A Predictive Energy Management for Hybrid Vehicles Based on Optimal Control Theory

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Abstract—In this paper we propose a predictive energy management for a hybrid electric vehicle with compound power-split powertrain configuration. The strategy relies on information on the future driving trip provided by modern navigation systems. Based on this information a simplified optimal control problem is solved via an indirect variation of extremals algorithm to determine a feasible start value of the adjugated variable. The powertrain controls are then determined from offline calculated maps using the value of the adjugated variables, the current vehicle speed and the requested wheel-torque. The strategy is implemented into a model-based simulation environment and has shown fuel savings on real world driving cycles. It has proven to be real-time applicable and very robust against low accuracy of the predicted driving trip.

I. INTRODUCTION

Hybrid powertrains have additional degrees of freedom (DOF) compared to conventional powertrains to reduce fuel consumption and exhaust emissions, respectively. Within certain limits these DOF can be used to operate the powertrain with a better overall system efficiency. It is the task of the energy management to coordinate these DOF, mathematically expressed as controls $u(t)$ of a dynamical system, in such manner that the systems efficiency is maximized and the fuel consumption as well as the closely related emissions of carbon dioxide minimized. Many applications to hybrid vehicles have been reported in the literature using a variety of approaches including heuristic and rule-based strategies [1], model predictive control [2], [3] and the well developed theory of Equivalent-Consumption-Minimization-Strategies (ECMS), originally proposed by [4] and enhanced by [5], [6], [7], [8] among others. Heuristic strategies have the advantage of being completely causal and hence directly applicable to given hybrid powertrain configurations. However, the obtained results depend strongly on a wide number of parameters. Therefore, the heuristic design process is time-consuming and cumbersome. The results are usually limited to one specific vehicle configuration. Model predictive control strategies depend on accurate sensor information and powerful electronic control units (ECUs).

Strategies based on optimal control theory and Pontryagins minimum principle (PMP) appear to have great potential for fuel savings. The fuel-optimal operation of a hybrid powertrain can in general be formulated as an optimal control problem (OCP). If a global solution can be found, the optimal controls derived from the solution can be fed to the powertrain, which then operates with the lowest fuel consumption possible. However, the following challenges arise with this approach:

1) The solution of the OCP might be time-consuming. At the same time it needs to be adapted to changing environmental conditions and thus needs to be repeated frequently. This might prevent an online-implemention of approaches based on optimal control theory.

2) In general, not all information necessary for solving the OCP, especially the future driving trip, is available during vehicle operation.

3) For many hybrid powertrain configurations the system contains both continuous and discrete dynamics such as switchings in drive modes. Hence, a hybrid optimal control problem (HOCP) might need to be solved. The solution of HOCPs is still subject to ongoing research ([9], [10], [11]).

In this paper we describe how these issues can be tackled for a given hybrid powertrain configuration so that an online energy management can be implemented. To make the OCP solvable online (challenge 1), a mild simplification is made as it is outlined in chapter VI. This simplification allows for storing valuable information in maps and a fast numerical solution. As a result the solution becomes suboptimal but investigations have shown that the deviation from the optimal solution is small and therefore negligible. Challenge 2 is overcome by building a future driving profile from information provided by modern navigation systems as will be described in chapters VIII and IX. Challenge 3 is avoided for this specific powertrain configuration by chosing the drive mode depending on vehicle speed and the requested wheel-torque. As a consequence, the switchings are defined before the OCP is solved. The optimal switchings between electric and power-split drive mode were researched a-priori and a strong correlation between the optimal switchings and these variables has been found. A rule-based logic is then developed based on the results. The drive modes are explained in more detail in chapter III. ECMS has strong analogies with Pontryagins minimum principle and consequently also potential for fuel savings [12]. Many advances made in the research area of ECMS-strategies can be applied to energy management based on the PMP as well. We follow [6] in developing a control structure that periodically solves an optimal control problem, based on a prediction of the driving mission provided by modern navigation systems. The approach is transferred to formulations of Pontryagins
minimum principle. The need to frequently resolve the problem with updated information is shown in [13] where the influence of certain road events on the ECMS-equivalence-factor is demonstrated.

