Control Strategy For An Off-Grid Hybrid Stirling Engine/Supercapacitor Power Generation System

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Abstract—In this paper, a novel control strategy is proposed for a hybrid Stirling engine/supercapacitor power generation system functioning in isolated sites rich in sunshine. The controller should drive the system through its electrical part to satisfy the varying load demand and maintain the state of charge of the supercapacitor at an appropriate level. The system includes a thermodynamic machine (Stirling Engine) that drives a Permanent Magnet Synchronous Generator (PMSG). An appropriate power electronics is interfaced between this PMSG and the AC load to be supplied with the desired power. In order to meet the load transient demand, a supercapacitor is used and its delivered power is controlled by mean of a bidirectional DC/DC converter. As the testbed is under construction, a simulation scenarios are proposed to assess the efficiency of the proposed control architecture.

I. INTRODUCTION

Currently, more than 1.4 billion people have no access to electricity; essentially in India, Indonesia, Bangladesh, Nigeria and Sub-Saharan Africa. However their need for electricity is real and increasing. The fuel based generators (mainly Diesel engines) typically used in this context are not well adapted because of their low reactivity to changes in the demand level, their high pollution, noise, and their low efficiency [1].

In order to bring solutions to the energy access for those populations, Schneider Electric launched a program called BIP BOP[1] [2] that aims at bringing safe, clean and affordable electricity to populations that do not have access to it. Within the framework of this program, the MICROSOL project aims at developing micro solar thermodynamic plants producing a minimum of 150 kWhel/day functioning 24h/24 thanks to an appropriate thermal energy storage. The aim of this paper is to present the currently adopted solution that addresses one of the major challenges of this project, namely the design of the control strategy that enables the varying power demand to be satisfied.

In the literature, many results can be found concerning the control of hybrid power generation systems but most of them focus on wind turbines, solar photovoltaic or fuel cells systems as primary energy source. For instance, in [3] and [4], nonlinear controllers based on sliding mode techniques and passivity principles have been developed to control a hybrid power generation system with wind turbine as a primary energy source, however the system is based on battery banks as an energy buffer while a supercapacitor associated to a DC/DC converter is adopted in our system. In [5], a PID based control strategy is presented for the control of a hybrid wind turbine/supercapacitor system while in [6] another PID based controller is used to control a hybrid fuel cells/supercapacitors power system for electrical vehicle applications. In the present work, a nonlinear controller based on backstepping design and model predictive control for optimal performances is developed. Its principal advantages consist in having few design parameters and an interesting way to generate the references for the key state variables.

This paper is organized as follows: section II describes the system under study. In section III, the mathematical models of the system’s components are given. The control strategy is described in section IV while section V is dedicated to some simulation results and discussions. Finally section VI concludes the paper and gives some hints for future investigation.

II. DESCRIPTION OF THE SOLAR MICRO PLANT

The hybrid energy generation system under study with its associated controllers is depicted in Figure 1. The system comprises a thermodynamic machine (Stirling Engine), a Permanent Magnet Synchronous Generator (PMSG), a diode bridge, a full bridge DC/DC converter, a DC bus, a single phase rectifier, a bidirectional DC/DC converter, a supercapacitor and two controllers. The first controller uses the information on the power demand, the input temperatures of the hot and cold fluids flowing through the internal heat exchangers of the Stirling Engine (see [7]) and computes mass flow rate setpoints of these hot and cold fluids ensuring an optimal thermoelectric efficiency [8]. These two mass flow rates and the two input temperatures define the Stirling engine torque. This engine drives a PMSG that provides a variable frequency and amplitude voltage (not directly usable by the load) which is rectified by a diode bridge. A DC/DC full bridge converter is introduced between the rectified voltage and the constant DC bus voltage compatible with the functioning of the inverter that provides the AC load with a 230V 50Hz voltage. In order to dynamically meet the load demand, the Stirling engine alone is not sufficient because of its high mechanical and thermal inertia. That is the reason why a supercapacitor is used to provide the energy needed during load power

