A Convergent Solution to the Multi-vehicle Coverage Problem

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Abstract—The paper presents a new solution to the multi-vehicle coverage problem. The proposed algorithm guarantees complete coverage and provides collaborative behaviors of vehicles, despite the fact that it does not explicitly exploit any computationally intensive optimization technique. The algorithm can deal with any mission domain, including regions with irregular shapes, multi-connected and disjoint regions. It gives reasonably good solutions even for partially connected multi-vehicle systems. The coverage problem for regions the shape of which change in time regardless the vehicle movement is also solved by the proposed algorithm.

I. INTRODUCTION

There exist several different definitions of the coverage problem. This has recently become a focus of the scientific community due to its increasing importance in a number of applications. Some applications, which have triggered the scientific community, where the coverage problem is inherently included and required to be solved, are: sensor deployment, routing, scheduling, power control, surveillance, monitoring, target tracking, hazard detection, planetary exploration, reconnaissance, rescue, mowing, cleaning, de-mining, snow removal, fire extinguishing and agricultural spraying, see [7], [5], [4], [8], [1] for a few examples.

Mobile sensor network coverage (sensor deployment problem or active sensing) usually refers to the problem of finding an optimal sensor location, from which sub(optimal) coverage is achieved, and a control to guide the sensors to such locations. In [6] and [15], the authors have introduced a density function to represent the frequency of random events taking place over the mission domain. They have developed a gradient-based algorithm to find a convergent sub(optimal) solution to the sensor network coverage problem. The approach presented in [23] solves the same problem by introducing the notion of 'virtual forces', while the solution given in [12] stochastically selects sensor locations by minimizing error covariance. In [8], the authors have developed a decentralized coverage control algorithm which is based on Voronoi partitions [9], [16].

Additionally, many approaches to guide vehicles to explore an unknown terrain to create a map exist. Most of these approaches are based on finding the closest unexplored areas (frontiers) during the execution. The work based on a single vehicle has been presented in [10], [17], [18], [20], [21], while algorithms for a multi-vehicle system, based on the frontier-like paradigm, have been introduced in [4], [5]. An interesting variant of such an approach, based on a market economy, is proposed in [22]. There also exist different heuristic-based approaches for exploring an unknown terrain, such as the one based on ants behavior [14]. Game theory based exploration in an unknown environment has been used in [19]. Finally, a receding horizon control approach to minimize spatio-temporal coverage of a known terrain, while satisfying collision avoidance and initial and final location, has been introduced in [2].

The multi-vehicle path planning coverage that is the focus of the paper addresses the following problem. Given a number of vehicles (or a sensor network), their initial positions as well as a known mission domain (area to be covered or explored), find a control strategy (vehicle trajectories) such that each location of the mission domain is visited by at least one vehicle. The problem does not consider collision avoidance constraints which is the case of multi-air vehicles operating at different altitudes. The problem may be formulated as an optimization problem and solved within the dynamic programming framework [11], [3], although such an approach easily becomes computationally intractable for a large number of vehicles and large mission domains. In such a case it can even be difficult to find an optimal solution offline to be used as a benchmark for approximate solutions. For this reason, a challenge is to find an approximate that can be intuitively accepted as a "good solution" at least for the cases in which one can visually evaluate and compare possible alternatives (i.e. trajectories). Another challenge, which is inherently present in this problem, is to find a control strategy that guarantees the task completion. The task completion in the multi-vehicle path planning coverage problem has been studied in [13]. Although the developed algorithm guarantees task completion, the obtained results seem somewhat far from "naturally good" solutions.

In this paper we present a new continuous control strategy for a multi-vehicle system that generates "intuitively good" trajectories and, at the same time, guarantees task completion. The proposed control strategy is significantly simpler than most of the state-of-the-art algorithms, so that the computational cost is not an issue. The control algorithm is decentralized in that each vehicle uses its own information to find the movement within the current iteration sample. Regarding the communication network topology, the algorithm can be used both for a fully connected (each vehicle has the same information on the environment) and a partially connected system (each vehicle has partial information on the environment). The control is based on the geometry of the currently uncovered region, so it gives good results for
different shapes of the mission region. We show that the proposed control strategy can also be used for a dynamic environment, where the area of interest is continuously changing such in case of bushfires. In addition, it is shown that the algorithm can be used for monitoring purposes.

