Abstract—In this work, control system design and implementation for active vibration control (AVC) of flexible structures with piezoelectric actuators is studied. The goal is to reduce the effect of harmonic disturbances with known time-varying frequencies acting on a system. As a test bed, a thin flexible aluminum cantilevered beam with two symmetrically bonded piezoelectric actuators is used. The harmonic excitation is generated by two DC motors each of them with an unbalanced mass. A discrete-time model is obtained through black-box system identification methods. The control algorithm design is based on a plant description with a disturbance model as a linear parameter varying (LPV) system in linear fractional transformation (LFT) form. This results in a gain-scheduled controller where the harmonic disturbance frequencies are the scheduling variables. The experimental real-time results show the effectiveness of the controller and its capability to suppress time-varying harmonic disturbances, whose frequencies are measured directly from the DC motors. The design method leads to a controller that stabilizes the closed-loop system even for arbitrarily fast changes in the disturbance frequencies. In the real-time experiment, the controller suppresses a disturbance consisting of two independent harmonics with frequencies that vary over a range of 15 Hz.

I. INTRODUCTION

The rejection of harmonic disturbances with time varying but known frequencies is a specific problem often encountered in AVC applications. On the other hand, lightweight flexible structures are easily prone to vibrate due to external forces or due to forces generated in the inner structure. A possible countermeasure is the use of active control techniques. For lightweight flexible structures, piezoelectric materials, embedded or bonded on the structure, are often used as actuators for active vibration control because they are light and can be easily integrated into the structures without significantly changing the structural dynamics of the system [1].

In industrial environments, the disturbances generated by machinery are often harmonic (e.g. automotive applications and aircrafts) or transient (impact forces or payload variations). In this work, the time-varying frequencies of two harmonic disturbances are measured and used for its rejection with an AVC system. The goal of this control system is to reduce the effect of harmonic disturbances with measured time-varying frequencies (modification of the forced harmonic response). Possible applications for such control systems could be flexible robotic structures that have rotating machinery as payloads.

For the suppression of harmonic disturbances adaptive methods are often used, for example the FxLMS (filtered-x least mean square) [2]. This approach was used in [3,4] to attenuate the response of a flexible beam. The FxLMS approach often works well in practice but might suffer from disadvantages like slow convergence and poor tracking performance. For time-varying frequencies the analysis of such an adaptive system is difficult because of its adaptive nature. Also, to date, only “approximate stability results” are available for the FxLMS algorithm [2].

Another alternative is feedback control. For good disturbance rejection, the feedback controller has to include a model of the disturbance [5]. This can be achieved, for example, through observer-based state-feedback controllers [6-9]. For practical real-time implementations it is suitable to carry out the control design in discrete time [6-14]. A discrete-time observer gain-scheduling controller was implemented in [6-8] and a discrete-time state feedback gain-scheduling controller in [8,9]. LPV control design techniques can also be used for AVC [7-14]. In [15], a continuous-time LPV approach was used for active control of a harmonic disturbance for a cantilevered flexible beam. However, in [15], the mass of the beam was the time-varying parameter and a harmonic disturbance with a constant frequency was considered. Here, the case of a harmonic disturbance with time-varying frequencies is treated (and the plant itself is time invariant).

The work presented here is motivated by [10-14], where the harmonic disturbances are suppressed by means of a discrete-time $H_\infty$-suboptimal controller for LPV-LFT systems. This approach is also used in this paper for AVC of a flexible beam.

To the best authors knowledge it is a novel application, where the harmonic disturbance is rejected by means of patch piezoelectric actuators with an LPV approach and the disturbance frequencies are directly measured from the motors.

Usually, AVC systems including piezoelectric actuators exhibit significant nonlinear behavior (actuator saturation, hysteresis [16]). However, for the application considered here, a linear model was suitable, partially due to the fact that the disturbance amplitudes were sufficiently small.

The remainder of this paper is organized as follows. In Sec. II, the method to model the disturbances and to build the generalized plant is discussed. The design procedure to calculate the gain-scheduling controller is explained in Sec. III. The experimental setup is explained in Sec. IV and
results are shown and discussed in Sec. V. Finally conclusions are given in Sec. VI.