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III. MODELLING OF THE ENERGY CONVERSION SYSTEM

A. Rotor dynamic

The dynamic of the rotor’s rotational speed $\Omega(t)$ is defined by the motor torque of the Stirling Engine $T_{mot}$ and the electromagnetic torque $T_{em}$ of the PMSG. Previous analysis [8] showed that the output torque of the Stirling engine can be expressed by:

$$T_{mot} = \alpha(\dot{m}_h, \dot{m}_c) \Omega(t) + \beta(T_h^{in}, T_c^{in})$$

where $\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of the hot and cold fluids at the inlet of the Stirling Engine, $T_h^{in}$ and $T_c^{in}$ are their respective temperatures. $\alpha(\cdot)$ and $\beta(\cdot)$ are functions that can be derived from experimental or knowledge based scenarios [8]. The dynamic of the mechanical rotational speed of the rotor is given by:

$$J_{rot} \frac{d\Omega(t)}{dt} = T_{mot} - T_{em} - T_{fr} \quad T_{fr} = D_{fr} \cdot \Omega(t)$$

where $D_{fr}$ is the friction coefficient of the rotor and $T_{fr}$ designates the friction torque. $J_{rot}$ is the total inertia of the rotating parts.

The control objectives is to regulate the DC bus voltage at the reference $V_{bus}^{ref} = 50V$ while maintaining the supercapacitor voltage at the nominal value of 125 V in a somewhat slower manner. In order to achieve those objectives, the duty ratios of the DC/DC full bridge and the bidirectional DC/DC converters have to be used as control variables. It is worth underlying that in the work described in the present paper, the two mass flow rates of the hot and cold fluids are not used as control variables but are considered as constant. Only the control of the electrical stage is considered here. Obviously, the two parts show separable time scales so that the solutions proposed in this paper can be reused in a global architecture that manages to monitor the efficiency of the Stirling engine through the manipulation of these two flow rates.

B. Association of the PMSG and the diode bridge

In the literature, the equivalence between the PMSG associated with a diode bridge and a DC machine is often used in order to obtain a model dedicated to the control design (see [9]). The equations describing the association of the PMSG and the diode bridge are given by:

$$\frac{dI_{red}}{dt} = \frac{R_s}{L_s} I_{red} - \frac{3}{2\pi} \Omega(t) I_{red} + \frac{3\sqrt{3}}{2\pi L_s} E_{m}(t)$$

where $E_{m}(t) = p \cdot \Omega(t) \cdot \phi_f$, $T_{em} = p \cdot \frac{3\sqrt{3}}{2\pi} \phi_f I_{red}$ while $V_{red}$ is the rectified voltage, $I_{red}$ is the output current of the diode bridge, $T_{em}$ designates the electromagnetic torque of the PMSG, and $E_{m}(t)$ its EMF (electromotive force). $R_s$ and $L_s$ are the stator resistance and the stator self-inductance respectively, $p$ is the number of pole pairs of the PMSG and $\phi_f$ the EMF constant of the motor.

C. bidirectional DC/DC converter and supercapacitor

The bidirectional DC/DC converter is a current reversible DC/DC converter. It can work as a buck converter when the current (of the bidirectional DC/DC converter inductance) flows from the supercapacitor to the DC bus. It works as boost converter when the current flows on the opposite direction. Hereinafter, it is considered that the current is positive when it flows from the supercapacitor to the DC bus. In order to avoid the discontinuous conduction mode of this converter, a complementary control (see [10]) of this device is preferred, where, during a switching period, both Insulated Gate Bipolar Transistors (IGBT) are used (but not simultaneously). Assuming that such precaution is taken in the hardware, the mean model (see [10]) used in the control
design can be given by:
\[
L_{bb}. \frac{di_{L_{bb}}}{dt} = V_{sc}.\alpha_{bb} - V_{bus}
\] (2a)
\[
C_{sc}. \frac{dV_{sc}}{dt} = -i_{L_{bb}}.\alpha_{bb}
\] (2b)
where \(\alpha_{bb}\) is the duty ratio of the bidirectional DC/DC converter, \(L_{bb}\) is the output inductance of this converter, \(C_{sc}\) is the supercapacitor capacitance, \(V_{bus}\) is the DC bus voltage, \(i_{L_{bb}}\) is the output current of the bidirectional DC/DC converter and \(V_{sc}\) is the supercapacitor voltage.