II. OPTIMAL CONTROL THEORY AND PONTRYAGINS MINIMUM PRINCIPLE

An OCP can be stated as follows:

\[
\min_{u(t)} \int_{t_0}^{t_f} f_0(x(t), u(t), t) dt \tag{1}
\]

\[
\dot{x}(t) = f(x(t), u(t), t) \tag{2}
\]

\[
x(t_0) = x_0 \tag{3}
\]

\[
\psi(x(t_f)) = 0 \tag{4}
\]

\[
c_u(u(t), t) \leq 0 \tag{5}
\]

\[
c_x(x(t)) \leq 0 \tag{6}
\]

where \( f_0 \in C^2(\mathbb{R}^{n+m} \times [t_0, t_f], \mathbb{R}) \) and \( f \in C^2(\mathbb{R}^{n+m} \times [t_0, t_f], \mathbb{R}) \). We denote \( x(t) \in \mathbb{R}^n \) the \( n \)-dimensional state vector and \( u(t) \in \mathbb{R}^m \) the \( m \)-dimensional vector of controls, both of them being piecewise continuous. The function \( \psi \in C^2(\mathbb{R}^{2n}, \mathbb{R}) \) defines the final state boundary condition for the dynamical system (2) and the functions \( c_u \in C^2(\mathbb{R}^m \times [t_0, t_f], \mathbb{R}^n) \) and \( c_x \in C^2(\mathbb{R}, \mathbb{R}^n) \) define general the control and state constraints \((n \in \mathbb{N}, m \in \mathbb{N}, r \in \mathbb{N}, s \in \mathbb{N})\). The set of feasible controls \( \mathcal{U} \) be defined as \( \mathcal{U} = \{ u | c_u(t) \leq 0 \} \).

With the definition of the Hamiltonian function as

\[
H(x(t), u(t), t) := f_0(x(t), u(t), t) + \lambda^T(t)f(x(t), u(t), t). \tag{7}
\]

Pontryagin’s minimum principle [14] supplies a set of necessary conditions for optimality in the time-range \([t_0, t_f]\)

\[
\dot{x}(t) = \nabla_x H(x(t), u(t), \lambda(t), t) \tag{8}
\]

\[
\dot{\lambda}(t) = -\nabla_\lambda H(x(t), u(t), \lambda(t), t) \tag{9}
\]

\[
u(t) = \arg \min_{u \in \mathcal{U}} H(x(t), u(t), \lambda(t), t). \tag{10}
\]

Necessary conditions for a constrained arc in the interval \([t_1, t_2]\) of the state variable, where \( c_x(t) = 0 \) are given in [15]. With the necessary conditions the OCP can be stated as two-point boundary-value-problem, which can be solved numerically in many cases using a variation of extremals approach.

III. POWERTRAIN DESCRIPTION

The compound-split powertrain configuration for which the OCP is to be solved is comprised of a power-split transmission that splits the power provided by the internal combustion engine (ICE) onto one mechanical branch and one electrical. The latter consists of two electrical machines (EM) (called electrical variator). In most driving situations one electrical machine works as a generator, the other one as a motor. In addition the vehicle can be propelled by a third electrical machine that is connected to the rear axle using a simple gear. In contrast to the well-known input-split transmissions in the Toyota Hybrid System [8], none of the electrical machines is directly connected to an input nor output axle. Therefore, the transmission provides an additional degree of freedom. The total number of degrees of freedom is 2. The configuration is depicted in Fig. 1. Besides the power-split mode, two additional drive modes are possible. In pure electric mode only EM3 is active, the power-split transmission is not connected to the final drivetrain which avoids drag power losses in the transmission. In serial drive mode the output of the power-split transmission is connected to the chassis. Thus both EM1 and EM2 can be operated as generators and transform the mechanical energy provided by the ICE into electrical energy that increases the State of Charge (SoC) of the battery. This introduces an additional constraint which reduces the DOF of the system to 1. The drive mode to be used at any \( t \in [t_0, t_f] \) is determined by the piecewise constant switching function

\[
\sigma(v_{veh}, T_{req}, SoC, t) = \begin{cases} 
-1 & \text{power-split mode} \\
0 & \text{pure electric mode} \\
1 & \text{serial mode}. 
\end{cases} \tag{11}
\]

