Self-excited Limit Cycles in an Integral-Controlled System with Backlash

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Abstract—The stability of systems with hysteresis, driven by developments in smart material applications, has been an important topic of research over the past two decades. Most results provide sufficient conditions for boundedness of the system states, but do not further investigate the steady state solutions. In this paper, we present an example of a system with hysteresis that possesses self-excited limit cycles. In particular, we consider an integral-controlled system with backlash (also known as play operator). A Newton-Raphson algorithm is formulated to calculate the limit cycles in the system. We then prove that the amplitude and period of these limit cycles have linear relationships to parameters within the system. These results are then confirmed in simulation, where we demonstrate our ability to predict and modify the properties of the limit cycles.

I. INTRODUCTION

The phenomenon of hysteresis has been considered by many fields of science, due to its prominence in various physical systems [1]. Most research into hysteresis focused on modeling the phenomenon until the 1970’s, when the theory of differential equations coupled with hysteresis was developed by introducing the notion of a hysteresis operator [2], [3]. This coupling of hysteresis operators and differential equations, especially ODEs, which we will refer to as a system with hysteresis, has been successfully employed as a model for many physical systems, particularly smart material-actuated systems.

While the methods used for modeling ODE’s are fairly well known and documented, hysteresis models possess a wide variety of forms. The most common hysteresis models fall under the category of Preisach-like operators, such as the Preisach operator [4], [5], Prandtl-Ishlinskii (PI) operator [6], and the Preisach- Krasnosel’skii-Pokrovskii (PKP) operator [7], [8]. The common thread in each of these operators is the use of a large number of hysterons, which are smaller elementary hysteresis units, combined in superposition to emulate a hysteresis phenomenon. Additionally, Preisach-like operators are phenomenological, in that they can describe the hysteresis phenomenon but provide no information regarding its physical origin. An alternative model is the model [9], [10], which is based on switching differential equations. Duhem models can be either phenomenological or based on physical laws, as was done for ferromagnetic materials by Jiles and Atherton [11].

The promising applications of smart material actuators have driven the development of many new control strategies for systems with hysteresis [12], [13]. A relevant example is the field of nanopositioning, where smart materials like piezoelectrics are used to generate nanometer-resolution motion [14]. Many controllers have been successively employed in such problems; a few examples include sliding-mode control [15], feed-forward hysteresis inversion [16], [17], adaptive control [18], two-degree of freedom control [19], and Iterative Learning Control [20].

In these papers, the authors aim to provide sufficient conditions under which a given controller structure guarantees stability of the system in question. In particular, results in the nanopositioning literature focus on proving boundedness of the tracking error. A natural question to consider would be to investigate the behavior of the system when these conditions are not satisfied; alternatively, what effects do hysteresis nonlinearities introduce into the steady-state solutions of the system? Several authors have remarked that hysteresis can lead to unwanted oscillations, perhaps most notably in the work of [18]. Further investigations into these limit cycles are limited. One result is in [21], where conditions are presented under which the method of harmonic balance predicts the existence of periodic solutions to systems with relay hysteresis. The authors of [22] utilized the describing function method to predict the existence of a limit cycle in a Terfenol-D-based actuator, and demonstrated its existence in experiments. These works focused fundamentally on the question of existence, and did not investigate any properties of the limit cycles in detail.

In this paper, we offer an in-depth exploration into the properties of self-excited limit cycles occurring in a system with hysteresis. In particular, we will focus on a scalar plant controlled by an integral controller, where a play operator [23] is present in the feedback loop. Such a system represents a crude model of a smart material actuator, and we will show that improper selection of controller gains can cause such a system to enter a self-excited limit cycle. Based on this motivating example, a Newton-Raphson algorithm is formulated to calculate the limit cycles of the system. This algorithm focuses on symmetric limit cycles with two monotonic portions per period. Using the setup of this algorithm, we are then able to prove that there exist linear relationships between several properties of the limit cycles and parameters of the system. These results are verified in simulation, where we also demonstrate the effectiveness of

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the Newton-Raphson algorithm at predicting the solutions of the system.

The remainder of this paper is organized as follows. The play operator is introduced in Section II. We then present our motivating example in Section III, which serves as the basis for the remainder of our work. Section IV contains the Newton-Raphson algorithm and analytical results on the limit cycles. Our results are then confirmed through simulation in Section V, and concluding remarks are provided in Section VI.