Section II gives the basic nomenclature used within the paper and introduces the problem formulation. The proposed ‘coverage’ algorithm is presented in Section III. Sections IV, V and VI illustrate simulation results for a fully connected system, a partially connected system and different cases with dynamic regions, respectively. The conclusion is outlined in Section VII.

II. NOMENCLATURE AND PROBLEM FORMULATION

Multi-vehicle system: \( \mathcal{A} \) is a \( N \)-multi-vehicle (agent) autonomous system, \( A_i \), a vehicle which belongs to this system, where \( i \in \{1, 2, ..., N\} \), and \( N \) is the number of vehicles.

Configuration space: \( C \) is the configuration space for each vehicle \( A_i \). In this paper, \( C \in \mathbb{R}^2 \) since we consider a vehicle to be a pointwise mobile robot fully determined by its position vector \( q_i \).

Region to explore: \( D \) is a subset of \( \mathbb{R}^2 \) which represents the region to be explored by \( \mathcal{A} \), \( S(t) \) the currently unexplored region, where \( S(t) \subseteq D, \forall t \geq 0 \), and \( S_i(t) \) the currently unexplored region seen from the agent \( A_i \).

Sensor range: Although there are always uncertainties in the measurement of the locations from far the vehicle, we assume that there is a sensor range, sufficiently close to the vehicle, where the uncertainties are not present. The sensor range of a vehicle \( A_i \), denoted by \( \mathcal{W}_i \), is defined as

\[
W(q_i) = \{ q_i' \in D : ||q_i - q_i'|| \leq d_i \}. \tag{1}
\]

The total sensing range for an \( N \)-multi-vehicle system at each time instant \( t \) is given by

\[
W(t) = \bigcup_i W(q_i(t)). \tag{2}
\]

The proposed algorithm is based on the current location of the center of mass (centroid) of the region to explore \( D \).

Centroid: \( q^{CD}(t) \) is the centroid location of the region \( S(t) \), namely

\[
q^{CD}(t) = \frac{\int_{S(t)} \rho(q) q dq}{\int_{S(t)} \rho(q) dq}, \tag{3}
\]

where \( \rho(q) \) is the mass density function at location \( q \).

Forgetting factor: it represents the factor whereby the system forgets the information on locations previously visited. In this paper, we take a uniform mass density function \( \rho(q) = 1 \) for the region \( S(t) \). This choice is suitable for all experimental cases where the system is not required to repeatedly monitor the same region \( D \). To cover also cases in which \( S(q(t)) = \{ q \in D : \rho(t; q) \geq p \}, \forall t \geq 0 \), and \( p \in [0, 1) \) is an arbitrarily selected threshold, we use the density function

\[
\rho(t; q) = \begin{cases} 
1, & t \in (0, t_1) \\
1 - e^{-\gamma(t-t_1)}, & t \in (t_1, t_{i+1}),
\end{cases} \tag{4}
\]

where the factor \( \gamma > 0 \) and \( t_i \) \( (i = 1, 2, \ldots) \) are the selected constant and the moment when the location \( q \) is visited for the \( i^{th} \) time by any agent, respectively. The higher \( \gamma \), the higher is the system forgetting.

Control: We consider the kinematic model of each vehicle. However, the results can be easily extended to a dynamic model, since the proposed algorithm does not depend on the vehicle model. For a pointwise mobile robot, the kinematic model is given by

\[
\dot{Q} = U, \quad \tag{5}
\]

where \( Q = (q_1, q_2, \ldots, q_N)^T \) and \( U = (u_1, u_2, \ldots, u_N)^T \). \( u_i \) is the control action of the vehicle \( A_i \).

Coverage Problem (P): Given a system \( \mathcal{A} \), initial values \( q_i(0), \forall i \in \{1, 2, \ldots, N\} \), a region to explore \( D \), find a control strategy such that each location \( q \in D \) is sensed in final time, that is, there exists \( T < \infty \) such that \( \int_{S(t)} dS = 0 \) for all \( t > T \).