A simple rule-based logic has been developed to select the drive modes depending on the inputs drive speed \( v_{veh} \), battery \( SoC \) and the required wheel torque \( T_{req} \). The gear ratios of the planetary gears are chosen, such that the powersplitting-factor

\[
\epsilon = \frac{P_{el}}{P_{mech}}, \tag{12}
\]

which resembles the ratio of the power on the electrical branch \( P_{el} \) and the power on the mechanical branch \( P_{mech} \), is small for vehicle speeds from 34.2 mph upwards. As a consequence good overall system efficiencies can be attained for this speed-range. At this speed the aerodynamic drag and hence, the required wheel torque is usually high enough to allow the ICE to be operated at higher loads and good efficiency. For lower speeds the electrical drive mode is provided. The serial drive mode serves as a backup if at low speeds the \( SoC \) decreases under the lower limit. It also provides flexibility in the catalytic converter light-off phase.
IV. POWERTRAIN MODEL

The nickel metal hydride battery is modeled using a simple circuit consisting of an ideal voltage source and an internal resistance. The open circuit voltage $V_{OC}$ of the ideal voltage source only depends on the SoC of the system. Thermal influences are disregarded as it is expected that the battery pack possesses an active thermo-management-system that keeps the cell-temperature on a desired constant level. The $V_{OC}$ for $SoC \in [0,100]$ is determined with the Nernst-equation

$$V_{OC}(SoC) = N_{cell} \cdot \left( E_0 + \frac{R \cdot T}{F} \cdot \ln \left( \frac{SoC}{100 - SoC} \right) \right)$$  \hspace{1cm} (13)

where $N_{cell}$ is the number of cells, $E_0$ is the standard cell potential, $R$ is the universal gas constant, $F$ is the Faraday constant and $T$ is the cell temperature. The internal battery resistance $R_{bat}$ is assumed to be constant in the allowed SoC-range. This assumption holds for the most modern battery types for hybrid vehicles. Applying Kirchhoff’s circuit laws to the simple circuit model, the battery current can be calculated by the following equation

$$I = -\frac{V_{OC}(SoC) + \sqrt{V_{OC}^2(SoC) - 4R_{bat} \cdot P_{bat}}}{2R_{bat}}.$$  \hspace{1cm} (14)

The state equation can then be written using (14) as

$$SoC = \frac{100}{Q_{bat}} \cdot I$$  \hspace{1cm} (15)

where $Q_{bat}$ is the battery capacity. $P_{bat}$ is the sum of all electrical powers $P_{EM_i}$ of the electrical machines and the power required to supply the electrical on board system:

$$P_{bat} = P_{EM_1}(T_{EM1}, \omega_{EM1}) + P_{EM_2}(T_{EM2}, \omega_{EM2}) + P_{EM_3}(T_{EM3}, \omega_{EM3}) + P_{on}$$  \hspace{1cm} (16)

where $T_{EM_i}$ are the torques and $\omega_{EM_i}$ are the angular speeds of the electrical machines. $P_{on}$ denotes the electrical power needed to supply the electrical on board system. The electrical powers are smooth functions of $T_{EM_i}$ and $\omega_{EM_i}$. The kinematic constraints within the power-split transmission are given by a linear system of two equations, which describes the static speed relationship as

$$\begin{bmatrix} \omega_{EM1} \\ \omega_{EM2} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} \omega_{ICE} \\ \omega_{wh} \end{bmatrix}$$  \hspace{1cm} (17)

