Constrained Inner-Loop Control of a Hypersonic Glider Using Extended Command Governor

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Abstract—The paper considers inner-loop control of a hypersonic glider. To maintain vehicle flight, pointwise-in-time constraints on vehicle state and control variables need to be enforced. To enforce constraints, an Extended Command Governor (ECG) is augmented to the nominal LQ-PI controller. Simulation results demonstrate the capability of the ECG to enforce the constraints while minimizing the degradation in the vehicle responsiveness.

I. INTRODUCTION

Hypersonic vehicles are an area of great interest for the United States. Several experimental vehicles have been launched; DARPA’s Falcon Project being an example. Two test vehicles were launched under the Falcon Project; both flights ended unsuccessfully. It was determined that the main technical challenges are the assumptions from known flight regimes are inadequate and the advanced thermal modeling and ground testing are not successful at predicting reality [15].

Conventional control techniques appropriate for subsonic and supersonic flight are not necessarily applicable for hypersonic flight. A hypersonic vehicle requires a control system that can maintain stability and active damping during flight, be robust to uncertainties, and cater to structural effects [6]. Current research projects into hypersonic vehicles can benefit from an advanced guidance, navigation, and control system with capabilities of real time optimization. The capability of determining a guidance and control solution onboard the vehicle would be an enabling technology to achieve the DoD and NASA’s hypersonic vehicle goals.

Due to the uncertainties of hypersonic flight, predictive and adaptive control for hypersonic vehicles has been a major area of research. Predictive control was researched in Refs. [16], [4], and [5]. In Refs. [18] and [17], a Model Predictive Control (MPC) approach was used. In a companion paper [1], we implement an MPC approach for guidance of the hypersonic vehicle. Adaptive control has also been actively researched for control of a hypersonic vehicle [14], [7], [8].

This paper presents an inner-loop control scheme for the longitudinal dynamics of a hypersonic glider vehicle with constraint enforcement capability. For the inner-loop, angle-of-attack command tracking is achieved using a Linear Quadratic Proportional plus Integral (LQ-PI) controller. An Extended Command Governor (ECG) is augmented to the nominal closed-loop system to handle the constraints. The ECG is an advanced reference type governor algorithm that manipulates the command to enforce constraints. Reference governors have been an active area of research since the 1990s. They are add-on schemes used to enforce constraints in stable closed-loop systems. In the late 1990s, the theory was further developed to include command governors and, more recently, to also include extended command governors, both of which give a larger constrained domain of attraction and have a potential for a faster response than the reference governor. See Ref. [13] and references therein.

In Ref. [19], a reference governor was implemented to avoid the occurrence of saturation of the control system and prevent windup. It was augmented to an existing adaptive controller, which was designed to track reference signals. In Ref. [10], a reference governor was used to enforce constraints on elevator deflection for conventional UAV gliders. In this work, the ECG ensures that the implemented control will keep the vehicle in a safe operating region; its added benefit is an increase in the domain of attraction for the set of initial conditions, and the potential for improving system responsiveness.

II. LONGITUDINAL EQUATIONS OF MOTION FOR A FLEXIBLE HYPersonic GLIDER

The flexible aircraft model used is from Ref. [3]. The longitudinal equations of motion for a flexible aircraft are as follows

\[
\dot{\alpha} = \frac{-D}{mV_t} - g \sin \gamma \cos \alpha - g \sin \gamma, \quad (1)
\]

\[
\dot{\dot{\alpha}} = \frac{-D}{mV_t} + \frac{g}{V_t} \frac{\dot{V}_t}{r} \cos \gamma + Q, \quad (2)
\]

\[
\dot{Q} = M \frac{\dot{\gamma}}{\dot{\gamma}}, \quad (3)
\]

\[
\dot{\Theta} = Q, \quad (4)
\]

\[
\dot{h} = V_t \sin \gamma, \quad (5)
\]

\[
\dot{\eta}_i = -2\xi_i \dot{\eta}_i - \omega_i^2 \dot{\eta}_i + N_i, \quad i = 1, 2, \ldots, n, \quad (6)
\]

where \(V_t\) is the true airspeed, \(T\) is the thrust, \(D\) is the drag, \(L\) is the lift, \(\alpha\) is the angle-of-attack, \(\gamma\) is the flight path angle, \(M\) is the pitching moment, \(Q\) is the pitch rate, \(h\) is the altitude, and \(r\) is the radius of the vehicle. The flexible dynamics of the vehicle are modeled in Eqn. 6, where \(\eta_i\) is the \(i\)th modal coordinate of the flexible dynamics, \(\xi_i\) is the damping ratio, \(\omega_i\) is the natural frequency, and \(N_i\) is the \(i\)th
generalized force. The modal method and its derivation is more completely described in Ref. [3]. This model is useful for preliminary control analysis and provides insight in the interactions between the flexible vehicle dynamics and the implemented control. The thrust is set to zero in the gliding phase.

