Observer Design for Motorcycle Lean and Steering Dynamics Estimation: a Takagi-Sugeno Approach

Dalil Ichalal, Habib Dalbadji, Hichem Arioui, Saïd Mammar and Lamri Nehaoua

Abstract—In this paper, a nonlinear motorcycle model is considered in order to estimate both the lean and steering dynamics. The model is transformed into a Takagi-Sugeno (T-S) form using the well-known sector nonlinearity approach. The first contribution of this work is the exactness of the obtained T-S model compared to the considered nonlinear model, where the weighting functions of the T-S model depend on unmeasured state variables. A novel approach to construct a nonlinear unknown input observer is proposed. The objective is the simultaneous reconstruction of the state variables and the rider’s torque. The observer’s convergence is studied using Lyapunov theory guaranteeing boundedness of the state and unknown input estimation errors which is expressed by the Input to State Practical Stability (ISpS). Stability conditions are then expressed in terms of Linear Matrix Inequalities (LMI). Finally, simulation results are provided to confirm the suitability of the proposed nonlinear observer.

I. INTRODUCTION

Currently, powered two-wheeled (PTW) vehicles is a mean of transport increasingly sought after, especially for the opportunities it offers to sidestep traffic congestion. This increase in vehicle fleet to PTW is accompanied by the blast of the number of accidents. For a long time, industrial societies record on their roadways thousands of deaths and other fatalities per year. Road safety institutions have launched several preventive actions (radar, tickets, etc.) and research programs headed for safety systems, mainly for cars. Some of these steps have brought their results, since the number of fatalities recorded a significant decrease over the past few years. However, if the number of fatalities experiences an overall decrease, the PTW remains a particularly dangerous transportation, [14], [2].

Development of safety systems for cars has reached a certain maturity. Unfortunately, for motorcycles it is not the case. The most prominent example is that of the ABS, that exists for 20 years, but is not still standard equipment for PTW and its use remains marginal, [8]. The direct transposition of security systems, from cars to motorcycles, is not obvious because of the complex motorcycle dynamics (highly nonlinear, [6], [9]), for example. In addition, ahead of the design of a safety system, we must ensure the availability of the relevant dynamic states in order to quantify the risk (loss of control, skidding, etc.). PTW dynamics can be estimated through suitable sensors and/or observers. The first way is generally avoided for several reasons: price, noise measurement, feasibility, etc. On the other hand, there are only few works dealing with the problem of motorcycle state observation.

In the literature, most studies have mainly concerned the estimation of the lean dynamics unlike the steering one. Different techniques have been proposed to estimate the roll angle: for example, frequency separation filtering [4] or extended Kalman filters, [19]. These techniques, performed under restrictive assumptions (dynamic steering is neglected, tire-road forces are linear, etc.), are not robust against the variations of the forward velocity.

The topic of estimation of the steering angle (not the rider’s torque reconstruction) is not well covered in literature as the lean angle estimation problem. However, a few results have been obtained in [7], where an LPV observer has been used to design single-sensor control strategies for a semi-active steering damper. The approach is a simple gain scheduling for an LTI motorcycle model under three constant forward velocities (50, 100 and 140 m/s). Unfortunately, no guarantees for stability or convergence of the LPV observer are given outwards these constant velocities.

To the best knowledge of the authors, the simultaneous estimation of the lean and the steering dynamics (rider’s torque) have never been addressed.

II. PROBLEM STATEMENT

Our long term objective points the identification of all pertinent inputs and dynamic states improving the risk quantification of the loss-of-control during cornering. Indeed, inadequate cornering is responsible for most motorcycle fatalities, especially for single motorcycle crashes. For safe cornering, riders should respect: 1) a suitable speed before starting the corner, 2) the road conditions (under weak friction) and 3) weather conditions do not allow optimal visibility when riding. Early warning systems are based generally on related work carried out for standard cars [16]. The goal is the synthesis of a function estimating the maximum safe speed at which a vehicle can be kept stable on the road while moving at a constant longitudinal velocity on a circular section. This velocity depends, among other factors, on the lateral friction $\mu_{lat}$ whose computation involves all the dynamic states of the PTW and a good modeling of the tire-road contact [16]. This makes the success of such warning systems strongly dependent on the availability of dynamic states of the motorcycle. Correlating this fact with the highly nonlinear dynamics, standard observation approaches are questionable and less efficient. To answer this real challenge, we address in this paper a nonlinear observer synthesis based on Takagi-Sugeno approach for PTW vehicles.

