A NEW STABILIZER FOR LTV INTERNAL MODEL BASED SYSTEM AND ITS APPLICATION TO CAMLESS ENGINE VALVE ACTUATION

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ABSTRACT

This paper focuses on the stabilization of an internal model based control system for reference tracking/disturbance rejection, where the physical plant is linear time invariant while the generating dynamics of the reference/disturbance is time varying. Given the inevitable linear time varying (LTV) nature of the internal model unit resulting from the time varying generating dynamics, a critical problem to be addressed is the design of a low order and robust stabilizer for the entire augmented system. While a parameter-dependent output injection based stabilizer was introduced in our previous work, the new method proposed in this paper reduces the conservativeness in the control synthesis and thus enables a more effective control design especially for the problem involving LTI physical plant with LTV internal model dynamics. The proposed approach is then validated through experimental investigations on a camless engine valve actuation system, where robust and precise tracking performance is demonstrated.

1. INTRODUCTION

The internal model principle based output regulation [1] has generally been recognized as a powerful tool for the reference tracking/disturbance rejection problem. For the problem setting of LTI plant and the regulation of signals with LTI generating dynamics, the control synthesis is well established [1]. One of the well-known examples is the repetitive control [2,3], which regulates the periodic signal by embedding multiple harmonics into the closed loop and has been widely applied.

However, for the regulation associated with either LTV plant or LTV generating dynamics, which requires the construction of an LTV internal model [4], the problem becomes much more difficult. Even the LTV internal model design by itself is non-trivial [4-6]. Very recently [5] proposes an LTV internal model construction approach, which enables an effective synthesis for a class of LTV systems. Once the time-varying internal model is constructed, a robust and low order stabilizer is required to enable the reliable real time implementation. Very limited studies have been conducted before, partly due to the previous unavailability of an effective internal model construction. Ref [6] directly applied the LTV/LPV stabilization techniques developed for a general LTV system, but the stabilizer designed is inevitably of very high order to enable the convex optimization based synthesis [7-9], which poses fundamental challenges to the real time implementation. To have a low order design, [10,11] proposes a parameter-dependent output injection based stabilization method by leveraging the uniqueness of the internal model system. The research outcome shows both the numerically efficient synthesis and the robust experimental results.

However, for the stabilizer design proposed in [10,11], two crucial LMI constraints are required for the synthesis. One is to add the perturbation terms on some of the control gains, and the other is to restrict the magnitude of the gains. While this method could work well for a certain class of LTV systems, the selection of appropriate perturbation values and also the gain constraining boundary is not intuitive. It’s also possible that for certain problems, these additional LMIs may result in a conservative controller with undesirable transient performance or in some cases make the synthesis infeasible. Therefore, in this paper, we propose a new stabilizer architecture, which can avoid the perturbation/gain constraining in synthesis and enable a more efficient and less conservative design process. The proposed method is specifically for a linear time invariant plant and a linear time varying internal model. While still keeping the parameter-dependent output feedback gains directly injected into the internal model as proposed in [10,11], an additional low order dynamic compensator is introduced in parallel with the internal model unit. Note that the order of the dynamic compensator is only required to be equal or close to the order of the physical plant, so that the low order nature of the whole stabilizer is still maintained.

In this paper, we will formulate the new stabilizer design in the discrete domain, reveal its challenges and finally address this problem by formulating feasible linear matrix inequalities (LMIs). The effectiveness of the method to ensure precise and robust motion control will be analyzed using simulations and experimental investigations on a camless engine valve actuation setup.

The rest of the paper is organized as follows. Section 2 presents the preliminaries of the control problem in the discrete setting. Section 3 formulates the stabilizer design problem and introduces the new design approach. Simulation and experimental results are presented in Section 4.
2. PRELIMINARIES

2.1 Problem Formulation

This paper focuses on the output regulation of signals with time varying generating dynamics for a linear time invariant plant. Consider an \( n \)-th order discrete linear time invariant plant of the form:

\[
x_p(k+1) = A_p x_p(k) + B_p u(k)
\]
\[
y_p(k) = C_p x_p(k)
\]

where \( k \) is the current step, \( x_p(k) \in \mathbb{R}^n \), \( y_p(k) \in \mathbb{R} \), the control input \( u \in \mathbb{R} \). The LTV generating dynamics (exo-system) of the signal \( d \) to be tracked or rejected (Figure 1(a)) can be described as:

\[
\delta(k+1) = S(o(k))\delta(k) + d(k)
\]

where, \( \delta \) is the exo-system state and \( o \) is the vector containing all the time varying parameters. (One example of the signal with time varying generating dynamics is the repetitive/cyclic signal with time varying frequency, which will be shown in the experimental section.) And the error \( e \) is defined as:

\[
e(k) = d(k) - y_p(k)
\]