D. DC/DC full bridge converter

The DC/DC Full bridge converter illustrated in Figure 3 is an isolated buck converter. It consists of two IGBT legs at the primary of the transformer and a diode full-wave rectifier at the secondary of the transformer [11]. The mean average model of this converter is given by:
\[
L_{fb}. \frac{di_{fb}}{dt} = k.V_{red}.\alpha_{fb} - V_{bus}
\] (3a)
\[
C_{f}. \frac{dV_{red}}{dt} = I_{red} - I_{fb}^n
\] (3b)
\[
I_{fb}^n = k.i_{L_{fb}}.\alpha_{fb}
\]
where \(\alpha_{fb}\) is the duty ratio of the Full bridge converter, \(L_{fb}\) is the output inductance of the DC/DC full bridge, \(C_{f}\) is the value of the input capacitor, \(k\) is the transformer winding ratio, \(i_{L_{bb}}\) is the output current of the DC/DC full bridge and \(I_{fb}^n\) is the current at the input of the converter.

E. DC bus equation

The equation of the DC bus is given by:
\[
C_{tot}. \frac{dV_{bus}}{dt} = i_{L_{fb}} + i_{L_{bb}} - I_{inv}
\] (4a)
\[
C_{tot} = C_{bb} + C_{fb} + C_{inv}
\] (4b)
where \(C_{bb}\) and \(C_{fb}\) are the values of the output capacitors of the bidirectional DC/DC converter and DC/DC full bridge converter respectively, \(C_{inv}\) is the value of the input capacitor of the rectifier and \(C_{tot}\) is the equivalent capacitance of the three capacitors in parallel. \(I_{inv}\) denotes the input current of the rectifier.

F. modelling of the single phase rectifier

The single phase rectifier is modelled by a simple static equation that reflects power conservation between the input of the rectifier and its output which is connected to the load that demands a power \(P_L\). This situation represents the worst case for the controller since a step change in \(P_L\) will be reflected at the input of the rectifier as a step on the power drawn by this rectifier. This leads to:
\[
P_L = \eta_{inv}.V_{bus}.I_{inv}
\] (5)
where \(\eta_{inv}\) is the electrical efficiency of the rectifier.

G. State space model of the power generation system

Gathering all the equations described so far leads to the following equations in the state space form:
\[
\dot{x}_1 = a_1.x_1 - a_2.x_2 + a_2
\] (6a)
\[
\dot{x}_2 = -a_4.x_2 - a_5.x_1.x_2 + a_6.x_1 - a_7.x_3
\] (6b)
\[
\dot{x}_3 = a_8.x_2 - a_9.k .x_3.u_1
\] (6c)
\[
\dot{x}_4 = -a_9.x_5 + k.a_9.x_3.u_1
\] (6d)
\[
\dot{x}_5 = a_{10}.(x_4 + x_6) - \frac{a_{10}}{\eta_{inv}}.P_L
\] (6e)
\[
\dot{x}_6 = -a_{11}.x_5 + a_{11}.x_7.u_2
\] (6f)
\[
\dot{x}_7 = -a_{12}.x_6.u_2
\] (6g)
where \(x_1 = \Omega, x_2 = I_{red}, x_3 = V_{red}, x_4 = i_{L_{fb}}, x_5 = V_{bus}, x_6 = i_{L_{bb}}, x_7 = V_{sc}\). The control variables are: \(u_1 = \alpha_{fb}\) and \(u_2 = \alpha_{bb}\) corresponding to both duty ratios of the DC/DC full bridge and bidirectional DC/DC converter respectively. The coefficients \(a_i\) used in the state equations are given by:
\[
a_1 = \frac{J_{rot}}{\alpha(m_a.m_c-D)J_{rot}}, a_2 = \frac{J_{rot}}{\alpha(TH_a.m_c.C_{tot})}, a_3 = \frac{p.3.\sqrt{3}.\Phi_r}{\pi \pi J_{rot}}, a_4 = \frac{R_s}{L_x}, a_5 = \frac{3p}{2.\pi}, a_6 = \frac{p.3.\sqrt{3}.\Phi_r}{2.\pi L_x}, a_7 = \frac{1}{TC_{rot}}, a_8 = \frac{1}{TC_{rot}}, a_9 = \frac{1}{L_{fb}}, a_{10} = \frac{1}{C_{tot}}, a_{11} = \frac{1}{C_{sc}}, a_{12} = \frac{1}{\eta_{inv}}.
\]