All the authors are with the IBISC Laboratory, Evry-Val-d’Essonne University, 40, rue de Pelvoux, 91020 Evry Courcouronne Cedex {dalil.ichalal}@ibisc.univ-evry.fr
Takagi-Sugeno fuzzy structure is one of the most interesting approaches to model nonlinear behaviors as proven in recent years [17], [18]. Indeed, it offers a way to represent nonlinear complex behaviors by a more tractable mathematical formulation inspired by linear models. It consists on decomposing the operating state space on several regions and each region is modeled by a linear model. Thanks to nonlinear weighting functions which satisfy the convex sum property, the overall nonlinear behavior of a system is characterized in a compact set of the state space. In recent years, some works are dedicated to the study of nonlinear systems via T-S models, especially in observers design [10], [3], [1]. The proposed observer in this paper is inspired from the linear adaptive one in [20] and adapted to T-S models with unmeasurable premise variables which constitutes an open and interesting field of research. Indeed, the most developed work in the literature is limited only to the case of T-S models with measurable premise variables. T-S models with unmeasurable premise variables have an interesting property, namely, the ability to transform a general nonlinear model into a T-S model by the use of nonlinear sector nonlinearity transformation with no loss of information (see [11] and references therein), and it is pointed out in [21] that T-S models with unmeasurable premise variables can describe a wider class of nonlinear systems.

The paper is organized as follows: Section III presents the formalism of a motorcycle and a Takagi-Sugeno formulation of the considered model. In Section IV, an unknown input and state observer for estimating the motorcycle lateral dynamics and the steering torque is synthesized. The convergence of the observer is studied with Lyapunov theory and an optimization problem under LMI constraints is provided to design the observer in such a way to guarantee the IPSP Property which illustrates the boundedness of the state and unknown input errors. Finally, Section V provides some simulations results and discussions on the proposed observer.

III. MOTORCYCLE MODEL DESCRIPTION

A. Nonlinear model of the motorcycle

The lateral dynamics of a motorcycle are represented by a model with four equations [15], [16] describing the lateral motion, due essentially to the effect of lateral forces from the front and rear wheels \((F_{f} \text{ et } F_{r})\) and the yaw and roll motions under rider’s steering actions. The study of such a model aims to reconstruct the dynamic states of a motorcycle in cornering situation.

These movements expressed by the following equations that correspond respectively to lateral, roll, yaw and steering motions:

\[
\begin{align*}
F_{f} + F_{r} &= M(\dot{v}_{y} + v_{y} \dot{\phi} + M_{f} k \dot{\psi} + (M_{f} + M_{b}) \ddot{\phi}) + M_{f} \ddot{e} \\
\Sigma M_{e} &= (M_{f} + M_{b}) \dot{v}_{y} + a_{1} \ddot{\phi} + a_{2} \dot{\psi} + a_{3} \ddot{\alpha} + a_{4} \psi \dot{y} \\
\Sigma M_{e} &= M_{f} (\dot{v}_{y} + v_{y} \dot{\psi}) + a_{2} \ddot{\phi} + b_{1} \psi + b_{2} \ddot{\alpha} - b_{3} v_{y} \dot{\phi} \\
\Sigma M_{e} &= M_{f} v_{y} + a_{3} \dot{\phi} + c_{1} \dot{\psi} + c_{2} \ddot{\phi} - a_{5} v_{y} \dot{\phi} + c_{3} v_{y} \dot{\phi} + + K \ddot{\phi} \\
\end{align*}
\]

where

\[
\begin{align*}
\Sigma M_{e} &= (M_{f} + M_{b}) g \sin(\phi) + (M_{f} g - \eta F_{f}) \sin(\delta) \\
\Sigma M_{e} &= (M_{f} + M_{b}) \dot{\psi} + (M_{f} g - \eta F_{f}) \sin(\phi) + (M_{f} g - \eta F_{f}) \sin(\delta) - \eta F_{f} \tau + \tau \\
\end{align*}
\]