Note that the coverage problem \( P \) differs from the persistent monitoring problem in which a given system \( \mathcal{A} \) is supposed to continuously monitor the region \( D \). In Section VI-B, we slightly adopt the algorithm proposed for the coverage problem to solve the persistent monitoring problem.

III. CONTROL ALGORITHM

Assume that a system of only one vehicle is given (i.e. \( N = 1 \)), hence the control equation (5) becomes

\[
\dot{q}_1 = u_1. \tag{6}
\]

The proposed algorithm is given by the equation

\[
u_1(t) = -k(q_1(t) - q_1^*(t)), \tag{7}
\]

where

\[
q_1^*(t) = \arg\min_{q \in S(t)} \|q_1(t) - q\|^2 + \frac{1}{\|q_1^{CD}(t) - q\|^2}, \tag{8}
\]

\( k > 0 \), and \( q_1^{CD}(t) \) is the currently computed centroid of the area \( S_1(t) \) obtained using the information available to the vehicle \( A_1 \) at the moment \( t \). Loosely speaking, the vehicle is guided to go towards the closest frontiers (uncovered locations) which are further away from the current centroid \( q_1^{CD}(t) \).

To use the control algorithm for one-vehicle system in the case of the \( N \)-multi-vehicle system, an arbitrarily fixed ordering of the vehicles is used. Namely, the control action for the vehicle \( A_i \), and for \( t \in (t_0 + (i-1)\Delta T, t_0 + (i-1)\Delta T + N\Delta T) \), is given by

\[
u_i(t) = -k(q_i(t) - q_i^*), \quad \tag{9}
\]

\[q_i^* = \arg\min_{q \in S_i(t)} \|q_i((t_0 + (i-1)\Delta T)) - q\|^2 + \|q_i^{CD}((t_0 + (i-1)\Delta T)) - q\|^2, \tag{10}\]

where \( q_i^{CD}((t_0 + (i-1)\Delta T)) \) is the computed centroid of the area \( S_i \) as in eq. (3) obtained using the information available to the vehicle \( A_i \) at the time, \( t_0 + (i-1)\Delta T \). \( \Delta T \) is the time cycle after which the successive vehicle taken from the
IV. SIMULATION RESULTS FOR FULLY CONNECTED SYSTEMS

A fully connected multi-vehicle system is a system in which all vehicles have the same information on the region to be explored \( \mathcal{S}(t) \), i.e. for all \( i \) and \( j \), \( \mathcal{S}_i(t) = \mathcal{S}_j(t) \).
In such a moment, only a small uncovered area $S(t)$ is left, and the vehicles are all attracted by the same centroid. This issue can be avoided by distributing the last part of the task to only a reduced number of vehicles. Fig. 3-f shows the convergence rate, in which $\mu(S(t))$ is approximated by the number of cells of the discretized region $D$.

Simulation results for triangular and trapezoidal regions $D$ covered with a one and two-vehicle systems are depicted in Figs. 4 and 5, respectively. The results show that the proposed algorithm performs satisfactorily regardless the shape of $D$. This is a desired feature of any coverage algorithm. Again, the feature is a natural consequence of the proposed algorithm that explicitly involves geometry of the region by continuously computing its centroid.

Another interesting feature provided by the proposed algorithm is the possibility to deal with multi-connected regions. One example of a doubly connected region is given in Fig. 6. The rectangular hole can be considered as an area that has been already covered. The paths obtained show minimal crossover with this area.

Finally, the algorithm can also deal with regions with several connected components, as shown in Fig. 7. This result is also a consequence of the continuous computation of the centroid of $S(t)$.

