Toward Energy-Optimizing Rotary Wing MAV Formations

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Abstract—A bio-inspired energy-optimizing control methodology for a formation of rotary-wing Micro Air Vehicles (MAVs) is proposed. The key idea is that the recirculating airflow induced by each vehicle can be used to provide an upward airflow to a neighbor in the formation. The recirculating airflow around each vehicle is modeled using a doublet field, and the effects of recirculation on each vehicle in the formation are accounted for. A separation-bearing controller is implemented for a leader-follower formation, and the optimal formation is determined. It is found that the effects of recirculation are relatively weak, and only a single vehicle at a time can be the beneficiary of the recirculating flow of the rest of the formation. Within the optimal configuration, significant reduction in hover power can be realized for the lead vehicle.

I. INTRODUCTION

A wide variety of Unmanned Aircraft Systems (UAS) are becoming more commonplace in many civilian and military applications [1]. Examples include surveillance and critical infrastructure monitoring [2]. Owing to lower cost, ease of deployment and maintenance, there is a growing realization that the class of UAS that will find the most users will be the portable (small or micro UAS) that typically weigh less than a few pounds (see [3] for a recent classification of UAS). Of this group, rotary wing UAS are particularly attractive in applications where an ability to hover, takeoff/land vertically and/or operate at street level in built-up areas are of great value. However, rotary wing micro air vehicles suffer from limited flight endurance and payload capacity.

Approaches for improving flight endurance and/or payload capacity in rotary wing MAVs fall into the broad categories of increasing energy storage or more efficient energy utilization. In the area of increasing energy storage, there are many initiatives underway to develop lightweight batteries that offer higher energy (or power) density than the ubiquitous Lithium Polymer (LiPo) batteries used by MAVs. These include some novel nanotechnology based materials and hybrid Lithium/supercapacitor based schemes. While it is the case that better energy storage will have the most significant impact on the flight endurance of MAVs, these schemes are still in the early stages of development [4].

In this paper, we study an approach for improving overall mission endurance of a heterogeneous team of rotary wing MAVs by developing formation control laws that place each MAV in an energy-optimal region of the flow field from neighboring MAVs for a finite time during the mission, enabling individual MAVs in the ensemble to be rotated in and out in order to minimize the energy required to operate. Our approach characterizes the vortex field shed by each MAV, especially the far field effect, to determine the energy-optimal location - i.e. the region in the flow field where another MAV would experience maximum lift. A leader-follower formation control scheme based on separation of bearings (see [5]) is then used to place each MAV at these locations in turn, in some ways, mimicking the behavior and aerodynamic benefits of flocks of bird in formation. This approach sidesteps issues such as designing more efficient micro-motors and higher density, low SWAP batteries in favor of developing algorithms to enable teams of existing rotary wing MAVs operate longer.

The rest of this paper is organized as follows: Section II describes the Samarai MAV and discusses the dynamic model used for these studies. The far field flow model used to determine energy-optimal locations in the flow field associated with each MAV is developed in Section III-B. Formation control laws to achieve energy-optimal operations are then described in Section IV-B followed by a discussion of results in Section V-C and some concluding remarks.

II. ROTARY WING MAV

We study the flow field properties and interactions in the context of the Samarai family of UAVs, two of which are depicted in Figure 1. Developed by Lockheed Martin Advanced Technology Laboratories, the Samarai MAV is a wholly rotating single-wing air vehicle with wing radius 30cm, weight 200g and nominal rotation rate of 600 rpm. The SUAS (small unmanned aircraft system) or DAV has a wing radius of 72cm and a weight of 600g. A tip-mounted electric motor driving a small propeller provides propulsion to the MAV. Its sole control surface is a trailing edge flap driven by servo torque-rod assembly mounted at the root end of the flap. The key to flight control for this vehicle using a single flap is to modulate the flap in such a way that it synthesizes a ‘virtual swashplate’ that makes the vehicle fly like a regular helicopter. Details of control system design and videos of flight demonstrations can be found in [6] and [7] respectively.