The lateral forces \(F_{f} \text{ and } F_{r}\) acting, respectively, on the front and rear wheels depend on the sideslip angles \(\alpha_{f} \text{ and } \alpha_{r}\) and camber angles \(\gamma_{f} \text{ and } \gamma_{r}\), are expressed by

\[
\begin{align*}
F_{f} &= -C_{f1} \alpha_{f} + C_{f2} \gamma_{f}, \quad F_{r} = -C_{r1} \alpha_{r} + C_{r2} \gamma_{r} \\
\end{align*}
\]

where \(\alpha_{f} = \frac{v_{y} \psi - \eta \ddot{\phi}}{v_{x}}\) and \(\gamma_{f} = \phi + \delta \cos(e)\). \(\gamma_{r} = \phi. M_{f}, M_{r} \text{ and } M\) are the masses of the front frame, the rear frame and the total mass of the motorcycle. \(j, k \text{ and } e\) are the distances from the center of gravity of the front frame to the ground, to the center of gravity of the rear frame and to the fork respectively. \(L_{f}, L_{r}\) and \(h\) are the distances from the center of gravity of the rear frame to the front wheel contact, to the rear wheel contact and to the ground respectively. \(e\) is the steering head angle. \(i_{y}, i_{z} \text{ and } I_{e}\) are the polar moments of inertia of the front wheel, of the rear wheel and the camber inertia of the rear wheel respectively. \(R_{f} \text{ and } R_{r}\) are the radius of the front wheel and the rear wheel respectively. \(g\) is the acceleration to the gravity. \(\sigma_{f}, \sigma_{r}, C_{f1}, C_{r1}, C_{f2} \text{ and } C_{r2}\) are the front and rear tire relaxation lengths, cornering stiffnesses and camber stiffnesses respectively. \(\eta\) is the pneumatic trail, \(v_{x}\) is the forward speed and \(Z_{f}\) is the front wheel load.

In this work, a normal riding is considered (without taking into account the limit situations) which justifies the linear form of the lateral forces \(F_{f} \text{ and } F_{r}\) with respect to both sideslip and camber angles.

By replacing the mathematical expressions of the forces in the dynamics model and by choosing the state vector as \(x(t) = [v_{y} \, \psi \, \dot{\phi} \, \delta \, \ddot{\delta}]^{T}\), the system is rewritten as follows

\[
E \dot{x}(t) = A(x(t), v_{y}) x(t) + B \tau(t)
\]

where \(E\) is a constant nonsingular matrix, \(B\) is a constant
matrix and \( A(x) \) is a nonlinear matrix given by
\[
A(x) = \begin{pmatrix}
    a_{11}(v) & a_{12}(v) & 0 & a_{14} & a_{15}(v) & a_{16} \\
    a_{21}(v) & a_{22}(v) & a_{23}(v) & a_{24} & a_{25}(v) & a_{26} \\
    0 & a_{32}(v) & 0 & a_{34}(\phi) & a_{35}(v) & a_{36} \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    a_{41}(v) & a_{42}(v) & a_{43}(v) & a_{44}(\phi) & a_{45}(v) & a_{46} \\
    0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

errors converge to zero, the term \( \Delta(t) \) converges also towards zero. In addition, since the weighting functions

premise variables are given by

where

It is important to notice that the motorcycle, contrarily to a

output equation

angle and the steering angle rate

interval where the motorcycle is stable, the bounds of the

by

(see [18] for more details), the obtained model is

\[
X^TY + Y^TX \leq X^TGX + Y^TG^{-1}Y \quad G > 0
\]  

The objective of this section is to provide a new approach in order to design observers for T-S fuzzy systems with unmeasurable premise variables which is the case for the considered motorcycle model. The observer aims to estimate, simultaneously, the state variables and the rider’s torque, especially, the lateral velocity \( v_y \), the roll angle \( \phi \) and the steering torque \( \tau \) which constitute the most important variables in synthesizing risk function for riding assistance.

