_1,h_2] \) for given \( h_1 \) in which

\[
\begin{align*}
D_{[h_1,h_2]} &= \{ k_0, k_1, k_2, \ldots, k_m \}, \\
D_{[h_a+1,h_2]} &= \{ k_1+1, k_2+1, k_3+1, \ldots, k_{m+1}+1 \}, \\
L_{[h_1,h_2]} &= \sum_{i=0}^{\infty} (k_{2i} - k_{2i+1}), \\
L_{[h_a+1,h_2]} &= \sum_{i=0}^{\infty} (k_{2i+1} - k_{2i}).
\end{align*}
\]

In order to derive our delay distribution dependent stability criterion, the following assumption is made for the time-varying delay function \( h(k) \).

Assumption 1. For the interval time-varying delay \( h(k) \) that satisfies (2), there exists positive scalar \( \gamma < +\infty \) such that

\[
\frac{L_{[h_a+1,h_2]}}{L_{[h_1,h_2]}} \to \gamma, \quad k \to +\infty.
\]

Then by letting \( \Delta V(x(k)) > 0 \) to some extent for the value of time-varying delay in subinterval \( [h_a+1,h_2] \), we can get the following Theorem.

**Theorem 2.** For given scalar \( \gamma > 0 \) and integers \( 0 < h_1 < h_a < h_2 \), choosing \( 0 < \alpha < 1 \) and \( \beta > 0 \) such that (16) is satisfied, system (1) with interval time-varying delay satisfying (2) and (15) is stable if there exist symmetric positive matrices \( P, Q, R_0, R_1, R_2, T_1, T_2 \) and matrices \( T_{i}(i=1,2) \) such that the following LMIs (17-18) hold:

\[
\begin{align*}
(1+\beta)^{\gamma}(1-\alpha) &< 1, \\
\Pi_0(\alpha) + \Pi_1(\alpha) &< 0, \\
\Pi_0(-\beta) + \Pi_2(-\beta) &< 0,
\end{align*}
\]

where

\[
\Pi_0(\beta) = \Pi_0(-\beta) + \Delta,
\]

\[
\Delta = (1+\beta)^{\gamma} \beta^3 e_4^T Q e_5 - (1+\beta)^{\gamma} \beta^3 e_3^T Q e_5 + (1+\beta)^{\gamma} \beta^2 e_4^T Q e_4
\]

\[
- (1+\beta)^{\gamma} \beta e_3^T Q e_4.
\]

**Proof.** From (19) we have \( \Pi_0(-\beta) < \Pi_0(-\beta) \). Then if (17) and (18) hold, by letting \( \rho = \alpha \) in (8) and \( \rho = -\beta \) in (9), for the value of \( h(k) \) in the first subinterval we can get

\[
V(x(k+1)) < (1-\alpha)V(x(k)),
\]

and for the value of \( h(k) \) in the second subinterval we can get

\[
V(x(k+1)) < (1+\beta)V(x(k)).
\]

From (15) and (16) we have

\[
\exists \epsilon > 0, \quad K > k_0, \quad \text{such that}
\]

\[
(1+\beta)^{\gamma+\epsilon}(1-\alpha) < 1,
\]

and

\[
\frac{L_{[h_a+1,h_2]}}{L_{[h_1,h_2]}} < \gamma + \epsilon, \quad k \geq K.
\]

Since

\[
L_{[h_1,h_2]} + L_{[h_a+1,h_2]} = k - k_0,
\]

from (23-24) we can get

\[
L_{[h_1,h_2]} > \frac{k - k_0}{\gamma + \epsilon}, \quad \text{and} \quad L_{[h_a+1,h_2]} < \frac{(\gamma+\epsilon)(k-k_0)}{\gamma + \epsilon + 1},
\]

then from above inequalities together with (20-22) we have

\[
V(x(k)) < (1+\beta)^{\gamma+\epsilon}(1-\alpha)\frac{L_{[h_1,h_2]}}{T_{1}^T T_{1}} V(x(k_0))
\]

\[
< (1+\beta)^{\gamma+\epsilon}(1-\alpha)\frac{k - k_0}{\gamma + \epsilon + 1} V(x(k_0))
\]

\[
= [(1+\beta)^{\gamma+\epsilon}(1-\alpha)]\frac{k - k_0}{\gamma + \epsilon + 1} V(x(k_0)).
\]

From it we can conclude that \( V(x(k)) \to 0 \) as \( k \to +\infty \). The proof is completed.

**Remark 2.** For getting Theorem 2, the terms \(- (1+\beta)^{\gamma} \beta e_3^T Q e_5 \) and \( -(1+\beta)^{\gamma} \beta^2 e_4^T Q e_4 \) are enlarged as \(- (1+\beta)^{\gamma} \beta e_3^T Q e_5 \) and \( -(1+\beta)^{\gamma} \beta e_3^T Q e_4 \) in (19), thus the obtained LMIs (17-18) are monotone increasing when \( h_2 \) increases.