II. LPV-LFT CONTROL DESIGN

An LPV system in LFT form is shown in Fig. 1. It consists of a generalized plant $G(z)$ that includes input and output weighting functions and a parametric uncertainty block $\theta_k$. For this general system, a gain-scheduling controller can be calculated following the method presented in [17]. In this method, two sets of linear matrix inequalities (LMIs) are solved. The first set of LMIs determines the feasibility of the problem which means that a bound on the control system performance in the sense of the $H_{\infty}$ norm can be satisfied. With the second set of LMIs, the controller matrices are calculated from the solution of the first set of LMIs.

![Fig. 1. General LPV system.](image)

As a result of applying this control design method, the gain-scheduling control structure of Fig. 2 is obtained. The time-varying plant parameter is directly used as the gain-scheduling parameter of the controller. This approach has been used in [10-14] and is also employed in this paper. The gain-scheduling parameter $\theta_k$ is calculated from the disturbance frequency. This is described in the following section.

![Fig. 2. LPV gain-scheduling control structure.](image)

III. DESIGN PROCEDURE

In this section, it is described how the design method outlined in the previous section can be applied for the rejection of harmonic disturbances. For this, it is necessary to combine the plant and a disturbance model and transform this to the LPV-LFT form shown in Fig. 1.

A signal consisting of $n$ harmonic disturbances with varying frequencies can be modeled as the output of the state-space model given as [12]

$$
\begin{align*}
\dot{x}_{\delta,k+1} &= A_{\delta}x_{\delta,k} + B_{\delta,w}w_{\delta,k} + B_{\delta,u}u_{\delta,k}, \\
q_{\delta,k} &= C_{\delta,w}x_{\delta,k}, \\
y_{\delta,k} &= C_{\delta,y}x_{\delta,k}, \\
w_{\delta,k} &= \theta_k q_{\delta,k},
\end{align*}
$$

with

$$
A_{\delta} = \begin{bmatrix} A_{\delta,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_{\delta,n} \end{bmatrix}, \quad A_{\delta,1} = \begin{bmatrix} 0 & 1 \\ -1 & \bar{\alpha} \end{bmatrix},
$$

$$
\bar{\alpha} = \cos(2\pi f_{i,max} T) + \cos(2\pi f_{i,min} T),
$$

$$
B_{\delta,w} = \begin{bmatrix} B_{\delta,w,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_{\delta,w,n} \end{bmatrix}, \quad B_{\delta,w,1} = \begin{bmatrix} 0 \\ \vdots \end{bmatrix},
$$

$$
p_i = \cos(2\pi f_{i,max} T) - \cos(2\pi f_{i,min} T),
$$

$$
C_{\delta,y} = \begin{bmatrix} C_{\delta,y,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_{\delta,y,n} \end{bmatrix}, \quad C_{\delta,y,1} = \begin{bmatrix} 0 & 1 \end{bmatrix},
$$

and

$$
\theta_k = \begin{bmatrix} \theta_{1,k} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \theta_{n,k} \end{bmatrix}, \quad \theta_{i,k} \in [-1, 1], \ i = 1, 2, \ldots, n.
$$

The sample time is denoted by $T$ and the effect of the time-varying frequencies can be described as

$$
\alpha_i(f) = 2\cos(2\pi f_i T) = \bar{\alpha}_i + p_i \theta_k(f)
$$

which is the approximation made in order to turn the original system into an LPV system.

To model a harmonic disturbance, an autonomous state-space model would suffice. Here, an input signal $w_{\delta}$ has been included to utilize the norm-optimal control framework. The time-invariant part of the disturbance model is denoted by $G_{\delta}(z)$ and the disturbance model then represents an LPV system in linear fractional transformation (LFT) form between $G_{\delta}(z)$ and $\theta_k$. 

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The disturbance model used here is only correct for constant frequencies. This is because the rate of change of the frequency is not included in the model. To correctly account for time-varying frequencies, a different model with a system matrix of the form

\[
A_i = \begin{bmatrix} \cos(2\pi f_i T) & \sin(2\pi f_i T) \\ -\sin(2\pi f_i T) & \cos(2\pi f_i T) \end{bmatrix}
\]

would have to be used. Nevertheless, the “incorrect” model is used here since it is slightly simpler. This does not affect the performance of the system for constant disturbance frequencies or the closed-loop stability. It might affect the disturbance attenuation for fast changes in the disturbance frequency. In most practical applications with time-varying frequencies, however, the measured frequency will not correspond exactly to the instantaneous frequency due to measurement delays and the transmission of the disturbance to the plant. It is then unclear whether using the correct model would result in better performance.