To model the Samarai MAV, we use the autorotating flight of nature’s maple seeds as the starting point drawing on the work of [8]. By including a rigid body representation of the Samarai with blade geometry and inertias based on vehicle CAD model, we develop a model whose inputs are flap deflections and engine throttle setting. These are mapped to forces (F) and moments (M) through tables of aerodynamic and propulsion coefficients such that the response of the vehicle to these inputs is described by the state vector...
Fig. 1. The Samarai family of UAVs is inspired by nature’s maple seeds and has been demonstrated experimentally at multiple sizes. In flight, the vehicle spins in the same manner as the maple seed, but also adds the ability to fly like a helicopter. This provides a flow field characteristics that can be analyzed for the purpose of optimizing energy usage by the vehicle.

\[
X = [P, V, \omega, q], \quad \text{where} \quad P = [P_x, P_y, P_z] \quad \text{is the position vector of the center of gravity,} \quad V = [V_x, V_y, V_z] \quad \text{is the translational velocity,} \quad \omega = [\omega_x, \omega_y, \omega_z] \quad \text{is the angular velocity vector and} \quad q = [q_0, q_1, q_2, q_3] \quad \text{is the attitude quaternion.}
\]

Vehicle state evolution is governed by the equations of motion

\[
\begin{align*}
\dot{P} &= RV_b \\
\dot{m}V &= \sum F - \dot{\omega} \times (mV) \\
I\dot{\omega} &= \sum M - \dot{\omega} \times I\omega \\
\dot{q} &= \frac{1}{2} Qtq
\end{align*}
\]

where \( V_b \) is the body frame velocity, \( m \) is the mass, \( \dot{\omega} \) is the skew symmetric matrix of angular velocities, \( I \) is the Inertia matrix, \( R \) the rotation matrix from body to inertial frame and \( Qtq \) is the quaternion wedge matrix defined in [6]. To study the flow effects due to the MAV, the model (2) is augmented by characterizing the wake induced around the blade in Section III-B. The numerical results obtained in this paper are based on this augmented model.

III. FAR FIELD FLOW MODEL

A. Wake-Induced Airflow near Monocopter

The importance of the vortex wake shed by a rotating helicopter blade is well known [9]. The wake significantly influences the airflow in the vicinity of the blade, thereby influencing its aerodynamic properties. In addition to this self-effect, the wake also determines the airflow pattern at greater distances from the vehicle.

Qualitatively, there is strong, downward-directed airflow in a cylindrical region (the wake core) directly below the rotor disc, as shown in Figure 2. This airflow couples to the inflow above the rotor disc, experiencing radial contraction (“wake contraction”) as it transitions from the inflow region to the wake core. Finally, the flow leaves the vortex core, travels upward through a wide expansion region, and eventually returns to the inflow region.

In this work, only low Mach-number flows far removed from viscous boundary layers are of interest. It is therefore a good approximation to treat the flow as incompressible and inviscid. Furthermore, it is assumed that the flow is irrotational, except at the surface of the vehicle and in the vortex wake itself. With these approximations in mind, we can write the equations for the flow velocity as:

\[
\begin{align*}
\nabla \cdot V &= 0 \\
\nabla \times V &= \Gamma(x)
\end{align*}
\]

For a known vorticity distribution \( \Gamma(x) \), (6) can be directly integrated to yield the flow velocity at the control point \( x \). The control point represents the location of the vehicle at which the induced flow velocity is to be computed. Since both the vehicle which represents the source of the wake (the primed coordinates) and the vehicle which is influenced by the wake are in motion (in general), the integral of (6) would have to be recomputed at each timestep. For multi-vehicle simulation in which the ultimate goal is control-law design, this is overly time consuming. Consequently, a more suitable approach is required. The key simplifying assumption is that the flowfield results are needed only “far” from the source vehicle, since the vehicles are not actually allowed to get too close. If the distance at which the flowfield is sought is much larger than the wake itself, a multipole expansion of the flowfield is appropriate.

B. Multipole Expansion

A multipole expansion [10] is an infinite series representation of an integral, such as the flow-field velocity of (6) [11]
The terms of the series are increasing powers of the inverse distance. Thus, at sufficiently large distances, the series may be truncated, since additional terms have a vanishingly small effect.

