Observer-Based Delayed Feedback Attitude Control for Single- and Multi-Actuator Maneuvers

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Abstract—Observer-based attitude controllers for single- and multi-actuator maneuvers are developed from the delayed state feedback control laws obtained by the Lyapunov-Krasovskii functional and inverse dynamics approaches, respectively. The TRIAD algorithm is employed to process the observations. The observer gain is selected based on the extended Kalman-Bucy (EKB) filter, and a novel delay estimation approach is used to provide the estimates of the delay. The observer-based delayed feedback control laws are shown to be capable of accurate control of spacecraft attitude from noise-corrupted attitude measurements.

I. INTRODUCTION

A variety of different attitude parametrizations are available which can be utilized in attitude control and estimation problems. Modified Rodriguez parameters (MRPs) [9], [10], [14], [15] are a minimal three-parameter set defined as a stereographic projection of the quaternions. Furthermore, if switching to the shadow set is employed when the principal angle passes through 180 deg, then the MRPs are unique and singularity-free [9], [14], [15]. Recently, using MRPs as the attitude parametrization in the problem of attitude estimation using Kalman filtering has attracted an increasing interest. Attitude estimation based on MRPs was first implemented in [6] where the problem of the normalization constraint associated with the quaternions was eliminated. The subtractive and multiplicative attitude estimation methods are studied and compared in [8] where both methods provided good attitude estimations, but no disturbances are made at the switching point in the case where multiplicative attitude estimation was used. The results also showed that the MRP-based extended Kalman-Bucy (EKB) filter [2] yields a faster initial convergence as compared to the multiplicative quaternion-based filter. However, the previous work in MRP-based attitude estimation has not considered the full dynamics of the spacecraft.

Time delay in a control system arises due to communication delays including delays in the measurement [5], or processing delays including delays which occur in the actuators [11]. Recently, a method known as Chebyshev spectral continuous time approximation (CSCTA) [3] has been developed for delayed systems that produces an equivalent system of ordinary differential equations (ODEs) which can be used for obtaining the time response or stability of the system of delayed differential equations (DDEs). This technique can be used for stability analysis of different DDEs including ones with either nonlinearities or multiple delays [3]. Recently, a novel approach exploiting CSCTA and optimal control filtering is proposed for state, parameter, and delay estimation of linear, nonlinear, and chaotic DDEs [18].

The observer-based delayed feedback attitude control of a rigid spacecraft with delayed single- and multi-actuator feedback control are studied in this paper where MRPs are used as the attitude parametrization. Two pairs of unit vectors, one observed in the spacecraft-fixed frame and the other observed in the Earth-centered inertial (ECI) coordinate frame (the latter being extracted from the star catalog) are used to measure the spacecraft attitude in terms of MRPs using the TRIAD method. An observer-based controller is designed based on delayed feedback control laws obtained by the Lyapunov-Krasovskii (L-K) functional and inverse dynamics approach. For the multi-actuator control law obtained by the inverse dynamics approach, which is relevant to the problem of desaturation maneuvers, it is assumed that only one actuator (e.g. reaction control thrusters) contains time delay. For this case, the CSCTA technique [3] is used to transform the resulting set of DDEs governing the closed-loop dynamics into a system of ODEs using Chebyshev collocation. The observer gain is selected based on continuous time EKB filter and a novel delay estimation approach is used to provide the estimates of the delay for the delayed feedback control law. To the authors’ knowledge, there has not thus far been an observer-based controller including time delay for spacecraft attitude dynamics and control.

II. ATTITUDE DYNAMICS CONTROL

Consider the attitude dynamics of a rigid spacecraft as

\[
\begin{align*}
\dot{\sigma}(t) &= \frac{1}{4} B(\sigma(t)) \omega(t), \\
\dot{\omega}(t) &= -J^{-1} \omega^T(t) J \omega(t) + J^{-1} u(t), 
\end{align*}
\]

where \(\omega \in \mathbb{R}^3\) represents the body-frame angular velocity vector, \(\sigma(t) \in \mathbb{R}^3\) is the MRP set, \(u(t) \in \mathbb{R}^3\) is the control input, \(J\) is the \(3 \times 3\) symmetric inertia matrix, and

\[
B(\sigma) = [(1 - \sigma^T \sigma) I_{3 \times 3} + 2\sigma^T + 2\sigma \sigma^T].
\]