IV. CONTROLLER DESIGN

A. Control objectives

Based on the notation above, the operational control objectives become:
- regulate \(x_5=V_{bus}\) around \(x_5^r=V_{bus}^r=50V\)
- maintain \(x_7=V_{sc}\) around \(x_7^r=V_{sc}^r=125V\)

This system is also subject to the following constraints:
- positivity constraints \(x_i \geq 0\) except \(x_6\): Indeed, the bidirectional DC/DC converter is a current reversible converter, therefore \(x_6 = i_{L_{bb}}\) can be either positive or negative. However, since the DC/DC Full Bridge is not reversible, its output current is always positive, therefore, its average current value must be greater than half of the ripple current in order to avoid discontinuous conduction mode, in which case the differential equations become more complicated (see for example [12]).
- strong saturations on control inputs since:
  - \(u_1 \in [0, 1]\): duty ratio of the DC/DC Full Bridge.
  - \(u_2 \in [0, 1]\): duty ratio of the bidirectional DC/DC converter.
However, the eigenvalues of $A(u_1)$ contain highly oscillatory modes resulting in oscillatory open-loop state trajectories that would almost systematically lead to positiveness constraints violation. Then, simple static model inversion (computing $u^{st}$ given the desired state $z^{st}$) will not stabilize the system without violating the constraints. The control law used to stabilize $z$ at some $z^{st}$ (defined by $u^{st}_1$ as it is shown later) is a one step\textsuperscript{2} predictive control \cite{13}, namely:

$$u_1^{opt} = \arg \min_{u_1 \in [0,1]} J(u_1) := \left[ \|z^+(u_1) - z^{st}\|^2_{P_0(u^{st})} \right]$$

(9)

where $P_{0}(u^{st})$ is the Lyapunov stability matrix of subsystem 1 (depending on $u^{st}_1$). Recall that the optimal problem (9) is defined for a given $u^{st}_1$ which is defined once $x_4^{st}$ is known (See Figure 4). The way $x_4^{st}$ is defined is explained in the sequel. In order to solve (9) during the short sampling period, the differential equation (7) is approximated by a second order approximation of the discretized equation:

$$z^+(u_1) = A_d(u_1)z + u_0(u_1)$$

The resulting scalar optimal problem can then be solved by standard SQP(Sequential Quadratic Programming)\textsuperscript{iterations}. The possibility to solve this scalar optimal problem has been assessed on a real time target with a sampling period of 100 $\mu$s.

\textbf{D. analysis and control of subsystem 2}

Recall that subsystem 2 is given by equations (6e)-(6g). Let us use a backstepping approach \cite{14} to derive a feedback law that stabilize $x_5=V_{bus}$ around $x_5^{ref}=V_{ref}=50V$. In order to do this, the variable $x_6$ is first used as a virtual control signal to regulate $x_5$ around its setpoint leading to a desired value for $x_6$ namely $x_6^{ref}$. The control input $u_2$ is then used to force $x_6$ to track its reference $x_6^{ref}$. This results in the following feedback law:

$$u_2 = \frac{1}{a_{11}x_7}(-a_{11}x_5-a_{10}(x_5-x_5^{st})+x_6^{ref}-\lambda_6(x_6-x_6^{ref}))$$

(10)

where

$$x_6^{ref} = \frac{P_L}{\eta_{inv}}x_4 + \frac{\lambda_5}{a_{10}}(x_5^{st} - x_5)$$

(11)

$\lambda_5$ and $\lambda_6$ are the design parameters of the backstepping controller. Since the backstepping controller given by equations (10) and (11) enables the system to rapidly reach the stationary (reference) value for $x_5$, it follows that one rapidly reaches $x_5 \approx 0$, leading to:

$$x_6 = -x_4 + \frac{P_L}{\eta_{inv}\cdot x_5^{st}}$$

(12)