It is assumed that the LTI system (1) is controllable and observable and the exo-system (2) is stable in the sense of Lyapunov in both forward and backward direction along the time axis [6]. (The signal \( d \) described is therefore persistent yet bounded).

\[
\begin{align*}
\dot{x}_m(k) &= \Phi_m(o(k))x_m(k) + \Psi_m(o(k))y_m(k), \\
x_p(k) &= \Gamma_m(o(k))x_m(k) + \Theta_m(o(k))e(k), \\
y_p(k) &= C_m(o(k))x_m(k)
\end{align*}
\]

Figure 1(a). Block diagram of the augmented system consisting of the LTI plant and the time varying internal model

\[
\begin{align*}
\dot{x}_m(k) &= \Phi_m(o(k))x_m(k) + \Psi_m(o(k))y_m(k), \\
x_p(k) &= \Gamma_m(o(k))x_m(k) + \Theta_m(o(k))e(k), \\
y_p(k) &= C_m(o(k))x_m(k)
\end{align*}
\]

Figure 1 (b). Stabilization with a parameter dependent output injection based stabilizer

The control target is to have the error \( e \) converge to zero. The controller design typically involves two parts. One is to have an internal model unit constructed, which will be briefly introduced in Section 2.2. The second part is the stabilizer design, which is the main focus of this paper.

2.2 Time Varying Internal Model Unit Construction

The time varying internal model design adopted from ref [5] is formed by two interconnected subsystems (Figure 1(a)) and can then be written as:

\[
\begin{align*}
\xi(k+1) &= \Pi(o(k))\xi(k) \\
u(k) &= \Upsilon(o(k))\xi(k)
\end{align*}
\]

Here we assume that the internal model (4) is of the order \( m \), which is typically much larger than the order of the plant \( n \). This is because the internal model needs to embed a high order generating dynamics to enable the precise tracking/rejection. Once the internal model is constructed, the critical challenge is to design a low order and robust stabilizer for the entire augmented system i.e., the internal model and the plant.

3. STABILIZER SYNTHESIS FOR DISCRETE TIME VARYING INTERNAL MODEL BASED CONTROL

The unforced time varying augmented system [10] (internal model + plant dynamics) can be written as:

\[
\begin{align*}
X_{aug}(k+1) &= A_{aug}(o(k))X_{aug}(k) \\
y_p(k) &= C_{aug}X_{aug}(k)
\end{align*}
\]

where

\[
A_{aug}(o(k)) = \begin{bmatrix}
A_p & B_p \Upsilon(o(k)) \\
0_{n \times m} & \Pi(o(k))
\end{bmatrix}
\]

and

\[
X_{aug} = \begin{bmatrix}
x_p \\
\xi
\end{bmatrix}
\]

is the state vector of the augmented system.

The control objective is to construct a time varying stabilizer for the entire augmented system. Very limited research has been conducted in this area before, partly due to the previous unavailability of an effective internal model unit construction. To enable a low order stabilizer design, [10,11] propose to use the parameter dependent output injection (Figure 1(b)) as the control mechanism, where the stabilization inputs can be written as:

\[
u_p = K(o)y_p = K(o)C_{aug}X_{aug}
\]

where \( K(o) \) is a parameter dependent gain vector as:

\[
K(o) = \begin{bmatrix}
k_1(o) & \cdots & k_n(o) & k_{n+1}(o) & \cdots & k_{n+m}(o)
\end{bmatrix}^T
\]

It is worthwhile to note that the output feedback gain based stabilization could be very conservative or even infeasible due to the lack of regulation flexibility comparing with a dynamic controller. However, it turns out to be an effective stabilization tool for the internal model based system, since the entire internal model is a virtual system as pointed out in [10]. The uniqueness of a virtual system lies in the fact that each state in the internal model is accessible. Motivated by this observation, refs [10,11] propose to directly inject each independent output from the gain stabilizer (7) into individual state equation as shown in Figure 1(b).