In terms of quaternions, the MRP attitude parametrization \(\sigma\) is defined as

\[
\sigma = \frac{\beta}{1 + \beta_0},
\]

where \(\beta_0\) is the scalar part of the quaternions, \(\beta = [\beta_1, \beta_2, \beta_3]^T\) is the vector part, and the quaternion constraint
\[ \sum_{i=0}^{3} \beta_i^2 = 1 \] holds. Figure 1 illustrates how the MRP set $\sigma$ is related to the principal angle of rotation and quaternions. Note that \( \mathcal{E} : \mathbb{R}^3 \rightarrow so(3) \) is the skew-symmetric mapping given by

\[
\Gamma^x = \begin{bmatrix}
0 & -\Gamma_3 & \Gamma_2 \\
\Gamma_3 & 0 & -\Gamma_1 \\
-\Gamma_2 & \Gamma_1 & 0
\end{bmatrix},
\]

where the space of $3 \times 3$ real skew-symmetric matrices is denoted by $so(3)$, the Lie algebra of the Lie group SO(3).

One method for controlling the attitude dynamics of a rigid body is to assume a linear control law which results in a nonlinear model for the closed-loop dynamics of the system \cite{1}, \cite{13}. Another method is to assume a nonlinear control law which results in a linear model for the closed-loop dynamics \cite{11}. Both approaches are considered in this paper. In the first approach, a linear state feedback controller is designed using a L-K functional approach and dependent stability criteria using a L-K functional approach.

A linear state feedback controller is designed to yield a delay-dependent stability criteria using a L-K functional approach and a linear matrix inequality (LMI) such that the solution is asymptotically stable. For this purpose, we choose the linear delayed state feedback control \cite{13}

\[
u(t) = J \left(-4K_1 \sigma(t) - K_2 \omega(t) - \Delta \right) - K_1 \sigma(t) - K_2 \omega(t) + \Delta \right),
\]

where $K_1$ and $K_2$ are positive definite matrices.

**Assumption 1:** The purely nonlinear function

\[
f(x(t)) = \mathcal{E}(\sigma(t), \omega(t)) = \begin{bmatrix} B(\sigma(t)) - 4x_3 \omega(t) - 4J^{-1}\omega(t)J\omega(t) \end{bmatrix} \in \mathbb{R}^6,
\]

satisfies the Lipschitz condition in a certain vicinity of the origin, that is

\[
\|f(\sigma, \omega)\| = \|f(x(t))\| \leq \gamma \|x(t)\|
\]

where $\gamma$ is some positive constant. The constant term $\gamma$ can be obtained by finding the upper bound of the nonlinear term $f(x)$, where $x(t) = [\sigma(t)\tau^T]T$ is the state variables of the system if we write the closed-loop dynamical system in the following matrix form

\[
x(t) = Ax(t) + A_d x(t - \tau) + f(x(t))
\]

where the matrices $A$ and $A_d$ can be easily obtained.

Define the L-K functional candidate $V(x(t))$ as

\[
V(x(t)) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds + \int_{t-\tau}^{t} \int_{\eta}^{t} \left[ \Phi(t)R\Phi(s) + f^T(s)R_1f(s) + \|R_2\| \|x(s)\|^2 \right]dsd\eta
\]

where $\Phi(t) = Ax(t) + A_d x(t - \tau)$. The time derivative of the above L-K functional along its trajectories results in the following stabilization theorem.

**Theorem 1:** There exists a linear state feedback controller in the form of Eq. (5) such that the solution of the closed-loop dynamical system is asymptotically stable for any given time delay $\tau$ satisfying $0 < \tau \leq \tau_{\text{max}}$, if there exists symmetric matrices $X, W, Z, Z_1, Z_2, Z_3 > 0$ and matrices $H, G, V, V_1$ such that the following LMI holds

\[
\Psi_3 = \begin{bmatrix}
\Omega_3 & \Gamma_3 & \tau_{\text{max}}X^T & X^T & \tau_{\text{max}}X^T \\
* & \Delta_3 & \tau_{\text{max}}H^T & 0 & 0 \\
* & * & -Z_{\text{max}} & 0 & 0 \\
* & * & * & -X & 0 \\
* & * & * & 0 & -\tau_{\text{max}}Z_2
\end{bmatrix} < 0,
\]

and

\[
\Sigma_3 = \begin{bmatrix}
V & G \\
G^T & -(Z + Z_1 + Z_2) + 2X_1
\end{bmatrix} < 0
\]

where

\[
\Omega_3 = \begin{bmatrix}
X^T + A^T + W & + \tau_{\text{max}}V + (G + G^T) \\
H - G,
\end{bmatrix}
\]

\[
\Gamma_3 = -W.
\]

The control gain matrices $K_1$ and $K_2$ can be obtained from matrix $A_d = HX^{-1}$.