II. THE PLAY OPERATOR

We begin this paper with a discussion of the play operator, a unit hysteretic element illustrated in Fig. 1. Each play operator \( W_r \) is parametrized by a parameter \( r \), representing the play radius or threshold. When the input \( v(t) \) is monotone and continuous, we can express the output \( u(t) \) of a play operator \( W_r \) as

\[
u(t) = W_r[v;u(0)](t) = \max\{\min\{v(t) + r,u(0)\},v(t) - r\}
\]

The output \( u(t) \) is also referred to as the state of the play operator \( W_r \). For general inputs, the input signal is broken into monotone segments, and the output is then calculated by setting the last output of one monotone segment as the initial condition for the next. Notice from Fig. 1 and (1) that there are two basic modes in which the state of a play operator can reside. The first is the linear region, in which \( u(t) = v(t) \pm r \). The second mode of operation is the play region, where \( u(t) \) is constant, represented in (1) by the term \( u(0) \). We will make use of the linear and play region terminology throughout the paper. Furthermore, we will also refer to the leftmost linear branch in Fig. 1 as the descending region, and the rightmost linear branch as the ascending region. The play operator is used in the Prandtl-Ishlinskii hysteretic operator, which uses a superposition of play operators and a linear gain to model a hysteresis phenomenon [23].

III. MOTIVATING EXAMPLE: LINEAR SYSTEM WITH HYSTERESIS

Let us consider a scalar system preceded by a play operator and a unity gain controlled using integral control and feedback. This represents a basic cascade of a Prandtl-Ishlinskii hysteretic operator with dynamics, a model that is commonly considered in piezoelectric nanopositioning. The system is written as,

\[
\dot{x}(t) = ax(t) + v(t) + \theta h W_r[v;0](t)
\]

\[
\dot{\sigma}(t) = x(t)
\]

\[
v(t) = -k_1 x(t) - k_2 \sigma(t)
\]

where \( r \) is the play radius and \( \theta h \) is a weighting term. For our simulations, we will let \( K = [k_1,k_2] = [1,1] \), and set \( a = 1.5 \) with \( \theta h = 1 \).

One technique used in nanopositioning control is to consider the hysteresis as a linear gain coupled with a bounded, time-varying uncertainty, as was done in [24]. For (2), this linear gain is equal to the coefficient of \( v \) added with the weight \( \theta \). Based on this idea, we reformulate the closed-loop system as

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{\sigma}(t)
\end{bmatrix} =
\begin{bmatrix}
a - 2k_1 & -2k_2 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\sigma(t)
\end{bmatrix} +
\begin{bmatrix}
S_r[v;0](t)
\end{bmatrix}
\]

where \( S_r[v;0](t) \) is a stop nonlinearity [25] which takes values in the bounded region \([-r,r]\). In particular, for monotone input \( v \),

\[
S_r[v;0](t) = \min\{r,\max\{-r,v(t) - v(0) + S_r[v;0](0)\}
\]

We can compute the eigenvalues of this system through the roots of the equation

\[
\det(sI - A) = s^2 - (a - 2k_1)s + 2k_2
\]

where

\[
A = \begin{bmatrix}
a - 2k_1 & -2k_2 \\
1 & 0
\end{bmatrix}
\]

These eigenvalues have negative real parts for \( k_1 > a/2 \) and \( k_2 > 0 \), both of which are satisfied for our choice of \( K \). Therefore the trajectories of the system remain bounded, since (3) is an exponentially stable linear system driven by a bounded input. However, when the play operator in (2) is in the play region, the \( \dot{x} \) equation becomes

\[
\dot{x}(t) = (a - k_1)x(t) - k_2 \sigma(t) + k_r
\]

where \( k_r \) is a constant value determined by the current state of the play operator. Using this, the eigenvalues of the closed-loop system obey

\[
\det(sI - A) = s^2 - (a - k_1)s + k_2
\]

where

\[
A = \begin{bmatrix}
a - k_1 & -k_2 \\
1 & 0
\end{bmatrix}
\]