In order to perform the desired multipole expansion, the flow-field equations are re-cast into potential form. Rather than solving directly for the flowfield \( \mathbf{V} (x) \), a velocity potential \( \mathbf{A} (x) \) is introduced:

\[
\mathbf{V} (x) = \nabla \times \mathbf{A} (x) \tag{7}
\]

The divergence of the newly-introduced potential \( \mathbf{A} \) is free to be specified. The standard choice is to simply specify \( \nabla \cdot \mathbf{A} = 0 \). With this choice, the equation for \( \mathbf{A} \) becomes:

\[
\nabla^2 \mathbf{A} = \Gamma (x) \tag{8}
\]

(8) is a vector form of Poisson Equation, with a known vortex distribution \( \mathbf{m} \). Defining the integral in (10) as the doublet moment of the vortex distribution \( \mathbf{m} (x) \), we have:

\[
\mathbf{m} = \frac{1}{2} \int_{\Omega} \mathbf{x} \times \Gamma (x') d^3x' \tag{11}
\]

\[
\mathbf{A} (x) = \frac{\mathbf{m} \times \mathbf{x}}{||\mathbf{x}||^3} \tag{12}
\]

Taking the curl of (12) in order to obtain the velocity field (as per (5)), the resulting expression for the far-field velocity is obtained as:

\[
\mathbf{V} (x) = \frac{3\mathbf{n} (\mathbf{n} \cdot \mathbf{m}) - \mathbf{m}}{||\mathbf{x}||^3} \tag{13}
\]

where \( \mathbf{n} \) is a unit vector in the direction of the observation point, and \( \mathbf{m} \) is defined in (10). (13) therefore represents the lowest-order term of the general multipole expansion. Due to the absence of of flow sources and compressibility \( (\nabla \cdot \mathbf{V} = 0) \), there is no monopole term, hence the lowest order term is a doublet. The qualitative behavior of the field is illustrated in Figure 3.

The doublet strength \( \mathbf{m} \) needs to be related to the properties and state of the source vehicle. The most straightforward way to do this is to match the mass rate of the flow through the vortex core to the return flow. The mass rate of flow through the contracted center of the vortex core is simply given by:

\[
\dot{M}_{\text{core}} = \rho V_{\text{ind}} R^2 \pi \tag{14}
\]

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\]

where \( R \) is the contracted radius of the core, and \( \rho \) is the mass density of air. The return flow can be obtained by integrating (13) over the entire center plane of the vortex core. Thus, the return flow is given by:

\[
\dot{M}_{\text{return}} = \int_R^\infty \int_0^{2\pi} \rho ||\mathbf{m}|| r' dr'd\phi = 2\pi ||\mathbf{m}|| R \tag{15}
\]

Equating the inflow (14) and return flow (15), the magnitude of the doublet moment is obtained:

\[
||\mathbf{m}|| = \frac{1}{2} V_{\text{ind}} R^2 \tag{16}
\]

Finally, momentum theory can be used to relate the induced velocity \( V_{\text{ind}} \) to thrust \( T \):

\[
V_{\text{ind}} = \sqrt{\frac{T}{2\rho A}} \tag{17}
\]

Combining (16) and (17), the relationship between basic properties of the source vehicle and the induced far-field velocity is obtained. As can be seen from (13), the magnitude of the velocity field drops of with the third power of distance from the core. Thus, it can be expected that the far-field velocities are quite low. Indeed, this is shown to be the case in Figure 4.

C. Wake Model in Simulation

Using a simple analysis method, the results of the dipole model are divided into the three velocity matrices of the coordinate frame: Height, Easting and Northing. The goal of this initial assessment is to find a position with a considerable
amount of upward velocity and a small amount of sideways velocity (Easting and Northing). Minimizing the side winds will result in an environment that will lessen the effort made by the formation controller. Since exact positioning will be important, finding a point where the vehicles do not experience side forces will have more robust results.

The first task of assessing the position of maximized upward flow is relatively straightforward. The closer a follower is to its leader’s position, the more wake each vehicle will experience. However, the areas directly above and below a vehicle are negative wake; this automatically eliminates these areas for consideration. Instead, we focus on the side area. Figure 5 shows a vertical slice of the magnitude of the vertical velocity of the dipole model. The symmetry of the model around the height of zero can be seen immediately. If the position is split into relative height and tip-to-tip distance, Figure 5 shows that a small change in relative height at some fixed radius can decrease the positive vertical velocity. Therefore, with a relative height of zero, the vertical velocity experienced by neighbours increases as the tip-to-tip radius decreases. In essence, the best position is the smallest relative distance that vehicles can be flown without risking collision. The distance chosen for this vehicle is 0.5 meters, approximately the same as the vehicle’s spinning radius. Also, at the relative height chosen, the Easting and Northing velocities fall close to zero, making it the perfect position for our experiments.