Which means, if there are feasible solutions for a given \( h_2 \), there are feasible solutions for \( h_2 < \tilde{h}_2 \).

**Remark 3.** It is generally assumed that large delay may cause system to be unstable, hence we let the difference of
Lyapunov functional have positive upper bound for the value of time-varying delay in the large delay interval in Theorem 2. For a given γ, it is easier to getting feasible solutions for LMIs (17)-(18) with the increasing β or decreasing α. So the larger value of γ may allow the existence of less conservative results. Fig. 2 shows an image of ψνγ = 1

\[ \psi^2 \gamma = 1 \]

Corollary 1

\[ \psi^2 \gamma = 1 \]

IV. NUMERICAL EXAMPLE

In this section, we consider one example to show the effectiveness of the obtained criteria.

Example 1. Consider the system (1) with

\[ A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}. \]

In the situation that the distribute information of time-varying delay is unknown, we compare our obtained asymptotic stability criterion to that of Ref.[8]. Some results are listed in Table 1, in which the value of \( h_u \) used for getting the admissible upper bound \( h_2 \) are listed in the last line. From it we can see that our method can get better results for small value of \( h_1 \). Moreover, the number of decision variables (ndv) used in Ref.[8] is \( \frac{5}{2}R \) with different value \( R \) and \( \alpha, \beta \) are listed in Table 2. From it we can see that for a fixed \( h_u \), the smaller value of \( \beta \) may allow a bigger upper bound of time-varying delay. While for different fixed pair of \( \alpha, \beta \), the optimal value of \( h_u \) may be different. Furthermore, by using Theorem 2 with \( h_u = 10 \) and \( (\alpha, \beta) = (0.001, 0.5) \), we can get max \( h_2 > 1750 \). We can’t get the

\[ \Xi(\beta) = (h_{12} + 1)e_1^T Q e_1 - (1 - \rho)e_1^T P e_1 - (1 - \rho)^2 e_2^T Q e_2 \]

After applying the similar transformation to (6), by using the following inequality [18]

\[ 2u^T v \leq \frac{1}{\hat{\epsilon}} u^T \hat{\epsilon} u + \hat{\epsilon} v^T \hat{\epsilon} v, \]

and Schur Complement, we can obtain Theorem 3.

\[ \Xi(\beta) + \frac{\rho}{\rho+1} v^T \Xi(\beta) v < 0. \]

TABLE I

<table>
<thead>
<tr>
<th>( h_1 )</th>
<th>4</th>
<th>12</th>
<th>16</th>
</tr>
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<tr>
<td>( h_{12} ) (m = 2)</td>
<td>17</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>( h_{12} ) (m = 4)</td>
<td>18</td>
<td>22</td>
<td>25</td>
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<tr>
<td>Corollary 1</td>
<td>19</td>
<td>23</td>
<td>24</td>
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<tr>
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exact value of admissible upper bound $h_2$ by using Matlab Toolbox for solving LMIs in this case. Since by increasing $h_2$, the value of $(1 + \beta)^{h_2}$ that appeared in $\lambda_2(\beta)$ gets large enough such that “NaN” value appeared. How to avoid this phenomenon needs our further research.

From Table 2 we can see that system (1) is stable with interval time-varying delay satisfying $\gamma = 0.1$, $h_1 = 4$, $h_u = 10$ and $h_2 = 51$. This type of time-varying delay $h(k)$ and the corresponding system’s state response under initial condition $x(0) = [5, -1]^T$ and $\phi(k) = [2, 2]^T$ are depicted in Fig. 3 and Fig. 4, respectively, which show the effectiveness of our obtained criterion.

### Table II

<table>
<thead>
<tr>
<th>$h_u$</th>
<th>6</th>
<th>10</th>
<th>15</th>
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<tbody>
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<td>$h_2(\gamma = 1)$ ($\alpha, \beta$)</td>
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<td>20</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(0.1, 0.1)</td>
<td>(0.01, 0.01)</td>
<td>(0.01, 0.01)</td>
</tr>
<tr>
<td>$h_2(\gamma = 0.1)$ ($\alpha, \beta$)</td>
<td>43</td>
<td>31</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>(0.01, 0.1)</td>
<td>(0.01, 0.1)</td>
<td>(0.01, 0.1)</td>
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<tr>
<td>$h_2(\gamma = 0.05)$ ($\alpha, \beta$)</td>
<td>149</td>
<td>246</td>
<td>181</td>
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<tr>
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<td>(0.01, 0.2)</td>
<td>(0.01, 0.2)</td>
<td>(0.01, 0.2)</td>
</tr>
</tbody>
</table>

### V. CONCLUSIONS

In this paper, the problem of stability analysis for discrete-time systems with interval time-varying delay has been investigated. By dividing the delay interval into two subintervals, exponential stability criterion and delay distribution dependent stability criterion are obtained. Numerical example is given to verify the effectiveness of our obtained criteria. The method used in this paper can be further extended to cope with the state and output feedback control problem for such systems.

### REFERENCES