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} -i_1 & i_{FD} \cdot (1 + i_1) \\ i_1 \cdot i_2 & -i_{FD} \cdot (i_1 \cdot i_2 - 1) \end{bmatrix}.$$  \hspace{1cm} (18)

Similar, a static torque relationship can be obtained by inversion of the transfer matrix (18), which yields

$$\begin{bmatrix} T_{EM1} \\ T_{EM2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} T_{ICE} \\ T_{req} \end{bmatrix}$$  \hspace{1cm} (19)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \left( \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \right)^{-1}.$$  \hspace{1cm} (20)

The parameters $i_1$ and $i_2$ are the stationary gear ratio of planetary gear 1 and 2, respectively, $i_{FD}$ is the final drive ratio, $\omega_{wh}$ is the angular wheel speed, $\omega_{ICE}$ and $T_{ICE}$ are the angular speed and the torque provided by ICE. The required wheel torque $T_{req}$ needed for a given driving situation is derived from a simple longitudinal vehicle dynamics model [16]. For the formulation of the OCP only the static torque relationships are considered. The fuel consumption of the ICE is smoothly approximated by a polynomial fit.

V. DEFINITION OF THE OPTIMAL CONTROL PROBLEM

To allow for a compact system description, the following expressions are introduced:

$$[x]^+ := \max\{x,0\}, \ [x]^− := −\min\{x,0\}$$  \hspace{1cm} (21)

With the a-priori defined switching function $\sigma(\cdot)$ the optimal control problem can then be defined as follows: Find the controls $u^*(t)$ that minimize the functional

$$J = \int_{t_0}^{t_f} b_{e,ICE}(T_{ICE}, \omega_{ICE}) \cdot T_{ICE} \cdot \omega_{ICE} dt$$  \hspace{1cm} (22)

where $b_{e,ICE}$ is the specific fuel consumption of the ICE, subject to the state differential equation (15) with the state boundary condition

$$\psi(SoC(t_f)) = SoC(t_f) - SoC_{t_f} = 0,$$  \hspace{1cm} (23)

whose $SoC_{t_f} \in \mathbb{R}$ is a fixed boundary value. We choose as vector of controls during the power-split driving mode

$$u = [T_{ICE} \quad \omega_{ICE}]^T.$$  \hspace{1cm} (24)

In serial mode, the controls are obtained from maps depending on the driving situation so that the overall efficiency of the powertrain is maximized and the battery SoC is continuously increasing. Hence, the optimization task remains to determine the controls during power-split mode. The remaining torques and angular velocities result from the kinematic constraints

$$\begin{bmatrix} T_{EM1} \\ T_{EM2} \\ T_{EM3} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} T_{ICE} \\ T_{req} \end{bmatrix} \cdot \sigma(\cdot) +$$  \hspace{1cm} (25)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i_3^{-1} \end{bmatrix} \cdot \begin{bmatrix} T_{EM1} \\ T_{EM2} \\ T_{req} \end{bmatrix} \cdot \sigma(\cdot) +$$  \hspace{1cm} (26)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -i_3^{-1} & 0 \\ i_3 & 0 & 0 \end{bmatrix} \cdot \sigma(\cdot) \cdot (1 - \sigma(\cdot)),$$

$$\begin{bmatrix} \omega_{EM1} \\ \omega_{EM2} \\ \omega_{EM3} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} \omega_{ICE} \\ \omega_{wh} \end{bmatrix} \cdot \sigma(\cdot) +$$  \hspace{1cm} (27)