**B. Inverse Dynamics Approach for Multi-actuator Maneuvers**

The second approach will be utilized in this section. In particular, an inverse dynamics approach common in robotics open-loop path-planning problems is utilized here, in which the desired closed-loop response is approximated by a set of second order delay differential equations. This approach
(without time delay) has been used in the attitude control problem using both quaternions [12] and MRP s [15].

We assume that the desired closed-loop system is given by
\begin{equation}
\dot{\sigma}(t) + k_1 \dot{\sigma}(t) + k_2 \sigma(t) = k_3 \sigma(t - \tau), \tag{12}
\end{equation}
where \(k_1, k_2,\) and \(k_3\) are scalar control gains. Note that the integral term in Eq. (12) is added to eliminate the steady state error caused by unmodeled torques due to noise processes. By following the inverse dynamics approach [11], it can be shown that the nonlinear delayed feedback control law given by
\begin{equation}
\sigma(t) = \frac{A^T - A}{\sqrt{\text{tr}(A) + 1}(\sqrt{\text{tr}(A) + 1} + 2)} \tag{17}
\end{equation}

On the other hand, one can obtain the measurement vector \(y\) with the process noise as a function of the attitude states as
\begin{equation}
y = h(\sigma, \omega) + \epsilon(t), \tag{18}
\end{equation}
with the measurement function \(h()\) being defined as
\begin{equation}
h(\sigma, \omega) = \begin{bmatrix} v^{1s} \\ v^{2s} \\ \omega \end{bmatrix}, \tag{19}
\end{equation}
where \(v^{is} = A(\sigma)v^{is}, i=1,2,\) and where \(A\) is expressed in terms of MRP s as [15]
\begin{equation}
A(\sigma) = I_{3\times 3} + \frac{8(1 - \|\sigma(t)\|^2)\sigma^X}{(1 + \|\sigma(t)\|^2)^2}, \tag{20}
\end{equation}

IV. OBSERVER-BASED CONTROLLER DESIGN

In developing the feedback control laws in Section II, both the MRP parametrization \(\sigma(t)\) (or \(\sigma(t - \tau)\)) and the angular velocity \(\omega(t)\) (or \(\omega(t - \tau)\)) of the rigid body are assumed to be available. A problem arises if the internal states of the system are not known, in which case we can design an observer or estimator that attempts to reconstruct the internal state vector of the plant using measurable noisy outputs in a way that is applicable in state feedback. Assuming the system is controllable and observable, the controlled model with measurements takes the form of
\begin{equation}
\begin{bmatrix} \sigma(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} -J^{-1}\omega^X(t)J\omega^X(t) + J^{-1}\hat{u}(t) \\ f(\begin{bmatrix} \sigma(t) \\ \omega(t) \end{bmatrix}) + b\hat{u}(t) + G\zeta(t), \end{bmatrix} \tag{21}
\end{equation}
where \(\zeta(t)\) and \(\epsilon(t)\) are zero-mean Gaussian noise processes, \(b = [0_{3\times 3} J^{-1}]^T,\) \(G = [0_{3\times 3} I_{3\times 3}]^T,\) and \(\hat{u}(t)\) is the control law given by Eqs. (5) or (13) where the current states \(\sigma(t),\) \(\omega(t),\) and the delayed states \(\sigma(t - \tau),\) \(\omega(t - \tau)\) are replaced with their corresponding estimates \(\hat{\sigma}(t),\hat{\omega}(t),\) and \(\hat{\sigma}(t - \tau),\hat{\omega}(t - \tau)\) obtained from the state observer. Note that the closed-loop system in Eq. (21) contains time delay through the control law \(\hat{u}(t)\) and thus the dynamic model is a delay-differential equation. Also note that the process noise is added only to the dynamics, not the kinematics.