IV. ENERGY OPTIMAL FORMATIONS

A. Formation Objective

After inspecting the wake around our aerial vehicle, we have a better understanding of the energy flow around the chosen vehicle. We can approximate where the best energy-optimal positions exist. In order to continually take advantage of the aerodynamically beneficial wind currents created by neighbouring vehicles, a robust formation controller needs to be established.

The formation needs to be able to maintain steady distances between all vehicles to remain in each other’s wakes and in the correct position. From the discussion earlier, we have determined the general configuration in which the vehicles will fly. The next step is to create a controller that will maintain the chosen formation.

B. The Separation-Bearing Formation Controller

Formations of aerial vehicles fall into three general categories: leader-follower, behavioural and virtual structure. The method of choice for this paper’s experimentation is the first, leader-follower. This choice takes a page from previous fixed-wing vehicle experiments that utilize formations for mutual aerodynamic benefit [12]. The leader-follower method allows for a leader to be established that creates the aerodynamically beneficial positions for its group of followers.

With most fixed-wing formations, an individual leader heads a formation and creates the positions of efficiency for a series of followers. It is well known that this class of vehicles achieve their aerodynamic benefit from flying in the tip vortices of a leading vehicle [13],[14]. However, this paper focuses on rotary wing vehicles, a group that is not well researched within energy-optimal formation flight. So, for this novel attempt, we are aiming at keeping the energy-receiving vehicle where aerodynamic benefit is maximized; the selected implementation is the leader-follower formation method.

The separation-bearing controller (SBC) was originally derived for maintaining formations of mobile ground robots in 2001 [15]. It has more recently been used with unmanned aerial vehicles, specifically a quadrotor UAV[5], and has also been utilized for this project. Though the Samarai is a single rotor vehicle with less control inputs, the result of the separation-bearing formation control can be modified for use with the Samarai simulation.

The general idea of the SBC is to define the desired position of any follower vehicle, $F$, in formation with respect to its leader $L$ by a separation distance and two bearing angles.
From previous work [6] the Samarai’s world coordinates are predefined in a height-East-North frame, i.e.
\[ p = [x, y, z] \]
After coordinate modifications are made to the original SBC position vector to correspond with the Samarai’s coordinate, the full position vector for each vehicle is defined as
\[ P = [h, E, N, \psi]^T \]
where \( \psi \) is the current heading, noted by the direction of motion. The instantaneous shape vector between a leader \( L \) one of its followers \( F \) as a function of the trajectory of leader \( L \) is defined as (shown in Figure 6).
\[ S = [d_{L,F}, \theta_{L,F}, \phi_{L,F}, \psi_{L,F}] \]
where \( d_{L,F} \) is the separation distance between leader and follower, \( \theta_{L,F} \) is the elevation angle, \( \phi_{L,F} \) is the relative azimuth angle and \( \psi_{L,F} \) is the difference in heading angle.

The shape vector \( S \) is essentially modified spherical coordinates in a right-handed system with the \( y \)-axis as the zenith direction and the origin at the center of gravity of the leader vehicle. To ensure the transformed \([x, y, z, \psi]\) coordinates from the \( S \) vector will always be unique, the follower’s shape vector will be restricted to \( S_1 > 0, S_2 \in [-\pi/2, \pi/2], S_3 \in [0, 2\pi] \) and \( S_4 \in [0, 2\pi] \). \( S_2 \) is defined such that positive indicates the follower will sit above leader (follower will have a grater \( x \) value) and negative indicates below.

The actual position vector of the follower with respect to the leader can be written as a modified spherical transformation using the separation-bearing vector \( S \) defined above
\[ x_L = x_L + S_1 \sin(S_2) \]
\[ y_L = y_L + S_1 \cos(S_2) \cos(S_3) \]
\[ z_L = z_L + S_1 \cos(S_2) \sin(S_3) \]
\[ \psi_L = \psi_L + S_4 \]

or
\[ P_F = P_L + f(S) \]
with
\[ f(S) = \begin{bmatrix} S_1 \sin(S_2) \\ S_1 \cos(S_2) \cos(S_3) \\ S_1 \cos(S_2) \sin(S_3) \\ S_4 \end{bmatrix} \]