V. SIMULATION RESULTS FOR PARTIALLY-CONNECTED SYSTEM

A partially connected multi-vehicle system is a system in which all vehicles use their own information on the environment, therefore, in general, $S_i(t) \neq S_j(t)$. In such a case, two vehicles $A_i$ and $A_j$ exchange their information only when $r(A_i, A_j)(t) \leq d_i + d_j$, where $r(A_i, A_j)(t)$ is the distance between the vehicles at time $t$, and $d_i$ and $d_j$ are the parameters of the sensor range in eq. (1). The proposed algorithm shows collaborative features even in cases in which the $d_i$ are selected to be small as in Figs. 8(a), 9(a) and 10(a), in which $d_1 = d_2 = d = 1$. For instance, in Fig. 8(a), at the beginning of the task execution both vehicles follow the boundary of the region. At the moment when the vehicles approach each other, they exchange their maps (see lower right-hand corner of Fig. 8(a)). If the system is a totally disconnected, meaning there is no map exchange between the vehicles, the vehicles would continue to follow the boundary of the region repeating the work already done by the other vehicle. What happens however, is that the vehicles exchange their maps just in the right moment when the redundancy could potentially occur. All other cases given in Figs. 8, 9 and 10 confirm the same conclusions.
Finally, the times needed to complete the task for different multi-vehicle systems are measured and compared. First, we compare the completion times obtained for a one-vehicle system and the worst considered case of the two-vehicle system (with \( d = 1 \)), by using various initial conditions. This comparison is depicted in Figs. 11(a) and 11(b). Here, the completion times of the worst considered case of a partially connected two-vehicle system (with \( d = 1 \)) are smaller than the completion times needed by a one-vehicle system. Similar conclusion can be drawn from Figs. 11(c) and 11(d), where a fully connected two-vehicle system is compared against the worst considered case of a partially connected three-vehicle system (with \( d = 1 \)). The average completion time of a partially connected three-vehicle system for \( d = 1 \) is much shorter than the time needed by a fully connected two-vehicle system. For the cases \( d = 2 \) and \( d = 3 \), the results have expectedly been improved in comparison to the case \( d = 1 \) (see Figs. 11(e) and 11(f)).

VI. SIMULATION RESULTS FOR DYNAMIC REGIONS

A. "Moving" regions

There exist cases in which the multi-vehicle system is required to monitor a "moving" region, as in a fire extinguishing task. This case is represented in Fig. 12. The region \( D \) identified by 1-1, transforms into the region 1-2. The region 1-2 then transforms to the region 2-3 and so on. For the Case I (Fig. 12(a)), in which the region moves slower than in the Case II (Fig. 12(b)), the one-vehicle system manages to catch the moving region after the region 2-3 moves into the region 3-4. For the Case II, the "catching" happens after the regions 1-1 moves into 1-2 (12(b)).

B. Persistent monitoring

Persistent monitoring of a given area is a type of coverage problem in which a multi-vehicle system is required to continuously cover the mission region \( D \). The proposed
algorithm can be easily utilized to solve this problem by introducing the forgetting factor $\gamma$, defined in eq. (4). Fig. 13 shows the simulation results obtained for a one-vehicle system using different values of the forgetting factor $\gamma$. The role of the threshold $p$ is to guarantee "coverage completion". Namely, in case $T_{\text{task}}^{\max} < T_p$, where $T_{\text{task}}^{\max}$ is the maximum time required to complete the "coverage" regardless the initial conditions, and $T_p$ is the time for which $\rho(T_p; q) = p$, the Theorem given in Section III still holds.

For $p = 0.9$, $T_p = \frac{\ln 10}{\ln 0.9}$, Fig. 11(a) roughly estimates $T_{\text{task}}^{\max} \approx 3200$, implying $\gamma < \frac{\ln 10}{T_{\text{max}}^{\max}} \approx 7 \cdot 10^{-4}$. Although even $\gamma = 8 \cdot 10^{-4}$ solves "the monitoring problem" (Fig. 13(a)), larger values do not manage to solve the problem resulting in an area which has not been covered by the vehicle. This is depicted in Fig. 13(b) with the stationary values obtained by different $\gamma$. However, the monitoring problem can be easily solved by a careful selection of the forgetting factor.

VII. CONCLUSION AND FUTURE WORK

The proposed continuous control algorithm provides a convergent solution to the multi-vehicle coverage path planning problem without using a computationally intensive optimization algorithm. The simulation results show that the solution is "natural", consisting of non-redundant paths and providing fast covering. The algorithm deals with various geometry of the mission region and generates good solutions to the coverage problem for "dynamic" regions.

Different constructions of the density function may give solutions in the presence of sensor uncertainties. The feature of avoiding "holes" present in multi-connected regions additionally gives a possibility to extend the proposed algorithm to include collision avoidance constraints, where the holes are considered as obstacles or "no-fly" zones. This would be possible if a receding horizon control scheme is used. These extensions will be the focus of future work.

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REFERENCES