Now, we employ CSCTA [3] and define the expanded \((6(N+1) \times 1)\) vector \(Y(t)\) as
\begin{equation}
Y(t) = \begin{bmatrix} Y_1^T(t), Y_2^T(t), \cdots, Y_{N+1}^T(t) \end{bmatrix}^T
\end{equation}
\begin{equation}
Y_i(t) = [\sigma^{T}(t - \tau_{i-1}), \omega^{T}(t - \tau_{i-1})]^T, i = 1, \cdots, N+1
\end{equation}
such that \( \tau_{i-1} = \tau_{i-1} \), where \( \tau_{i-1} = \cos \frac{(i-1)\pi}{N} \).
Using the expanded state vector definition, the controlled system described in Eq. (21) can be cast in the form of the following set of ODEs and algebraic equations:

\[
\dot{Y}(t) = \begin{bmatrix} 0_{6 \times 6} & 0_{6 \times 6} & \cdots & 0_{6 \times 6} & 0_{6 \times 6} \\ \frac{2}{\pi} D(7:6,N+1) : & b \\ 0_{6N \times 1} \end{bmatrix} \dot{Y}(t) + \begin{bmatrix} f(Y_1(t)) \\ 0_{6N \times 3} \end{bmatrix} \dot{u}(t) + \begin{bmatrix} G \\ 0_{6N \times 3} \end{bmatrix} \zeta(t) = \mathcal{F}(Y(t)) + \mathcal{B} \dot{u}(t) + \mathcal{G} \zeta(t)\]

\[
y = h(Y_1(t), t) + \epsilon(t) = \mathcal{H}(Y(t)) + \epsilon(t), \quad (23)
\]

where the differential operator \( \mathcal{D} \) is obtained as the
Kronecker product \( D_{(N+1) \times (N+1)} \) \( \otimes \) \( I_{6 \times 6} \), and
where \( D_{(N+1) \times (N+1)} \) is the Chebyshev spectral differentiation matrix defined as

\[
D_{11} = \frac{2N^2 + 1}{6}, \quad D_{qq} = -\frac{t_q}{2(1 - t_q^2)}, \quad q = 2, \ldots, N, \]

\[
D_{pq} = c_p(-1)^{p+1}, \quad p \neq q, \quad p, q = 1, 2, \ldots, N + 1, \]

\[
c_p = \begin{cases} 2, & p = 1, N + 1 \\ 1, & \text{otherwise} \end{cases} \quad (24)
\]

Then, it can be shown that the optimal observer is given by the set of ODEs of the form

\[
\dot{\hat{Y}}(t) = \begin{bmatrix} 0_{6 \times 6} & 0_{6 \times 6} & \cdots & 0_{6 \times 6} & 0_{6 \times 6} \\ \frac{2}{\pi} D(7:6,N+1) : & b \\ 0_{6N \times 1} \end{bmatrix} \hat{Y}(t) + \begin{bmatrix} f(Y_1(t)) \\ 0_{6N \times 3} \end{bmatrix} \hat{u}(t) + \begin{bmatrix} G \\ 0_{6N \times 3} \end{bmatrix} \hat{\zeta}(t) = \mathcal{F}(\hat{Y}(t)) + \mathcal{B} \hat{u}(t) + \mathcal{G} \hat{\zeta}(t)\]

\[
y(t) = \mathcal{H}(\hat{Y}(t)) + \epsilon(t), \quad (25)
\]

where \( \hat{Y}(t) \) denotes the augmented estimated states. The observer described in Eq. (25) delivers the estimated states \( \hat{\omega}(t), \hat{\sigma}(t), \hat{\sigma}(t-\tau) \), and \( \hat{\omega}(t-\tau) \) which are required to form the control input \( \hat{u}(t) \) in Eq. (21). Therefore, the observer-based controller can be expressed as

\[
\dot{Y}(t) = \mathcal{F}(Y(t)) + \mathcal{B} \dot{u}(t) + \mathcal{G} \zeta(t), \quad (26)
\]

\[
y(t) = \mathcal{H}(Y(t)) + \epsilon(t), \quad (26)
\]

where, according to Eqs. (5) and (13), \( \hat{u}(t) \) can be either of the following forms:

\[
\hat{u}(t) = J [-4K_1 \hat{\sigma}(t-\tau) - K_2 \hat{\omega}(t-\tau)], \quad (27)
\]

\[
\hat{u}(t) = \hat{\omega}^T(t) J \hat{\omega}(t) - J K_1 \hat{\omega}(t) - J \left[ \frac{4k_2}{1 + \| \hat{\sigma}(t) \|^2} \right] \hat{\sigma}(t) + \frac{4 \mathcal{J} \hat{\sigma}(t)}{1 + \| \hat{\sigma}(t-\tau) \|^2}, \quad (28)
\]