Then the closed loop augmented system becomes:

\[
\begin{align*}
X_{aug}(k+1) &= [A_{aug}(o(k)) + B_{aug}(o)K(o)C_{aug}]X_{aug} \\
y_p &= C_{aug}X_{aug}
\end{align*}
\]
where
\[ B_{\text{aug}} = \begin{bmatrix} 0_{m \times n} & 0_{m \times m} \\ 0_{n \times m} & I_{m \times m} \end{bmatrix} \]  
(9)

Note that there is an identity matrix term in (9), which is contributed by the fact that the output from the parameter-dependent output injection stabilizer can be directly injected into each stated space equation of the internal model. As mentioned previously, the internal model dynamics order usually is much higher than the plant dynamics, therefore the identity matrix is a major portion in (9). As will be revealed in the following section, the high dimensional identity matrix will be the key enabler for the parameter-dependent gain synthesis.

### 3.1 Stabilization using Parameter Dependent Output Injection

As presented in [11], the necessary and sufficient condition for system (8) to be stable can be formulated into an infinite dimensional Matrix Inequality constraint. Two critical issues should be addressed to enable the synthesis of the control gain \( K \). First is to convert the infinite dimensional matrix inequality into a series of finite dimensional matrix inequalities. This procedure, which involves the manipulation of the Lyapunov matrices, has been presented in great detail in our previous work [11] and will not be repeated here. The second issue is to formulate the stabilization condition into Linear Matrix Inequalities (LMIs) constraints, so that the control gain \( K \) could be synthesized through convex optimization. As explained in [10,11], if the input matrix \( B_{\text{aug}} \) is an identity matrix, the LMIs formulation and therefore the control synthesis will be straightforward. With this observation, reference [11] proposes a method of embedding perturbation terms in the stability constraint and at the same time restricting the control gain magnitude. This method, however, is sensitive to the choice of the perturbation terms and for certain cases may lead to a conservative synthesis. Therefore, in the following section, an alternative approach is proposed, which avoids adding perturbations and reduces the conservativeness of the gain synthesis.

### 3.2 Stabilization Using a Low Order Dynamic Compensator Together with Gain Injection

#### 3.2.1. Stabilizer Design for Minimum Phase Plant

A new stabilizer structure is proposed as shown in Figure 2. While retaining the original parameter-dependent output feedback terms which are directly injected into the internal model’s states, an additional dynamic compensator is constructed in parallel to the internal model unit. The order of the compensator is equal to \( n-1 \), where \( n \) is the order of the physical plant dynamics Eq.(1). The regulation error is also fed back to another \( n \) sets parameter-dependent gains, the outputs of which are injected into the compensator’s state space as shown in Figure 2. The output of the dynamic compensator is then added with the internal model output to be the control input \( u \) for the physical plant.

#### Output Injection

The compensator dynamics is determined based on the zero dynamics of the plant model (1). Let’s first consider the case where the zero dynamics order of (1) is \( n-1 \), which is one order lower than its pole dynamics. This is the common case when the discrete system is obtained from sampling continuous physical plant with zero order hold. Suppose the plant zero dynamics \( B_p(z) \) can be written as:

\[ B_p(z) = b_1 z^{n-1} + b_2 z^{n-2} + \cdots + b_{n-1} z + b_n \]  
(10)

Then the compensator dynamics can be designed as:

\[ x_c(k+1) = A_c x_c(k) + B_c u_c(k) \]

\[ u_{\text{com}}(k) = C_c x_c(k) + D_c U_c(k) \]  
(11)

where

\[ A_c = \begin{bmatrix} -b_2 & 1 & 0 & 0 & \cdots \\ -b_1 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ -b_{n-1} & 0 & 0 & \cdots & 1 \\ -b_n & 0 & 0 & \cdots & 0 \end{bmatrix} \]  
(12)

\[ B_c = \frac{1}{b_1} \begin{bmatrix} -b_1 / b_1 & 1 & 0 & 0 & \cdots \\ -b_2 / b_1 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ -b_{n-1} / b_1 & 0 & 0 & \cdots & 1 \\ -b_n / b_1 & 0 & 0 & \cdots & 0 \end{bmatrix} \]  
(13)

\[ C_c = [1 \ 0 \ \cdots \ 0]_{n \times (n-1)} \]  
(14)

\[ D_c = [1 / b_1 \ 0 \ \cdots \ 0]_{3 \times n} \]  
(15)

\[ U_c = [u_1 \ u_2 \ \cdots \ u_{n}]^T \]

\[ = [k_1(\omega)e \ k_2(\omega)e \ \cdots \ k_n(\omega)e]^T \]  
(16)