Since \( a - k_1 = 0.5 \), the system dynamics are unstable when the operator lies in the play region. Fig. 2 shows the behavior of (2) for varying play radii. We clearly see that the system enters a limit cycle for each value of the play radius. This occurs because the controller gains \( K \) are not chosen to account for the nonlinear behavior of the hysteresis. Furthermore, these limit cycles are self-excited, in that there is no external input driving the system.
IV. LIMIT CYCLES IN SYSTEMS WITH HYSTERESIS

We will now attempt to analyze the self-excited limit cycles observed in the system (2). We will begin our analysis by providing a coordinate transform in order to place (2) into a switched system form. From (2), define

\[ \alpha(t) = -k_2 \sigma(t) + \theta_0 W_r[v; 0](t) \]  

(6)

The derivative of \( \alpha \) requires us to define the derivative of a play operator, which is in general discontinuous. Let \( \Pi \) denote the set of all closed intervals of \( t \in \mathbb{R} \) in which \( W_r[v; 0](t) \) lies in a linear region, and let \( \Pi^c \) denote its complement. We therefore have a piecewise definition for \( W_r \), given by

\[ W_r[v; 0](t) = \begin{cases} v, & \text{if } t \in \Pi \\ 0, & \text{if } t \in \Pi^c \end{cases} \]  

(7)

where

\[ \dot{v}(t) = -k_1 [(a - k_1)x(t) + \alpha(t)] - k_2 x(t) - k_1 \alpha(t) \]  

(8)

Using (6)-(8), we can derive a switched system form for (2):

\[ \dot{\gamma}(t) = A_i \gamma(t), \quad i = 1, 2 \]  

(9)

\[ A_1 = \begin{bmatrix} a - k_1 & 1 \\ -k_2 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} a - k_1 & 1 \\ -2k_2 - k_1 & -k_1 \end{bmatrix} \]

where \( \gamma = [x, \alpha]^T \), and \( r \) is the play radius. The matrix \( A_1 \) characterizes the systems behavior when the system is in the play region of the hysteresis, while \( A_2 \) does so for the linear region of the hysteresis. To define the switching behavior, we will define the operator

\[ i = \Omega[W_r[v; 0](t)] \]  

(10)

\[ \Omega[W_r[v; 0](t)] = 1 \text{ when } W_r \text{ is in the play region, and} \]

\[ \Omega[W_r[v; 0](t)] = 2 \text{ when } W_r \text{ is in the linear region.} \]

A. Identification of the limit cycle

Based on our simulation results, we will now try to characterize the limit cycles generated from the evolution of (9)-(10), assuming that \( A_1 \) is an unstable matrix, and \( A_2 \) is a Hurwitz matrix. Because of the switching properties of the system (9), formulating a general closed-form solution to analyze any limit cycles is difficult. We will instead calculate the limit cycles numerically, and identify some key properties regarding the amplitude and frequency of the oscillations, assuming the system enters a symmetric limit cycle with no offset. We will refer to this as a simple limit cycle, which we have illustrated in Fig. 3.

Our search for the solution of the limit cycles begins from an initial state \( \gamma(0) \) such that the system is in the increasing linear section, and let \( \bar{t}_1, \bar{t}_2, \bar{t}_3 \), and \( \bar{t}_4 \) denote the times when the system switches between \( A_1 \) and \( A_2 \). Note that \( \gamma(\bar{t}_4) = \gamma(0) \), therefore \( \bar{t}_4 \) also represents the period of the limit cycle. From the description of the play operator, for any simple limit cycle, the control \( v(t) \) at these switching times obey the equations,

\[ \dot{v}(\bar{t}_1) = 0 \]  

(11)

\[ v(\bar{t}_1) - v(\bar{t}_2) = 2r \]  

(12)

\[ v(\bar{t}_3) = 0 \]  

(13)

\[ v(\bar{t}_3) - v(\bar{t}_4) = -2r \]  

(14)

implying that \( \gamma(0) = \gamma(\bar{t}_4) \). Furthermore, symmetry allows us to only consider the conditions (11) and (12). We will let \( t_1 = \bar{t}_1 \) and \( t_2 = \bar{t}_2 - \bar{t}_1 \); these values will be referred to as the switching intervals. We can then translate these equations into functions of the initial conditions of \( \gamma \).

\[ H_1(\gamma(0), t_1) = \tilde{K} e^{A_{1t_1}} \gamma(0) = 0 \]  