Assuming that the disturbance enters at the plant input, the state-space model of the plant is

\[
x_{h,k+1} = A_p x_{h,k} + B_p u_{h,k} + B_{d,k} y_{d,k},
\]

\[
y_{h,k} = C_p x_{h,k}.
\]

The plant and the disturbance model can then be combined as shown in Fig. 3, where \(G_d(z)\) and \(G_d(z)\) are the transfer functions of the plant and the time-invariant part of the disturbance model, respectively.

The state space model of this combined system is

\[
x_{h,k+1} = Ax_{h,k} + Bu_{h,k} + Bu_{d,k} w_{d,k},
\]

\[
y_{h,k} = Cx_{h,k}.
\]

with

\[
A = \begin{bmatrix} A_p & B_p C_{d,p} & 0 & 0 \\ 0 & A_{d,p} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{d,p} \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 \\ B_{d,p} \\ C_p \\ 0 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},
\]

\[
D = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.
\]

\[
G_p(z) = \begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix}.
\]

The disturbance model, the plant and the additional weighting functions are combined as shown in Fig. 4. The output of the plant and the control signal are weighted by

\[
W_u = \begin{bmatrix} A_{w_u} & B_{w_u} \\ C_{w_u} & D_{w_u} \end{bmatrix},
\]

\[
W_q = \begin{bmatrix} A_q & B_q \\ C_q & D_q \end{bmatrix},
\]

and

\[
W_y = \begin{bmatrix} A_{w_y} & B_{w_y} \\ C_{w_y} & D_{w_y} \end{bmatrix}.
\]

respectively. The state-space description of \(G(z)\) of the generalized plant in Fig. 4 is given as

\[
\begin{bmatrix} x_{h,k+1} \\ q_{h,k} \\ \theta \\ y_{h,k} \end{bmatrix} = \begin{bmatrix} A & B & B_p & B_d \\ C_p & D_p & D_{w_p} & D_{w_d} \\ C_q & D_q & D_{w_q} & D_{w_d} \end{bmatrix} \begin{bmatrix} x_k \\ q_k \\ \theta_k \\ y_k \end{bmatrix},
\]

\[
\begin{bmatrix} w_{h,k} \\ w_{q,k} \\ w_{\theta,k} \end{bmatrix} = \begin{bmatrix} w_{u,k} \\ w_{d,k} \end{bmatrix},
\]

with

\[
x_k = \begin{bmatrix} x_{p,k}^T \\ x_{d,k}^T \\ x_{w,q,k}^T \\ x_{w,y,k}^T \\ x_{w_{\theta,k}}^T \end{bmatrix}^T,
\]

\[
A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{d} & B_{d,p} & C_{w_d} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 & 0 & B_{w_p} \\ B_{w_q} & B_{d,q} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_{w_d} \end{bmatrix},
\]

\[
C = \begin{bmatrix} 0 & 0 & 0 \\ D_{w_p, q} & D_{w_p, y} & D_{w_q, y} & D_{w_y, y} \end{bmatrix},
\]

\[
D = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.
\]
The overall structure of the LPV control system is shown in Fig. 4 and corresponds to the structure of Fig. 1.

![Fig. 4. LPV gain-scheduling structure with the model of the disturbance and the weighting functions.](image)

Eventually with the state-space LPV-LFT representation of the augmented plant a gain scheduling controller is calculated according to the procedure described in Sec. II.

IV. EXPERIMENTAL SETUP

To study the suitability of this design approach for AVC of flexible structures, a test bed was designed. The scheme of this experimental setup is shown in Fig. 5 and Fig. 6 and a photograph of the system in Fig. 7. It consists of a clamped aluminum beam with dimensions 400 x 35 x 2 mm, two piezoelectric actuators (P 876.A12 DuraAct), two piezo amplifiers (PI E413.D2), a rapid control prototyping hardware (dSPACE 1104) and an accelerometer (ASC 4411LN). An anti-aliasing filter is used for the signal of the accelerometer and two reconstruction filter for the output signal of the controller. The harmonic disturbance is generated using two DC motor-encoder combinations (Maxon Motors) and two four quadrant P+I speed velocity control (ESCON 36/2 DC). The amplitude of the disturbances can be calculated according to

\[ F_d = m_p r_0 \left(2\pi f_i\right)^2 \cos(2\pi f_i t) \]  

(31)

presenting extreme values for 0.123 N @ 40 Hz and 0.050 N @ 25 Hz.