III. CONSTRAINTS

Given the model in the previous section, constraints were added to reflect requirements relevant to maintaining vehicle flight. The angle-of-attack, pitch rate, elevator deflection, elevator deflection rate, and flexible deflections were constrained. The elevator deflection is used to control the vehicle by changing the pressure distribution on the vehicle and thereby altering the lift and drag forces affecting the vehicle. The rate of elevator deflection is constrained to reflect the actuator physical limits and to ensure safe control of the vehicle by preventing control maneuvers that can lead to unstable flight. The angle-of-attack is constrained so the vehicle stays within its flight envelope during hypersonic flight. If large angle-of-attack values are allowed, the vehicle may transition to uncontrollable flight. These constraints are represented by

\[
\begin{align*}
\alpha_{\text{min}} & \leq \alpha \leq \alpha_{\text{max}}, \\
\delta_{e_{\text{min}}} & \leq \delta_e \leq \delta_{e_{\text{max}}}, \\
\Delta \delta_{e_{\text{min}}} & \leq \Delta \delta_e \leq \Delta \delta_{e_{\text{max}}}.
\end{align*}
\]

(7) (8) (9)

The pitch rate is constrained as follows

\[ Q_{\text{min}} \leq Q(t) \leq Q_{\text{max}}. \]

(10)

Due to high speeds of the vehicle in the hypersonic flight regime, structural dynamics, including aeroelastic effects, play a significant role; constraints must be imposed to ensure acceptable elastic deflections. Specifically, at specified distances back from the nose of the vehicle, the contributions of the three modal frequencies are constrained to be less than a specific value,

\[ ED_{\text{min}} \leq \sum_{i=1}^{3} \Phi_i(x_{\text{loc},i})\eta_i(t) \leq ED_{\text{max}}, \]

(11)

where \( ED \) is the total elastic deflection of the vehicle and \( \Phi_i \) is the \( i \)th mode shape at the specified location on the vehicle, \( x_{\text{loc},i} \). This constraint enables the designed controller to keep the vehicle’s structure safe during the gliding portion of flight. The selected locations are at 1/10, 3/10, 5/10, 7/10 and 9/10 of vehicle length from the nose.

IV. LQ-PI CONTROLLER WITH EXTENDED COMMAND GOVERNOR

Given the model and constraints proposed above, a nominal LQ-PI controller is augmented with an extended command governor to handle the constraints. The equations of motion are linearized about a trim point assuming zero thrust for the vehicle. This yields an 11 dimensional system with the first five states given above (velocity \( V \), angle-of-attack \( \alpha \), pitch rate \( \dot{\alpha} \), altitude \( h \) and pitch attitude \( \theta \)) and the last six are the modal coordinates and their time derivatives \((\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3)\). The control input for the system is the elevator deflection, \( \delta_e \).

A. LQ-PI controller

The LQ-PI controller is designed using the short period subsystem extracted from the continuous-time linearized system model. This subsystem includes the angle-of-attack, pitch rate, the modal coordinates, and their derivatives. The controller is designed for set-point tracking of angle-of-attack commands. To treat the case when the elastic states are not measured or estimated, a second LQ-PI controller was designed based on a second order subsystem with states being the angle-of-attack and pitch rate. In this case, the LQ-PI controller uses feedback on the integral of the angle of attack tracking error, angle-of-attack and pitch rate.

B. Extended Command Governor Overview

An ECG is implemented to enforce state and control constraints by modifying the set-point commands to the closed-loop system, as shown in Fig. 1. In Fig. 1, \( \dot{x} \) denotes the state estimate, \( y \) is the system output constrained by specifying the set inclusion conditions \( y(t) \in Y \) for all \( t \), \( \omega \) is the disturbance/uncertainty, \( v \) is the ECG output, and \( r \) is the reference command. In this paper, \( w(t) = 0 \) as the characterization and treatment of uncertainty is left to future publications.

Fig. 1: Extended command governor applied to a closed-loop system.

The ECG uses the set, \( \hat{O}_\infty \), of states and parameterized inputs such that the constraints are satisfied for all time. The ECG output is defined by

\[ v(t) = \rho(t) + \hat{C} \hat{x}(t), \]

(12)

where \( \rho(t) \) and \( \hat{x}(t) \) are solutions to the following optimization problem

\[
\begin{align*}
\text{minimize} & \quad ||\rho(t) - r(t)||_Q + ||\hat{x}(t)||_S, \\
\text{subject to} & \quad (\rho(t), \hat{x}(t), x(t)) \in \hat{O}_\infty.
\end{align*}
\]

(13)

The \( \hat{x} \in \mathbb{R}^n \), \( \bar{n} \geq 0 \), and \( \rho \) are states of a stable auxiliary dynamic system which evolve over the semi-infinite prediction horizon according to

\[ \dot{x}(t + k + 1) = A \hat{x}(t + k), \quad k \geq 0 \]

\[ \rho(t + k) = \rho(t). \]

(14)

Compared to reference governors and conventional command governors (the case \( \bar{n} = 0 \) above), the ECG yields a larger
domain of recoverable initial states and faster response. It
is also able to react to mitigate constraint violation even if
there are no changes in \( r(t) \).