(15)

\[ H_2(\gamma(0), t_1, t_2) = [-k_1, 1][I - e^{A_{1t_2}}] e^{A_{2t_2}} \gamma(0) = 2r \]  

(16)

where \( \tilde{K} = [-k_1 (a - k_1) - k_2, -k_1] \). Finally, because we are seeking symmetric limit cycles, we also have the constraint equation

\[ \Sigma(\gamma(0), t_1, t_2) = (I + e^{A_{1t_2}} e^{A_{2t_1}}) \gamma(0) = 0 \]  

(17)

which is derived from the forward time solution of the switched system from 0 to \( \bar{t}_2 \).
Equations (15)-(17) yield four equations with four unknowns, \( \gamma(0) = [\chi(0), \alpha(0)]^T, t_1, \) and \( t_2 \). We will refer to solving this set of simultaneous equations as the limit cycle problem. Due to the nonlinearity of these equations, we will utilize the well-known Newton-Raphson method to find the solution to the limit cycle problem. Denote our unknowns as \( \Phi = [y^T(0), t_1, t_2]^T \), and let \( \Sigma_1 = [1, 0] \Sigma \) and \( \Sigma_2 = [0, 1] \Sigma \). These are used to define

\[
\mathcal{P}(\Phi) = [\Sigma_1(\Phi), \Sigma_2(\Phi), H_1(\Phi), H_2(\Phi)]^T
\]

We can now apply the Newton-Raphson method to obtain a solution to the above equation using the iterative formula

\[
\Phi^{i+1} = \Phi^i + J^{-1}(\Phi^i) \mathcal{P}(\Phi^i)
\]

where

\[
J(\Phi) = \begin{bmatrix}
\frac{\partial \Sigma_1}{\partial \Sigma_1} & \frac{\partial \Sigma_1}{\partial \Sigma_2} & \frac{\partial \Sigma_1}{\partial \Sigma_1} & \frac{\partial \Sigma_1}{\partial \Sigma_2} \\
\frac{\partial \Sigma_2}{\partial \Sigma_1} & \frac{\partial \Sigma_2}{\partial \Sigma_2} & \frac{\partial \Sigma_2}{\partial \Sigma_1} & \frac{\partial \Sigma_2}{\partial \Sigma_2}
\end{bmatrix}
\]

These partial derivatives can be readily calculated in closed form based on (15)-(17); expressions have been omitted from this paper. The limit cycle is then characterized by the solution \( \Phi^* \) of the equation

\[
\mathcal{P}(\Phi^*) = 0
\]

Note that \( \Phi^* \) completely characterizes the behavior of the limit cycles, as once the switching times and initial conditions are known, the closed form solution of the limit cycle can be computed from successive solutions of linear systems.

**B. Properties of the limit cycles**

While the solution of the limit cycle problem \( \Phi^* \) must be calculated numerically, we can utilize the equations (15)-(17) to prove some properties of the limit cycles corresponding to the solution \( \Phi^* \). First, we will see how the solution \( \Phi^* \) varies with the play radius \( r \).

**Proposition 1:** Let the solution of the limit cycle problem with \( r = r^* \) be denoted by \( \Phi_{r^*} = [\gamma^T(0), t_1^*, t_2^*]^T \). Then, if \( r = r^* c_1 \), where \( c_1 > 0 \), the corresponding solution to the limit cycle problem is \( \Phi_r = [c_1 \gamma^T(0), t_1^*, t_2^*]^T \).

**Proof:** We begin by directly computing (16) evaluated at \( \Phi_r = [c_1 \gamma^T(0), t_1^*, t_2^*]^T \) with \( r = r^* c_1 \), which can be written as

\[
[-k_1, 1][I - e^{A t_2^*}] e^{A t_2^*} c_1 \gamma(0) = 0 = A c_1
\]

By dividing both sides by \( c_1 \), we arrive at the solution of \( H_2(\Phi_r) \). For Equations (15) and (17), note that each of these will contain linear terms of \( \gamma^*(0) \) on one side, and zero on the opposite; therefore the \( c_1 \) term can be immediately divided out, proving \([c_1 \gamma^T(0), t_1^*, t_2^*]^T \) solves the limit cycle problem.