The following nonrestrictive assumptions can be made

• the state \( x(t) \) is bounded (stable or stabilized motorcycle)
• \( rank(CB) = rank(B) = n_u \) (\( n_u \) is the dimension of the unknown input vector)
• the torque derivative \( \dot{\tau}(t) \) is bounded by \( \tau_{max} \)
• the pairs \( (A_i,C) \) are observable

the first assumption holds in open loop for a reduced longitudinal speed range and also assuming that the motorcycle is under driver control. In the case of the considered motorcycle model, \( \text{dim}(\tau) = n_u = 1 \). The third assumption is not restrictive since the variation of the rider’s steering torque is bounded.

A. State estimation

Let us consider the T-S model given by the equations (7) and (8). The state observer is then given by the following equations

\[
\begin{align*}
    \dot{\hat{x}}(t) &= \sum_{i=1}^{8} \mu_i(v_x)A_i\hat{x}(t) + B\tau(t) & (7) \\
    \hat{y}(t) &= C\hat{x}(t) \\
    \dot{\hat{y}}(t) &= \Gamma F(e_y(t) + \sigma_e(t)) \\
    e_y(t) &= y(t) - \hat{y}(t) & (10)
\end{align*}
\]

Let us consider the state estimation error \( e(t) = x(t) - \hat{x}(t) \) and torque estimation error \( e_\tau(t) = \tau(t) - \hat{\tau}(t) \). The matrices

(10)

Using equations (7), (8) and (10), the state estimation error obeys the following differential equation

\[
\dot{e}(t) = \sum_{i=1}^{8} \mu_i(v_x)\Phi_i e(t) + Be_\tau(t) + \Delta(t) & (11)
\]

where \( \Phi_i = A_i - L_iC \) and \( \Delta(t) = \sum_{i=1}^{8} (\mu_i(v_x,x) - \mu_i(v_x,\hat{x}))A_i x(t) \). Notice that if the state estimation errors converge to zero, the term \( \Delta(t) \) converges also towards zero. In addition, since the weighting functions
are bounded and the state vector $x(t)$ is also bounded (see assumption 1), the term $\Delta(t)$ is thus bounded. The objective is to design the matrices $L_i$, $F$ and the scalars $\Gamma$ and $\sigma$ guaranteeing an accurate estimation of the state and the unknown input by minimizing the bound of the state and unknown input estimation errors.

**B. Observer’s convergence study**

From Assumption 1 and the fact that the functions $\mu_i$ are bounded, the term $\Delta(t)$ is bounded. Indeed, the system is stable which provides bounded states for bounded input $\tau(t)$. In this context the definition 1 is used.

**Definition 1:** ([13], [12]) The system (11) verifies the Input To State Practical Stability (ISpS) if there exists a $\mathcal{L}_1$ function $\beta : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$, a $\mathcal{L}_1$ function $\alpha : \mathbb{R} \rightarrow \mathbb{R}$ and a constant $d$ such that for each input $\Delta(t)$ satisfying $\|\Delta(t)\|_\infty < \infty$ and each initial conditions $e(0)$, the trajectory of (11) associated to $e(0)$ and $\Delta(t)$ satisfies

$$\|e(t)\|_2 \leq \beta(\|e(0)\|_2 + \alpha(\|\Delta(t)\|_\infty) + d$$

(12)

Under bounded perturbation term $\Delta(t)$, the observer (10) is synthesized by solving the optimization problem under LMI and LME constraints given in the Theorem 1 in order to ensure the convergence of the observer and the convergence region defined by $d$ and $\alpha(\|\Delta(t)\|_\infty)$.