The set \( \mathcal{O}_\infty \) is a finitely determined inner approximation to
the set of all \( (\rho(t), \bar{x}(t), x(t)) \) that do not induce subsequent
constraint violation when the input sequence \( v(t + k) \) is
determined by the fictitious dynamics per (12) and (14). In
the case when \( Y \) is polyhedral, computational procedures
exist that lead to polyhedral \( \mathcal{O}_\infty \), see [13]. Without the
fictitious states, i.e., when \( \bar{n} = 0 \), the ECG becomes a
simple command governor [9]. The optimization problem can
be solved online using conventional quadratic programming
techniques. The iterative procedures can be avoided by using
explicit multi-parametric quadratic programming [2], leading
to \( \rho \) and \( \bar{x} \) being given as a piecewise-affine function of the
state \( x(t) \) and reference \( r \).

Various choices of \( \bar{A} \) and \( \bar{C} \) can be made. The shift
sequences used in Ref. [9] are generated by the fictitious
dynamics with,

\[
\bar{A} = \begin{bmatrix}
0 & I_m & 0 & 0 & \cdots \\
0 & 0 & I_m & 0 & \cdots \\
0 & 0 & 0 & I_m & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\tag{15}
\]

\[
\bar{C} = \begin{bmatrix}
I_m & 0 & 0 & 0 & \cdots
\end{bmatrix},
\]

where \( I_m \) is an \( m \times m \) identity matrix. Another approach
uses Laguerre sequences [12]. These sequences possess
orthogonality properties and are generated by the fictitious
dynamics with

\[
\bar{A} = \begin{bmatrix}
\alpha I_m & \beta I_m & -\alpha \beta I_m & \alpha^2 \beta I_m & \cdots \\
0 & \alpha I_m & \beta I_m & -\alpha \beta I_m & \cdots \\
0 & 0 & \alpha I_m & \beta I_m & \cdots \\
0 & 0 & 0 & \alpha I_m & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\tag{16}
\]

\[
\bar{C} = \sqrt{\beta} \begin{bmatrix}
I_m & -\alpha I_m & \alpha^2 I_m & -\alpha^3 I_m & \cdots (-\alpha)^{N-1} I_m
\end{bmatrix},
\]

where \( \beta = 1 - \alpha^2 \) and \( 0 \leq \alpha \leq 1 \) is a selectable parameter
that corresponds to the time-constant of the fictitious dynam-
ics. Note that with the choice of \( \alpha = 0 \), (16) coincides with
the shift register considered in Ref. [9].

C. Extended Command Governor Design

For this work, the constraints are on angle-of-attack, pitch
rate, elevator deflection, elevator deflection rate and
elastic deflection. The discrete-time model of the closed-loop
system necessary to implement ECG is based on the LQ-PI
controller (one of two designs depending on whether elastic
modes are used for feedback), the short period subsystem
(with elastic modes), and a sampling period of 0.01 sec.
This model has the following form

\[
x(t + 1) = A x(t) + B v(t)
y(t) = C x(t) + D v(t) \in Y.
\tag{17}
\]

The limits were as follows: \( -\alpha_{\text{min}} = \alpha_{\text{max}} = 0.3 \) rad for
the angle of attack; \( -Q_{\text{min}} = Q_{\text{max}} = 0.2 \) rad/sec for the
pitch rate, \( -\delta_{e,\text{min}} = \delta_{e,\text{max}} = 0.349 \) rad for the elevator
deflection (20 deg), \( -\Delta \delta_{e,\text{min}} = \Delta \delta_{e,\text{max}} = 0.349 \) rad/sec
for the elevator deflection rate, and \( -E D_{\text{min}} = E D_{\text{max}} = 1.0 \) for the
elastic deflections.