The compensator dynamics can also be represented in the \( z \) transform form as:

\[ u_{\text{com}} = \frac{z^{n-1}}{B_p(z)} u_1 + \frac{z^{n-2}}{B_p(z)} u_2 + \cdots + \frac{z}{B_p(z)} u_{n-1} + \frac{1}{B_p(z)} u_n \]  
(17)

There are primarily two benefits of adding the dynamic compensator in the control loop. Firstly, more
degrees of freedom will be available in the synthesis of the parameter-dependent control gains and therefore the transient performance of the tracking/rejection could be further optimized comparing to the previous approach of only injecting the output feedback gains into the internal model. Secondly, this control architecture can also enable an effective control gain synthesis as explained below.

The closed loop augmented system with the compensator as well as the parameter dependent output feedback gains can be written as:

\[
X_{\text{aug}}(k+1) = \begin{bmatrix} A_{\text{aug}}(\omega(k)) + B_{\text{aug}} K(\omega) C_{\text{aug}} \end{bmatrix} X_{\text{aug}}(k) + \begin{bmatrix} B_p^T & 0_{1,m} \end{bmatrix} u_{\text{COM}}(k)
\]

**Theorem 1:** If there exists a parameter-dependent output feedback gain vector

\[
K(\omega) = \begin{bmatrix} k_1(\omega) & k_2(\omega) & \cdots & k_{n+1}(\omega) & \cdots & k_{n+m}(\omega) \end{bmatrix}^T
\]

which can stabilize the following closed loop system:

\[
X_{\text{aug}}(k+1) = A_{\text{aug}}(\omega(k)) X_{\text{aug}}(k) + K(\omega) C_{\text{aug}} X_{\text{aug}}(k)
\]

then the same set of control gain \( K(\omega) \) can also stabilize the closed loop augmented system Eq. (18) shown in Figure 2.

**Proof:** It can be verified that the minimum realization of the state space dynamics from input \( [u_1 \ u_2 \ \cdots \ u_{n+1} \ \cdots \ u_{n+m}]^T \) to the plant output \( y_p \) in Figure 3 is identical to that in Figure 4. Setting control inputs as:

\[
\begin{bmatrix} u_1 \ u_2 \ \cdots \ u_{n+1} \ \cdots \ u_{n+m} \end{bmatrix}^T = \begin{bmatrix} k_1(\omega)e & k_2(\omega)e & \cdots & k_{n+1}(\omega)e & \cdots & k_{n+m}(\omega)e \end{bmatrix}^T
\]

then the closed loop system in Figure 3 is exactly the closed loop augmented system Eq.(18), and the closed loop system in Figure 4 is Eq.(20). Therefore the same set of control gain can stabilize both the systems.

3.2.2. Robust Stabilization Considering Unstructured Uncertainty

To enable reliable real time implementation, robust stabilizer considering the unstructured uncertainty should be addressed as well. The unstructured uncertainty could come from either the un-modeled dynamics of the plant or the incomplete pole-zero cancellation in the compensator.

\[
X_{\text{aug}}(k+1) = A_{\text{aug}}(\omega(k)) X_{\text{aug}}(k) + U_{\text{aug}}(k)
\]

\[
y_p(k) = C_{\text{aug}} X_{\text{aug}}(k)
\]

**Theorem 2:** If the following linear matrix inequality holds,

\[
\text{LMIs, which could be resolved using convex optimization tools.}
\]

In addition, for the case when the order of the plant zero dynamics \( B_p(z) \) is lower than \( n-1 \), corresponding number of delay operators \( z^{-1} \) could be added to Eq. (17) to avoid non-causality. The resulted uncertainty term between systems (18) and (20) could be addressed by the robust stabilization formulated in the following section.
where $P_k$ and $P_{k+1}$ are Lyapunov matrices at step $k$ and $k+1$ respectively and $L_s$ is a vector variable introduced to enable control synthesis (definition of $L_s$ can be found in [11]), then

(1) The nominal closed loop system is asymptotically stable.