**Theorem 1:** Under the Assumption 1, given a positive scalars $\sigma$ and $\gamma$ and $\alpha \in [0,1]$, if there exists a symmetric and positive definite matrix $P$, gain matrices $M_i$ and positive scalars $G, \gamma, \eta$ and $S$ solution to the following optimization problem, $i = 1, \ldots, 8$

$$\min_{P, M, \eta, \gamma} \alpha \eta + (1 - \alpha)\gamma$$

s.t.

$$\begin{bmatrix} B^TP & FC \\ -\frac{1}{\beta}^T P & -\frac{1}{\beta} P \end{bmatrix} < 0$$

(13)

where

$$\begin{align*}
\Omega_i &= A_i^TP + P A_i - M_i C - C^T M_i^T \\
\Psi &= -\frac{1}{\alpha} B_i^T P + \frac{1}{\alpha} G, \quad Q = \text{diag}(P, \frac{S}{\gamma})
\end{align*}$$

(14)

then the state and torque estimation errors are bounded. The gains of the observer are computed from $L_i = P^{-1} M_i$, $\Gamma = S^{-1}$ and $F$ is obtained directly by solving the above optimization problem. The attenuation level of the transfer from $\Delta(t)$ to state and unknown input estimation errors is bounded and given by the quantity $\sqrt{\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}}$. In addition, If $\Delta(t) = 0$ the state and unknown input estimation errors converge to a set around the origin with a size defined by the quantity $\sqrt{\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}}$. The parameter $\Gamma$ is then chosen sufficiently large in order to have a minimal value of $\delta$ which guaranteeing a more accurate estimations. Otherwise, if $\Delta(t) \neq 0$, the state estimate errors guaranty the Input To State Practical Stability (ISpS) given by the following property

$$\|e_\tau(t)\|_2 \leq \sqrt{\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}} \left( e^{-\frac{\eta t}{\alpha}} + \sqrt{\frac{\gamma}{\alpha}} \|\Delta(t)\|_\infty + \sqrt{\frac{\delta}{\alpha}} \right)$$

(18)

where $e_\tau^T(t) = [e^T(t) \ e^T_\tau(t)]^T$.

**Remark 1:** Of course, the equation (13) is a Linear Matrix Inequality and it can be solved easily with Yalmip toolbox.

**Proof:** In order to prove the convergence of the state and torque estimation errors, let us consider the following Lyapunov function

$$V(t) = e^T(t) P e(t) + \frac{1}{\sigma} e^T_\tau(t) \Gamma^{-1} e_\tau(t)$$

(19)

where $P = P^T > 0$ and $\Gamma > 0$. In the framework of the motorcycle model, the dimension of $\tau$ is 1 then the parameter $\Gamma$ is a scalar. (For more general case, we have $\Gamma \in \mathbb{R}^{n_\tau}$ where $n_\tau$ is the dimension of the unknown input vector).

According to the equation (11), the time derivative of $V(t)$ is given by

$$\dot{V}(t) = \sum_{i=1}^{8} \mu_i(v_i, \hat{x}(t)) e_i^T(t) (\Phi_i^T P + P \Phi_i) e(t)$$

$$+ \frac{2}{\sigma} e_i^T(t) \Gamma^{-1} e_i(t) + 2 e_i^T(t) P \Delta(t) + 2 e_i^T(t) B \hat{P} e(t)$$

(20)

Knowing that $\dot{e}_i(t) = \dot{\tau} - \hat{\tau}(t)$ and given the expression of $\hat{\tau}(t)$ (10), the time derivative of the Lyapunov function (20) becomes

$$\dot{V}(t) = \sum_{i=1}^{8} \mu_i(v_i, \hat{x}(t)) e_i^T(t) \Omega_i e(t)$$

$$- \frac{2}{\sigma} e_i^T(t) F \hat{\tau}(t) + 2 e_i^T(t) P \Delta(t)$$

(21)

where $\Omega_i = \Phi_i^T P + P \Phi_i$. Using the output equations, we have $e_i(t) = C e(t)$ and $\hat{e}_i(t) = \hat{C} e$, then, equation (21) can be rewritten under the following form

$$\dot{V}(t) = \sum_{i=1}^{8} \mu_i(v_i, \hat{x}(t)) (e_i^T(t) \Omega_i e(t) - \frac{2}{\sigma} e_i^T(t) F C \Phi_i e(t))$$

$$- \frac{2}{\sigma} e_i^T(t) F C \Delta(t) - 2 e_i^T(t) F C \hat{e}(t) + \frac{2}{\sigma} e_i^T(t) \Gamma^{-1} \hat{\tau}(t)$$

$$- \frac{2}{\sigma} e_i^T(t) F C B \hat{e}_i(t) + 2 e_i^T(t) P \Delta(t)$$

(22)