$$\begin{bmatrix} b_{11} & 0 \\ b_{21} & 0 \\ i_3 & 0 \end{bmatrix} \cdot \sigma(\cdot) \cdot (1 - \sigma(\cdot)),$$
where $i_3$ is the gear ratio of EM3 of the electrical rear axle. The functions
\[ c_x(SoC(t)) = \begin{cases} c_{x1}(t) = \frac{(SoC(t) - SoC_{max})}{SoC_{min} - SoC(t)} & \text{if } c_x(t) < 0 \\ c_{x2}(t) = \frac{(SoC_{max} - SoC(t))}{SoC_{min} - SoC(t)} & \text{if } c_x(t) = 0 \\ c_{x3}(t) = \frac{(SoC(t) - SoC_{min})}{SoC_{max} - SoC(t)} & \text{if } c_x(t) > 0 \end{cases} \] (27)
and
\[ c_u(u(t)) = \begin{cases} u(t) - u_{max}(u(t), t) & \text{if } u(t) > u_{max}(u(t), t) \\ u_{min}(u(t), t) - u(t) & \text{if } u(t) < u_{min}(u(t), t) \end{cases} \] (28)
impose upper and lower limits on controls and state. If $\sigma(\cdot)$ is known, the OCP can be solved with different numerical methods, e.g. a variation of extremals algorithm, where an initial value problem is defined as follows:
\[ \dot{y} = \begin{bmatrix} SoC \\ \lambda \end{bmatrix}, \quad y_0 = \begin{bmatrix} SoC_0 \\ \lambda_0 \end{bmatrix}. \] (29)
Herein $\lambda_0$ is an initial guess of $\lambda(t_0)$. With the definition of the Hamiltonian function
\[ H = [\sigma(\cdot)]b_{e,ICE}(T_{ICE}, \omega_{ICE}) \cdot T_{ICE} \cdot \omega_{ICE} + \lambda \cdot \frac{100}{Q_{bat}} I \] (30)
the differential equation of the adjungated variable becomes
\[ \dot{\lambda} = \begin{cases} -\lambda \cdot \frac{100}{Q_{bat}} \cdot \frac{\partial T_{ICE}}{\partial SoC}, & c_x(t) < 0 \\ -(\lambda + \mu) \cdot \frac{100}{Q_{bat}} \cdot \frac{\partial I}{\partial SoC}, & c_{x1}(t) = 0 \\ -(\lambda - \mu) \cdot \frac{100}{Q_{bat}} \cdot \frac{\partial I}{\partial SoC}, & c_{x2}(t) = 0. \end{cases} \] (31)
The control $u(t)$ in power-split driving mode is determined as
\[ u(t) = \begin{cases} \arg\min_{u \in U} H, & c_x(t) < 0 \\ \arg\min_{u \in U, t \leq 0} H, & c_{x1}(t) \geq 0 \\ \arg\min_{u \in U, t \geq 0} H, & c_{x2}(t) \geq 0. \end{cases} \] (32)
The constrained minimization problem in (32) is well suited for nonlinear programming methods such as SQP. An optimal $\lambda(t_0)$ is then found by solving the nonlinear equation
\[ \psi(\lambda(t_0)) = SoC(t_f) - SoC_0 = 0. \] (33)
This is done numerically by finding a sequence $\lambda_{0,k}, k = 1, 2, \ldots$, for $\lambda(t_0)$ such that $\psi$ approaches zero up to a desired exactness. A dampened Newton-method has proven to perform well on the given problem.

VI. SIMPLIFICATION OF THE OCP
This method for solving the OCP has the advantage of having low memory requirements. However, calculation time needs to be reduced drastically to make the optimization procedure applicable online in a vehicle. This can be achieved with the following simplification: We assume the battery current $I$ to be independent of the SoC. This assumption holds since the SoC is restricted to be in a small boundary, where the $V_{OC}$ changes only slightly. This relation is widely assumed in the literature, for example in [17] and [18]. The assumption affects optimality condition (9) since now $\frac{\partial}{\partial SoC} = 0$ applies and therefore $\lambda(t) = \lambda = const$. Another consequence of this simplification is, that for the Hamiltonian function $H(T_{req}, \omega_{wh}, \lambda)$ instead of $H(T_{req}, \omega_{wh}, \lambda, SoC)$ applies. In this way the minimum of the Hamiltonian function and the resulting control can be calculated offline and then stored in maps with three dimensions, namely the wheel-speed $n_{wh}$, $T_{req}$ and $\lambda$. One of these maps for a specific value of $\lambda$ is shown in Fig. 2.