The controller calculated in Sec. II uses the measured frequencies from the encoders and the signal from the accelerometer to generate the control signal.

The patch piezoelectric actuators are symmetrically placed to the flexural neutral axis of the beam in order to obtain a maximal applied moment, where the control signal is inverted for the other actuator. The longitudinal location of the actuators was defined according to the principle of maximal modal deformation [18].

V. EXPERIMENTAL RESULTS

The controller obtained from the design procedure of Sec. II with the disturbance model of Sec. III is validated experimentally on a test bed described in Sec. IV.

![Fig. 5. Schematic control architecture of the experimental setup.](image)

![Fig. 6. Schematic diagram of the experimental setup.](image)

![Fig. 7. Experimental test bed](image)

The discrete-time transfer function \( G_p(z) \) (12th order) between the output and the input of the control unit is obtained using standard black-box techniques for system identification. A sampling frequency of 1 kHz was chosen such that the Nyquist frequency of 500 Hz is well above the highest disturbance frequency.

The disturbance is the sum of two independent harmonic disturbances generated by two DC motors in the frequency range from 25 Hz to 35 Hz and 30 Hz to 40 Hz, respectively.
A first-order low pass filter is used as weighting function for the output. The control signal is weighted by a constant gain, as well as the disturbance input.

The open-loop and closed-loop amplitude frequency response for a disturbance with two fixed frequencies 30 Hz and 35 Hz are shown in Fig. 8. The amplitude frequency responses are scaled to acceleration over voltage, since the voltage is approximately proportional to the actuator moment.

Real-time result for the rejection of a disturbance with two harmonics of 30 Hz and 35 Hz, generated by the DC motors, is shown in Fig. 8. As expected from the amplitude frequency response, excellent disturbance rejection is achieved.

In order to demonstrate the effectiveness of the controller for the rejection of a disturbance with time-varying frequencies, the results of another experiment are shown. The change of the frequencies over time is shown in Fig. 10. The frequencies start in the middle of the ranges, increase to their maxima, then decrease to the minima and finally return to the initial values. The acceleration signals with and without control are shown in Fig. 11.

A third experiment was performed in order to show the capability of the controller to react and remain stable for fast changes in the frequencies of the harmonic disturbances (see Fig. 12 and Fig. 13).

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**Fig. 8.** Amplitude frequency response in open loop (gray) and in closed loop (black) for a fixed disturbance frequencies of 30 Hz and 35 Hz.

**Fig. 9.** Acceleration measured with the controller OFF-ON-OFF for a harmonic disturbance with two fixed frequencies 30 Hz and 35 Hz.

**Fig. 10.** Linear Frequency variations of the harmonic disturbances (black) and measured frequency variations (gray).

**Fig. 11.** Acceleration measured on the beam in open loop (gray) and closed loop (black) for a disturbance with linearly time-varying frequencies.

**Fig. 12.** Step linear frequency variations of the harmonic disturbances (black) and measured frequency variations (gray).
A very good rejection of the disturbance is obtained for the case of time-varying frequencies even when the two components of frequency are all at the same time. This has also been confirmed by other experiments.

VI. CONCLUSION

The experimental results show that excellent disturbance rejection is achieved for constant frequencies and for time-varying frequencies. The control approach is seen as a valuable alternative to adaptive filtering approaches. An advantage over adaptive filtering and other heuristic controller interpolation approaches [6] is that closed-loop stability is guaranteed. The gain-scheduling controller was calculated in discrete time, it provides a controller which is ready to implement in the hardware. Also this discrete-time design does not have the problems associated with discretization at each sampling instant of frequency distortion resulting from approximate discretization. The controller is capable to suppress time-varying disturbance for slow changes as well as for fast changes in the frequencies of the disturbance.

Further work in this area will be focused in the implementation of such AVC algorithm in a lightweight robotic structure of one degree of freedom.

REFERENCES