Using lemma 1 and the fact that the first derivative of $\tau$ is bounded by $\tau_{\max}$ (assumption 1), we obtain

$$\frac{2}{\sigma} e_i^T(t) \Gamma^{-1} \hat{\tau}(t)$$

$$\leq \frac{1}{\sigma} e_i^T(t) \Gamma^{-1} G \hat{e}_i(t) + \frac{1}{\sigma} \hat{\tau}(t) \Gamma^{-1} G^{-1} \Gamma^{-1} \hat{\tau}(t)$$

$$\leq \frac{1}{\sigma} e_i^T(t) \Gamma^{-1} G \hat{e}_i(t) + \frac{1}{\sigma} \tau_{\max} \lambda_{\max}(G^{-1} \Gamma^{-1})$$

(23)
and using assumption 1. Since the rank(CB) = rank(B), it is possible to obtain matrices F and P such that BTP = FC ...

Note that this phenomenon can be reduced by reducing the value of \( \gamma \). A compromise can then be obtained.

It follows

\[
V(t) \leq -\alpha V(t) + \gamma \Delta(t)\|\Delta(t)\|^2 + \delta
\]

where \( \alpha \) and \( \gamma \) are positive scalars, \( Q = \text{diag}(P^{-1}, \alpha) \), and \( \delta = \frac{1}{\sigma^2} \max_{i=1}^{8} \lambda_{\text{max}}(\Gamma^{-1}G^{-1}) \).

Knowing that \( \lambda_{\text{min}}(Q)\|e_a(t)\|^2 \leq V(t) \leq \lambda_{\text{max}}(Q)\|e_a(t)\|^2 \), one obtains the inequality

\[
\|e_a(t)\|_2 \leq \sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \left( e^{-\alpha t} + \sqrt{\frac{\gamma}{\alpha}} \|\Delta(t)\|_\infty + \sqrt{\frac{\delta}{\alpha}} \right)
\]

According to Lyapunov formulation of Input To State Practical Stability (ISpS), the state and unknown input errors converge to a region which will be minimized in order to achieve a more accurate estimation of the states of the vehicle and the torque applied on the handlebar. This ball is smaller as the constant \( \delta \) and the attenuation level of the transfer from \( \Delta(t) \) to the state estimation errors are smaller. To enhance the performances of the observer, a minimal value of these quantities are obtained by the following reasoning:

Let us consider the quantity

\[
\sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \leq \sqrt{\eta}
\]

where \( \eta \) is a positive scalar. It is then sufficient to minimize the term \( \eta \) and assuming \( \lambda_{\text{min}}(Q) \geq 1 (Q > I) \), one obtains

\[
\sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \leq \sqrt{\eta}
\]

which is transformed easily into \((\alpha \eta)^2 I - Q^T Q > 0\). Using Shur’s complement lemma, the inequality (15) is obtained. The second quantity \( \sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \) is minimized by choosing the parameters \( \alpha \) and \( G \) sufficiently large for minimizing the term \( \delta \) and the term \( \sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \) is treated above. Finally, the bound of this term will be \( \sqrt{\delta/\alpha} \) multiplied by \( \eta \). Always in the purpose of minimizing the two quantities, in theorem 1, the chosen objective function is a linear combination between \( \eta \) and \( \gamma \). By choosing the change of variables \( S = \Gamma^{-1} \) and \( M_t = PL_t \), the linear matrix inequalities in theorem 1 are obtained.