A similar approach is proposed by [7], where the controls for a given ECMS-equivalence factor $s$ and the driving situation $T_{req}, \omega_{wh}$ is determined from a map. The advantage is twofold: During the determination of an appropriate value for the adjungated variable $\lambda$, the minimization of the Hamiltonian function is replaced by a map-interpolation, which reduces the computing time for solving equation (33) by more than 90 %. Also, once a feasible $\lambda$ has been found, determining the controls at any instant does not require instantaneous minimization of the Hamiltonian function any more but a much faster map-interpolation only. With this simple and fast solution algorithm, a control structure can be designed as described in the following section.

VII. OUTLINE OF THE PREDICTIVE CONTROL STRATEGY
The predictive control strategy proposed in this paper consists of the following parts and is depicted in Fig. 3:
- a prediction of the future driving route
- a driver model to estimate the velocity trajectory on the predicted route
- an optimization routine that solves an optimal control problem over the prediction horizon
- a map that sends optimized controls to the powertrain depending on the current driving situation and the calculated $\lambda$
The prediction of the driving route will be given by modern navigation systems. Those systems also provide information about speed limits, traffic signs and the current traffic density. From this information a segmentation of the route can be performed. Those segments \((s_{seg}, v_{lim,seg})\) are then transferred to the driver model that generates a velocity trajectory \(v_{veh,pred}(t)\). From this trajectory, \(T_{req,pred}(t)\) can be calculated with the longitudinal dynamics model. Both trajectories are then passed on to the optimization routine that solves the optimal control problem and determines a feasible value for \(\lambda\). Before the problem can be solved, the switching function \(\sigma\) needs to be determined over the optimization horizon. The transitions between the driving modes electric and power-split depend on \(v_{veh,pred}(t)\) and \(T_{req,pred}(t)\) only. Consequently, the switching-times in the optimization horizon are no optimization variables. However, if long intervals at low speeds occur, a switching from electric to serial mode might be necessary to prevent low SoCs. Thus, a \(SoC\)-trajectory needs to be known a-priori. An approximate \(SoC\)-trajectory is therefore calculated using an average value for \(\lambda\). From this trajectory the approximate switching point can be attained. The final value for \(\lambda\), obtained from the OCP, is stored in the ECU. Together with the actual \(v_{veh}(t)\) and \(T_{req}(t)\) the controls at any time are then interpolated from a map. The process is repeated, as soon as the predicted \(SoC_{pred}(s)\) on a given place \(s(t)\) on the real trip deviates from the actual \(SoC\) more than a given threshold or as soon as the predicted driving trip changes.

VIII. Route-prediction

For means of simulation, the routing information is provided by an electronic horizon provider which uses a database with numerous real-world driving-cycles. The driving-cycles cover information about the topology including road inclination, traffic signs and speed limits. This information has been measured with the Floating-Car method using a car equipped with GPS device. The traffic signs were entered manually during these measurements. The route is then divided into segments based on the changes of speed limits and the road inclination. The topographic data and traffic signs are provided by digital maps stored in a database and are assumed to be exact for simulation purpose. In future this electronic horizon provider will be part of modern navigation systems. The segments and the corresponding information is then forwarded to the driver-model.

IX. The Driver-Model

Task of the driver-model is to calculate approximately the vehicle speed over time, based on the available information of the route-prediction. We use parts of the intelligent-driver-model proposed in [19] as it is easily adaptable to different driver-types. This model provides a set of differential equations for different driving situations. An acceleration to a desired speed level \(v_{des}\) for instance is modeled as following

\[
\dot{v} = a \cdot \left(1 - \left(\frac{v}{v_{des}}\right)^\delta\right),
\]

where \(a\) and \(b\) denote driver-dependent average accelerations/decelerations-values, \(\delta\) defines, when the acceleration is reduced when approaching the desired speed and \(r\) denotes the distance to the standstill position. To determine, when a braking procedure starts, we enhance the model by a range-of-sight

\[
r_s := p_0 \cdot \dot{v}^2 + p_1 \cdot v
\]

where \(p_0\) and \(p_1\) again are driver-dependent values. If an obstacle is within the range-of-sight, braking is initiated.