To illustrate how the imposition of constraints progres-
sively affects the response, we considered a subset of con-
straints and a full set of constraints. The subset of constraints
contained just the limits on the elevator deflection and elastic
deflections (8), (11). The full set of constraints contained all
of the constraints (7)-(11)

V. Simulation Results

The simulation results are based on vehicle model lin-
earized at a dynamic pressure of 1500 psi and an altitude
of 95000 ft with zero thrust. We first show the closed-loop
responses for the case of the nominal LQ controller that
includes elastic states in feedback. With a pitch command
of \( r = \pm 0.29 \), very close to the imposed angle-of-attack limit,
constraints on angle-of-attack, pitch rate, elevator deflection,
elevator deflection rate, and elastic deflections are all violated
in transients, see Fig. 2. Note that the elastic deflections are
largest near the nose of the vehicle, near the tail of the
vehicle, and near the middle of the vehicle, with smaller
deflections elsewhere. Enforcing the constraints on elastic
deflections is thus important to ensure that the vehicle does
not break apart during the flight. The response with ECG
designed for \( \bar{n} = 5, \alpha = 0.0 \) to enforce constraints on angle-
of-attack, pitch rate, elevator deflection rates of change, and
elastic mode deflections is shown in Fig. 3. The imposed
constraints are enforced and the response of the system with
ECG is slowed down. The LQ-PI controller does not use
elastic states for feedback and the response of the system is
similar for the cases when the LQ-PI controller is designed
with elastic states for feedback.

Different design choices can lead to differences in the
speed of response of the closed-loop system. Fig. 4 shows
the case of \( \bar{n} = 1, \alpha = 0 \), which essentially is the case
of the conventional command governor. While in all cases
(\( \bar{n} = 1, 5, 10 \)) the system is protected against the constraint
violation, there is a considerable speed up in responses as
\( \bar{n} \), i.e., the dimensionality of the auxiliary system increases.
Non-zero \( \alpha \) helps to speed up the response further. With
\( \bar{n} = 10, \alpha = 0.4 \) as shown in Fig. 5 versus \( \bar{n} = 5, \alpha = 0.0 \) as shown in Fig. 3, the response is significantly
more aggressive, and more constraints become active as the
ECG has considerable flexibility to improve the performance
under constraints, see Fig. 5. The computational complexity
of the ECG increases, however, with the increase in \( \bar{n} \).

VI. Towards On-Board Implementation

The preceding sections demonstrated the potential of the
ECG to handle constraints. The development of ECG for
on-board implementation require several additional steps.
The ECG based on partial state information and reduced
order model (without the elastic dynamics) can be developed
using the reduced order reference governor techniques [11].
Both observer errors and contributions of the omitted states
Fig. 2: The response without the ECG for (a) angle-of-attack, (b) pitch rate, (c) elevator deflection, (d) elastic deflection, (e) elevator deflection time rate of change.

Fig. 3: The response with the ECG formulated for $\bar{n} = 5$, $\alpha = 0$, LQ-PI controller that does not use elastic mode measurements/estimates for feedback and full set of the constraints: (a) angle-of-attack, (b) pitch rate, (c) elevator deflection, (d) elastic deflection, (e) elevator deflection time rate of change.
Fig. 4: The response of the system with ECG formulated based on $\bar{n} = 1$, $\alpha = 0.4$, LQ-PI controller that does not use elastic mode measurements feedback and full set of the constraints: (a) angle-of-attack, (b) pitch rate, (c) elevator deflection, (d) elastic deflection, (e) elevator deflection time rate of change. This is the case of the conventional command governor.

Fig. 5: The response of the system with ECG formulated based on $\bar{n} = 10$, $\alpha = 0.4$, LQ-PI controller that does not use elastic mode measurements feedback and full set of the constraints: (a) angle-of-attack, (b) pitch rate, (c) elevator deflection, (d) elastic deflection, (e) elevator deflection time rate of change.
to the constrained outputs can be handled by imposing an additional constraint on $\delta v(t) = v(t) - v(t - 1)$. The uncertainties in forces and moments in hypersonic flight can be handled by representing these uncertainties as additive set-bounded disturbances. The existing extended command governor theory can handle these uncertainties by employing disturbance invariant sets. Finally, if explicit implementation of ECG is used (in the form of Piecewise Affine controller that represents a multi-parametric solution to the QP (13)), the ECGs designed for different trim points during the flight can be merged into a single Piecewise Affine explicit solution applicable in the full flight range. If the model is periodically re-identified or updated on-board during the flight, the condition $(\rho(t), \bar{x}(t), x(t)) \in \mathcal{O}_\infty$ with $\mathcal{O}_\infty$ computed off-line, will need to be replaced by the conditions on the predicted response of the system, $y(t + k|t) \in \mathcal{Y}$, for $k = 0, \ldots, n_c$, where $n_c$ is the constraint horizon. We leave these developments as next steps for future publications.

VII. CONCLUDING REMARKS

The paper has proposed an augmentation of an inner-loop LQ-PI controller with an Extended Command Governor (ECG) to handle constraints faced during hypersonic vehicle flight. The advantages of ECG, which is based on higher order auxiliary dynamics and Laguerre’s sequence generation have been demonstrated for the actuator rate limited case. Future research will address steps which facilitate the on-board implementation of ECG design. These include implementation based on reduced order models and based on models with uncertainties in forces and moments during the hypersonic flight.

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REFERENCES