**V. SIMULATION RESULTS**

The nonlinear system, including longitudinal and lateral dynamics of two-wheeled vehicle is used. It requires three inputs: the rider’s steering torque applied on the handlebars \( \tau \) (see Figure 2) and the angular velocities of both front and rear wheels \( \omega_f \) and \( \omega_r \). The observer estimating the lateral dynamics and steering torque using only the measured states \( \psi, \phi \) given by the inertial unit and \( \delta \) and \( \delta \) obtained from a suitable encoder. The gains of the observer were calculated by solving the optimization problem under LMI constraints proposed in theorem 1. The obtained attenuation level, from the term \( \Delta(t) \) to the state and unknown input estimation errors is \( \gamma = 2.6119 \). The initial conditions of the system are \( x(0) = [0 0 0 0 0 0] \) and those of the observer are \( \hat{x}(0) = [1 1 0 1 0 0.1]^T \). The adaptation law providing an estimation of the steering torque is designed in such a way to have a fast convergence of \( \hat{\tau}(t) \) to \( \tau(t) \). The chosen parameters are \( \alpha = 1, \sigma = 5 \) and \( a = 0.99 \). If \( \Delta(t) = 0 \), the state and unknown input estimation errors \( e(t) \) and \( e_a(t) \) converge to a ball around the origin having the size \( \sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \approx 0.1 \), for \( \tau_{\text{max}} = 40 \). In the steady state \( \|\Delta(t)\|_\infty \) is less than 0.07, the attenuation of \( \Delta(t) \) is then given by \( \sqrt{\frac{\lambda_{\text{max}}(Q)}{\lambda_{\text{min}}(Q)}} \leq \sqrt{\eta} \|\Delta(t)\|_\infty \approx 2.78 \). A satisfactory state and unknown input estimation results are then obtained as shown in figures 3 illustrating the state estimation errors and in 2 illustrating the torque estimation.

In order to test the observer in the presence of measurement noise, let us consider the same observer’s parameters and assume that the measurement signals are affected by a centered and random noise with magnitude 5% of the maximal values of the measured variables. The estimation of the steering torque (see 2 (bottom)) is acceptable, but the effect of the measurement noise is visible. This is due to the presence of a derivation in the adaptation law leading to estimate the steering torque and also due to the large value of \( \Gamma \). Note that this phenomenon can be reduced by reducing the value of \( \Gamma \). A compromise can then be obtained.
by an adequate $\Gamma$ selected sufficiently large to ensure an accurate estimation and sufficiently less to reduce the effect of the measurement noise. Other possibility to enhance the robustness of the observer with respect to measurement noise is to use a sliding mode differentiator to estimate the first derivative of the output error.

Of course the ISpS property is weaker than ISS or asymptotic stability, but the proposed observer with the torque estimation adaptation law can only guarantee the ISpS property. Nevertheless, the simulation results are satisfactory. Furthermore, some parameters in the proposed design approach are fixed a priori, in future work, this will be treated in order to take them into account in the optimization problem leading to optimal values which give better results and smaller than theoretic bounds values.

VI. Conclusion

In this paper, an unknown input and state observer for estimating the lateral dynamics and the steering torque in motorcycles is proposed. A nonlinear model is considered with some nonlinearities and time-varying longitudinal velocity. Based on the obtained T-S model, an adaptive observer is proposed. The convergence of the observer is studied using Lyapunov theory and LMI conditions are given to ease the design. It is pointed out that this observer guaranty the ISpS property. The bound of the convergence region is studied and an optimization technique is used to minimize the radius of this region in order to enhance the estimation quality. Simulation results are provided which illustrate the effectiveness of the proposed observer in estimating both the states and the steering torque. In future work, the T-S model will be refined by taking into account some other nonlinear behaviors, namely, the nonlinear form of the lateral forces (Pacejka’s model). The observer will be then redesigned for more accuracy. In addition, the lateral and longitudinal forces estimation will be considered for estimating the road adhesion. This is for synthesizing risk functions which inform the rider on dangerous situations. Validation results in real situations will be published in future papers.

REFERENCES