**Proposition 1** indicates that increasing \( r \) linearly increases the amplitude of the limit cycles generated in (9)-(10), which can also be observed in Fig. 2. Next, we consider variation of the limit cycles with the plant time constant \( a \), letting \( K = [k_p a, a^2/2] \), where \( k_p \in (.5, 1) \). The system matrices are then

\[
\dot{\gamma}(t) = A_i \gamma(t), \quad i = 1, 2
\]

where

\[
A_1 = \begin{bmatrix}
(1 - k_p) a & 1 \\
-a^2/2 & 0
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
(1 - k_p) a & 1 \\
-a^2 - k_p(1 - k_p) a^2 & -k_p a
\end{bmatrix}
\]

Let us focus on the system in the linear region of operation, i.e., \( \dot{\gamma}(t) = A_2 \gamma(t) \). The characteristic equation of this system is

\[
s^2 - \text{Tr}(A_2) s + \text{Det}(A_2) = 0
\]

with

\[
\text{Tr}(A_2) = (1 - 2k_p) a, \quad \text{Det}(A_2) = a^2
\]

where \( \text{Tr} \) and \( \text{Det} \) denote the Trace and Determinant respectively. Let us consider the state \( x \) as the output of the system, and formulate a canonical form transformation. Let \( \chi_1 = x \) and \( \chi_2 = \dot{x} \). This transforms the system equations based on \( A_2 \) with our specified \( K \) into

\[
\dot{\chi}_1(t) = x_2(t)
\]

\[
\dot{\chi}_2(t) = -a^2 x_1(t) + (1 - 2k_p) a x_2(t)
\]

Next, let \( \eta_1 = a \chi_1 \), and let \( \eta_2 = \chi_2 \). The \( \dot{\eta} \) equations are then

\[
\dot{\eta}_1(t) = a \eta_2(t)
\]

\[
\dot{\eta}_2(t) = -a \eta_1(t) + (1 - 2k_p) a \eta_2(t)
\]

Finally, let the time variable \( t = a \tau \), which implies that

\[
\frac{d}{d \tau} = \frac{1}{a} \frac{d}{d t}
\]

Equation (27) now becomes

\[
\frac{d \eta_1}{d \tau}(\tau) = \eta_2(\tau)
\]

\[
\frac{d \eta_2}{d \tau}(\tau) = -\eta_1(\tau) + (1 - 2k_p) \eta_2(\tau)
\]

which is independent of \( a \). The same transform can be applied to the system governed by the \( A_1 \) matrix, which then also becomes independent of \( a \). The resulting system equations in these transformed coordinates are

\[
\eta(\tau) = A_{\eta_1} \eta(\tau), \quad i = 1, 2
\]

\[
A_{\eta_1} = \begin{bmatrix}
0 & 1 \\
-1/2 & (1 - k_p)
\end{bmatrix}, \quad A_{\eta_2} = \begin{bmatrix}
0 & 1 \\
-1 & (1 - 2k_p)
\end{bmatrix}
\]

These same transformations can be applied to the solution of the limit cycle problem \( \Phi^* \). Therefore, when \( K = [k_p a, a^2/2] \), the effect of increasing \( a \) is to scale down the amplitude and scale up the frequency of the resulting oscillations. Equivalently, the switching intervals \( t_1 \) and \( t_2 \) are scaled by \( 1/a \). This result allows us to present the following proposition.

**Proposition 2:** Let the solution of the limit cycle problem with \( a = a^* \) be denoted by \([\gamma^*(0)^T, t_1^*, t_2^*]^T \). Then, if \( a = a^* c_1 \), where \( c_1 > 0 \), the corresponding solution to the limit cycle problem is \( 1/c_1 [\gamma^*(0)^T, t_1^*, t_2^*]^T \).
Finally, we can show that the bias of the limit cycles can be directly controlled. Consider the system
\[
\dot{x}(t) = ax(t) + v(t) + \theta y_0 W_r[v;0](t)
\]
\[
\dot{\sigma}(t) = x(t) - y_r
\]
\[
v(t) = -k_1 x(t) - k_2 \sigma(t)
\]
where \(y_r\) is a constant reference. Define the coordinate transform
\[
x'(t) = x(t) - y_r
\]
Placing (32) into these coordinates yields
\[
\dot{x}'(t) = (a - k_1) x'(t) - k_2 \sigma(t) + (a - k_1)y_r + \theta y_0 W_r[v;0](t)
\]
\[
\dot{\sigma}(t) = x'(t)
\]
(33)
Next, redefine the coordinate transform \(\alpha\) as
\[
\alpha(t) = -k_2 \sigma(t) + (a - k_1)y_r + \theta y_0 W_r[v;0](t)
\]
(34)
Since \(y_r\) is constant, the derivative of \(\alpha\) possesses the same form as that used in (9). Therefore, (32), (10) possesses the same limit cycle as (9)-(10), with the exception of a constant shift in the coordinates \(\alpha\) and \(x\). We present this result as the following proposition.