X. Results

The strategy was implemented on a dSPACE MicroAutoBox and then connected via CAN-Bus with a MATLAB®/Simulink® Simulation-environment for the entire hybrid vehicle. As benchmark routes, GPS-measured test tracks, containing urban, suburban and highway traffic-situations were used. The approach was then tested on these tracks. The measurements were performed by different drivers. However, the driver-model was not adapted to each one of them. Instead, the parameters were determined to fit the measured data of an average driver. As benchmark-strategy serves a rule-based energy management that uses optimized operation points of the ICE and only shifts the optimal operation point if the \(SoC\) approaches the upper or lower limit. The controls for the serial mode are obtained from the same maps for both strategies. On the given benchmark-route, fuel-savings of up to 1% compared to the already very well calibrated rule-based strategy could be achieved, as shown in table I.

<table>
<thead>
<tr>
<th>Velocity Profile</th>
<th>consumption predictive strategy [l/100km]</th>
<th>consumption rule-based strategy [l/100km]</th>
<th>fuel savings [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>7.59</td>
<td>7.06</td>
<td>0.78</td>
</tr>
<tr>
<td>Profile 2</td>
<td>7.06</td>
<td>7.14</td>
<td>0.91</td>
</tr>
<tr>
<td>Profile 3</td>
<td>7.40</td>
<td>7.48</td>
<td>1.07</td>
</tr>
<tr>
<td>Profile 4</td>
<td>6.83</td>
<td>6.87</td>
<td>0.58</td>
</tr>
</tbody>
</table>

TABLE I
COMPARISON OF FUEL CONSUMPTIONS OF THE PREDICTIVE AND THE BENCHMARK STRATEGY

Since only information on speed limits, road inclination and holding situations were fed to the driver model, the generated velocity profile is inaccurate where changes in velocity caused by dense traffic situations occur. However, investigations have shown that the determined values of the adjungated variable for the approximated velocity profile are very close to the values experimentally determined for the actual velocity profile. As the minimization problem in equation (32) does not need to be solved online to determine the controls for a given driving situation and an assumed value for \(\lambda\), but can rely on information calculated offline.
To test the robustness of the operation strategy in terms of inaccuracy of the estimated velocity profile, the value of the traffic limitation as well as the configuration parameters of the driver model were disturbed with variables subject to a normal distribution and a standard deviation of 20% of the original value. Then, simulations over the same driving trip were repeatedly performed with these uncertainties and the results statistically analyzed. The deviations between predicted and actual drive trip result in a far more frequent re-calculation of the adjungated variable. However, the fuel consumptions of the disturbed system deviate only slightly from the consumptions of the undisturbed system, as it is shown in the boxplot in Fig. 5. Herein the red star depicts the consumption of the undisturbed system and the box the disturbed system’s statistical distribution of fuel consumptions.

Fig. 5. Boxplot of the fuel consumption of the disturbed system

XI. CONCLUSION

The predictive control strategy proposed in this paper has great potential for application in a hybrid vehicle. Even though only little information (speed limit, road inclination and traffic signs) was provided to generate the future driving mission, the fuel consumptions achieved can already compete with state-of-the-art rule-based strategies. An additional improvement is to be expected, if more detailed profiles of the driving trip can be built. An identification of the driver and an adaption of the respective model can also yield further improvements. The proposed strategy can be used for other hybrid powertrain configurations as well. As a next step, the developed strategy will be implemented on a real parallel hybrid vehicle. Results will be presented accordingly.

REFERENCES


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