(2) The induced $L_2$ norm between $y_p$ and $w$ is smaller than $\gamma$, thus the system is robustly stable with the existence of unstructured stability. ■

**Proof:** The proof is omitted. ■

### 3.2.3. Stabilizer Design for Non-minimum Phase Plant

When the plant model (1) is non-minimum phase, the zero phase error concept introduced in [12, 3] could be adopted to construct the compensator. The difference between the zero phase compensator and the plant model zero dynamics is lumped into an uncertainty term at high frequency, which could be addressed using robust stabilization approach introduced in previous section.

### 4. CONTROL IMPLEMENTATION

This section presents the results and analysis from the implementation of the proposed control framework on a camless engine valve actuation system.

#### 4.1 System Description

A schematic of the camless engine valve actuator used for the experimental investigations is shown in Figure 6 (a) and the hardware setup is shown in Figure 6(b). The reader can refer to [13] for background of camless engine valve actuation. It's basically one of the key enablers for more efficient and clean engine. It consists of a spring-loaded hydraulic actuator whose motion is controlled by regulating the high pressure fluid using a voice-coil actuated 3-way spool valve. The open-loop system dynamics i.e., between the voice coil current “i” and the actuator position “y” is marginally stable since the voice coil current directly affects the actuator velocity. Hence the system is first stabilized using a proportional controller as shown in Figure 6(a). This stabilized “inner-loop” is the plant for the time-varying internal model based controller design. The control signal “u” from the internal model will make the actuator output “y” track the desired reference signal “r” as shown in Figure 6(a).

#### 4.2 Simulation & Experimental Results

Figure 7 shows the simulation results of implementing the designed controller to track an engine valve opening/closing profile. The first row corresponds to the reference signal, which is split into two sections to show the zoomed in portions at the start and end of the experiment. The frequency of the signal goes up continuously throughout the simulation, and the frequency difference between the two reference profiles at the start and the end shows the trend of frequency variation. It can be seen that, after the initial transient, the error $e$ shown in the second row of Figure 7 is maintained at zero in the presence of the frequency variation. However, a closer inspection of the control signal shows that the amplitude of control input signal from the internal model unit ($u_{IM}$ in Figure 7) and that from the compensator ($u_{COM}$ in Figure 7) during the initial transient are beyond the acceptable limits of the experimental setup. To avoid saturation, the outputs from the two controller units should be restricted in the control synthesis. To constrain $u_{IM}$ from the internal model unit (Figure 2), the LMI (23) in theorem 2 can be used by replacing $C_{aug}$ in (23) with $C_{aug}$ as,

$$C_{aug} = \begin{bmatrix} 0_{t \times n} & 1 & 0 \cdots & 0 \end{bmatrix}$$

(24)

This will constrain the induced $L_2$ norm between the output from the internal model and the reference/disturbance signal $d$. For the transient control input $u_{COM}$ out of the compensator, it is observed that its magnitude closely correlates with the size of the gain $k_1$ in Eq. (16). Hence a technique based on the results presented in [10,11] can be used to restrict the gain $k_1$. For details, please refer to [10,11].

Figure 6 (a). Schematic of the valve actuation system

Figure 6 (b). Camless engine valve actuation system setup

Figure 8 shows the experimental results. The engine valve opening/closing reference profile has a continuously varying frequency from 12 Hz to 20 Hz at the rate of 0.5
Hz/sec. Again due to the large number of cycles in the engine valve reference signal, only the zoomed in portions of the reference profile $r$ during the start (0-0.5 sec) and end (16-16.5 sec) of the experiment are shown in the first row. The tracking error quickly converges to a small interval as shown in the second row. In addition, a zoomed in portion (13.5 sec to 14 sec) of the experimental data is shown to highlight the tracking at high frequency in the third row. The system output $y$ in red color closely tracks the desired valve profile $r$.

**CONCLUSION**

This paper presents a new stabilizer design for the linear time varying internal model based output regulation and its application to the camless engine valve motion control, where a precise tracking of a cyclic signal with time varying frequency is critical. Compared to the existing stabilizer design introduced in our earlier work, the new control construction is more effective for the problem setting where the plant is LTI while the signal generating dynamics is time varying. Instead of only injecting output feedback gains into the internal model unit, a low order dynamic compensator is constructed in parallel with the internal model. Simulation analysis and experimental investigations are used to demonstrate the effectiveness of the proposed method.

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