This last equation suggests that it is possible to regulate the stationary value of $x_6$ via $x_4$ in a somewhat slow manner to charge or discharge the supercapacitor to maintain its voltage because of the limitation on the sampling time that is fixed to 100 $\mu$s.
Fig. 5: Global control architecture for the complete system.

at a desired value $x_7^{st} = 125V$. This can be done by asking subsystem 1 to adopt the following reference value for $x_4$:

$$x_4^{st} = -k_6 \cdot \tanh(\beta.(x_7 - x_7^{st})) + \frac{P_L}{\eta_{tanh}x_6^{st}} \quad (13)$$

since according to (12), this would lead to a stationary value of $x_6$ given by:

$$x_6^{st} = k_6 \cdot \tanh(\beta.(x_7 - x_7^{st}))$$

which obviously [see (6g)] stabilizes $x_7$ at $x_7^{st}$.

E. Global control of the system

The whole control architecture described so far is depicted in Figure 5. This Figure shows a final modification of the control architecture according to which $x_4^{st}$ is filtered in order to avoid negative values of $x_4$ during transients. It can be rigorously shown that there is sufficient low pass filter such that this positivity constraint is satisfied. The arguments are not detailed here because of the lack of space.

It is worth noting that even if the optimal controller of subsystem 1 given by (9) will be affected by some model uncertainties leading to tracking error of the variable $x_4(t)$ around $x_4^{st}$, this will not affect significantly the global performances of the system. Indeed, this tracking error will force $x_7(t)$ to deviate from $x_7^{st}$ leading $x_4^{st}$ to adapt accordingly [see (13)].

V. SIMULATION RESULTS

In order to validate the control architecture, the closed-loop system is simulated under step changes in the power demand of the AC load as shown in Figure 6. One can see from Figure 8a that the proposed controller tightly regulates the DC bus voltage to its set-point value, allowing a good behavior of the rectifier (that supplies the AC load with 230V 50Hz voltage). Figure 8b shows that the controller also maintains the state of charge of the supercapacitor (through its voltage) at its nominal reference value of 125V allowing the system to always have a sufficient amount of energy during an increase of the load power demand. The nominal setpoint value of the supercapacitor voltage is also chosen to permit absorption of the extra energy during a decrease in the load power demand.

Figure 7a gives the mechanical rotational speed of the rotor and its stationary desired value $x_1^{st}$. One can see that the controller (especially the one step predictive controller for subsystem 1) always stabilizes this state variable to its desired stationary value (which depends on the load power demand).

In this figure, the current respects the positiveness constraint and is significantly above zero allowing a continuous conduction mode of this converter as required.

In Figure 7c, one can see the output current of the bidirectional DC/DC converter $i_{L fb}$ and its stationary desired value $x_4^{st}$. In this figure, the current respects the positiveness constraint and is significantly above zero allowing a continuous conduction mode of this converter as required.

VI. CONCLUSION AND FUTURE WORK

In this paper, the problem of satisfying in real time the load demand in isolated sites through a hybrid Stirling
The design parameters used for the controller are given by: \( \lambda_5 = 600, \lambda_6 = 6000, k_6 = 20, \beta = 0.5 \).

**REFERENCES**


**APPENDIX**

The hybrid power generation system studied was simulated with the following characteristics: PMSG nominal power: 1.8 kW, \( J_{rot}=0.133 \text{ Kg.m}^2 \), \( D_f=0.0002 \), \( \alpha(m_h,m_e)=-0.0111, \beta(TH_{in}, TC_{in})=37.4971 \), \( \phi_f=0.6014 \), \( R_e=5 \), \( L_s=0.28 \text{ mH} \), \( p=8 \), \( C_f=2350 \mu F \), \( L_{in}=1.5 \text{ mH} \), \( L_{bb}=223 \mu H \), \( C_{fb}=0.5 \mu F \), \( C_{bb}=4 \times 340 \mu F \), \( C_{inv}=200 \mu F \), \( C_{SC}=6 \mu F \).

The design parameters used for the controller are given by: \( \lambda_5=600, \lambda_6=6000, k_6=20, \beta=0.5 \).