Since there will be more than two vehicles in the formation, each relevant leader-follower pair will have its own desired separation-bearing shape vector, i.e.
\[ S_{i,j}^{des} = \begin{bmatrix} \tilde{d}_{i,j}^{des} \\ \tilde{\theta}_{i,j}^{des} \theta_{i,j}^{des} \phi_{i,j}^{des} \psi_{i,j}^{des} \end{bmatrix} \]

One aspect of the separation-bearing controller that is useful for this paper’s experimentation is the ability to dynamically control the position of every follower with respect to its leader. The controller allows a leader to be defined as the creator or the beneficiary of the energy-efficient position. The experimentation that will be reviewed in this paper will define the leader as the center of a horizontal circle formation. This will allow that single leader to be the recipient of the recirculating wake of its surrounding followers.

The leader of the formation will dictate the motion of the whole formation. As the leader moves, a trajectory will be created for its followers such that they remain at their pre-defined separation-bearing distances. If the followers are able to stay in formation, then the leader will receive constant upward flow from the recirculating wake.

V. RESULTS AND DISCUSSIONS

A. Numerical Simulator

All of the experiments were done in a simulated environment in MATLAB/Simulink. This simulation of the Samarai vehicle was previously developed at Lockheed Martin Advanced Technologies Laboratory (ATL) and is based on empirical data [6]. Using this simulation allows the methodology to be tested without having the limitation of a real-world situation. The simulation also allows for full control of environmental conditions, such as wind, which severely affect the flight of the MAV. All sensing, including position tracking, internal avionics and collision avoidance is assumed to be accurate.

For the purposes of mutual interaction of the vehicles wakes, the simulator was modified to include multiple vehicles as well as a wake-to-vehicle calculations. At each time step, the relative position of each follower vehicle with respect to the leader is used to determine the wake. A lookup table is used in Simulink to calculate the velocities of the follower’s wake. This velocity is added to the leader’s velocity and integrated to get the new wake-induced position.
B. Simulation Descriptions

We define the formation leader to be the MA V Samarai while its followers will be the DAVs. As discussed in Section IV-B, the leader MA V will control the movement of the whole formation. It is the job of the separation-bearing formation controller to build trajectories for the followers to maintain their pre-defined positions.

The leader will rise to a height of 6.0 meters and then will hover for the remainder of the experiment, a total of 400 seconds or 6.5 minutes. During its ascent, five DAV followers will also climb to their individual position in an even, horizontal circle around the leader. Figure 7 shows a comparison of a simulation with and without the DAV followers.

![Simulation Comparison](image)

Fig. 7. In these graphs, the red signifies a MAV flying alone while the blue is a formation flight with the MAV leader and five DAV followers. In clockwise order from the top left, the height, power draw, collective flap and climb rate of the MAV leader.

C. Results Discussion

The leader first ascends to the desired height, from the start time to approximately 20 seconds. During this ascent, the singular flight and formation flight graphs are very similar for all four variables displayed in Figure 7. However, after the five DAVs are in position, the energy saving effects can be seen in the leader’s height and power draw.

Once hover is started at 20 seconds, the MAV in formation uses less power while staying at a higher height. In the singular flight, the leader drops to about 5 meters and draws 35 watts of power. However, in formation the leader remains at a height of 6 meters with a power draw below 30 watts. Moreover, the collective flap remains the same for both simulations. This means that the drop in power draw is due to a lower throttle position. In a steady state hover there is a measurable energy-saving effect for the leader vehicle in a robust formation. Though the followers are affected by the wakes of their neighbors and the leader, the influences are relatively negligible.

VI. CONCLUDING REMARKS

Improving flight endurance and payload capacity is prevalent in small aerial vehicles, due to limitations of the size and range of such vehicles. Instead of changing the physicals of the vehicle such as weight, aerodynamics or battery density, this paper focused on creating a controller that reduced energy consumption over time utilizing the wake of neighboring vehicles. We have shown, through simulated experimentation, that it is possible to create formations of rotary wing vehicles such that there exists areas of recirculated flow for energy-efficient flight. The simulated environment is still necessary to attempt the vehicle to vehicle proximity that is required for the success of the formation; the major limiting factor for real-world testing are the precision and refresh-rate of the vehicle’s sensors. To get around this hindrance, the first experiments done in field could be done in a controlled motion-capture environment.

ACKNOWLEDGMENT

Thanks to the University of Pennsylvania and IUCR/C for their contributions to this project.

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