**Proposition 3:** Let \(\Phi_0 = [\gamma^T(0), t_1, t_2]^T\) denote a solution of the limit cycle problem for (9)-(10). Then, the solution to the limit cycle problem for the system (32), (10) is equal to \(\Phi^* = [\gamma_1(0) + y_r, \gamma_2(0) - (a - k_1)y_r, t_1', t_2']^T\), where \(\gamma(0) = [\gamma_1(0), \gamma_2(0)]\).

V. Simulation Results

We now continue our examination of the limit cycles through simulation. First, we explore the variation of the solution \(\Phi^*\) to the limit cycle problem with respect to the controller gain \(k_p\). These simulations are performed on a system obeying (9), where \(K = [k_p a, a^2/2]\). Outside of determining stability of the subsystems, the effect of the gain \(k_p\) on the limit cycle solution is difficult to determine analytically; we instead explore its effect in simulation. Simultaneously, we verify the proposed Newton-Raphson method by comparing its results to those observed in simulation.

Fig. 4 show the switching times of the limit cycle as computed by both the Newton-Raphson algorithm and directly from simulation. The range of \(k_p\) considered was 0.55 to 0.99. There are several features of note on these graphs. First, we are able to confirm the algorithm’s effectiveness at computing the solution to the limit cycle, as the simulation results agree very closely with the algorithm results. Second, looking at Fig. 4, we see that as \(k_p\) approaches 1, the system spends more and more time in the play region (denoted \(t_2\)) versus the linear region (denoted \(t_1\)). This is because the eigenvalues in the linear region are significantly faster than those in the play region when \(k_p\) is high, meaning the system must spend more time in the play region to keep the system in steady state. Accordingly, smaller values of \(k_p\) mean the system spends more time in the linear region than the play region.

Fig. 5 shows the variation of the initial conditions with \(k_p\). Again, we see that the simulation and algorithm calculations
are in tight agreement. Fig. 5 also indicates that as \( k_p \) approaches 0.5, the initial conditions rapidly grow in size. This signals a rapid growth in the amplitude of the limit cycles for \( k_p \), with the system becoming unstable for \( k_p > 0.5 \). Furthermore, Fig. 6 shows that the amplitude of the oscillations is strongly correlated with the size of \( \alpha(0) \). Interestingly, \( x(0) \) seems to level out around \( k_p = 0.65 \), after which \( x(0) \) stays in a small neighborhood of \(-0.5\) to \(-0.6\) for higher \( k_p \) values. For an estimate of the period, we can look to the damped frequency of the closed-loop system in the linear region, \( \omega_d \), which equals

\[
\omega_d = \sqrt{3/4 + k_p - k_p^2}
\]

Such an estimation is motivated by the expectation that the dynamics of (9) to tend towards a linear oscillator as \( k_p \) approaches 0.5. Fig. 7 compares the period computed by the algorithm with the period indicated by \( \omega_d \). The resulting periods are in reasonably close agreement. We can also see from this graph that though the switching times greatly vary with \( k_p \), as seen in Fig. 4, the period of the oscillations vary over a much narrower window.

VI. CONCLUSIONS AND FUTURE WORK

We have demonstrated the ability to predict and manipulate self-excited limit cycles generated by a system with hysteresis. Of particular interest is our result that the amplitude and periods of these limit cycles have linear relationships with system parameters. We also showed that a Newton-Raphson algorithm can be used to compute the closed-form of the limit cycles.

Future work will include an analytical proof of the existence of limit cycles for the system considered here. We will also look to consider other more complicated dynamics and hysteresis operators.

REFERENCES