Finally, the EKB filter is used in the observer to estimate the states and the delay to propagate the state error covariance of the delayed feedback controller from noise corrupted indirect measurements of the states. Therefore, the observer gain based on the EKB filter is \( L(t) = P(t) \tilde{H}^T(t) R^{-1} \) where \( R \) is the covariance of the measurement noise. \( P(t) \) is the error covariance propagated along with Eq. (26) as

\[
\tilde{P}(t) = \mathcal{F}(t) P(t) + P(t) \mathcal{F}^T(t) - L(t) \tilde{H}(t) P(t) + \mathcal{G}(t) Q \mathcal{G}^T(t), \quad (29)
\]

where \( \mathcal{F}(t) \) and \( \tilde{H}(t) \) are the Jacobian matrices

\[
\tilde{F}(t) := \frac{\partial \mathcal{F}(Y(t))}{\partial Y(t)} \bigg|_{Y=Y}, \quad \tilde{H}(t) := \frac{\partial \mathcal{H}(Y(t))}{\partial Y(t)} \bigg|_{Y=Y}. \quad (30)
\]

If parameters of the system are not known the observer can be designed to estimate the parameters along with the states in a way that is applicable in the state feedback. The control laws in Eqs. (5) and (13) requires the current states \( \sigma(t) \) and \( \omega(t) \) and the delayed states \( \sigma(t-\tau) \) and \( \omega(t-\tau) \). It can be shown that the approach used for designing the observer in the observer-based controller is also capable of obtaining an estimate \( \hat{\tau} \) for the delay \( \tau \). Another observer can be designed to estimate the current states \( \hat{\sigma}(t) \), \( \hat{\omega}(t) \), and the time delay \( \hat{\tau} \). Therefore, the estimated delayed states can be obtained by first finding the estimated current states and then moving back in time to the extent of the estimated delay. Obviously, the observer error in this case is expected to be more than in the case where both the current and the delayed states are directly estimated and the fixed time delay is assumed to be known. However, it is not always a realistic assumption to consider the delay to be known.

V. Simulation Results

The observer-based controller of Eq. (26) is applied in separate simulations with the linear and nonlinear control laws developed in Section II for attitude control of a rigid spacecraft with the inertia matrix \( J = \text{diag}[30, 20, 10] \text{kg m}^2 \). Two different simulations are considered for each control law.

In the first case, the TRIAD algorithm is used to obtain MRPs from the noise-corrupted measurements of vectors \( \mathbf{v}^1 \) and \( \mathbf{v}^2 \) in the body frame. The angular velocity is assumed to be measured directly with some uncertainty. The initial condition used for the first simulation is \( x(0) = \hat{x}(0) = [-0.5, -0.5, 0.5, 0.1, 0.2, 0.5]^T \). The angular velocities are in terms of radians per second. In other words, the observer’s initial state is assumed to be identical to that of the controller. The observer in the first case of the simulations estimates the current states \( x(t) \) and the delayed states \( x(t-\tau) \), where the time delay is assumed to have a known value of \( \tau = 0.5 \text{sec} \). The process noise covariance is assumed to be \( Q = 0 \) and the covariance of the measurement noise is assumed to be \( R = 1 \times 10^{-3} \text{I} \). The initial error covariance is considered as \( P_0 = 1 \times 10^{-3} \text{I} \). The results of the first simulation for the linear control law given in Eq. (5) and the nonlinear control law given in Eq. (13) are
depicted in Figs. 3 and 4, respectively. Note that the control gains for the linear control law given in Eq. (5) in this case are $K_1 = 0.2451I$, $K_2 = 0.9834I$, and the control gains for the nonlinear control law given in Eq. (13) are $k_1 = 8$, $k_2 = 16$, $k_3 = 8$. Note that the values of the measurement vector $y$, as in Eq. (19), contain measurements of $v^1s$ and $v^{2s}$.

Next, both $\sigma$ and $\omega$ are assumed to be directly measurable with some measurement noise. Therefore, the measurement function $h$ is just an identity function for the second case. The initial state of the controller is $x(0) = [-0.3, -0.4, 0.2, 0.2, 0.2, 0.2]^T$, where the angular velocities are in terms of radians per second. However, the initial state of the observer is assumed to be 50% deviated from that of the controller. The observer in this case estimates the current states $x(t)$ as well as the time delay $\tau$. The delayed states required for the controller are thus obtained from the estimated delay $\hat{\tau}$. Therefore, the observer-based controllers in this case assume that the time delay is unknown and estimated by the observer. The true value of the unknown delay for the linear and nonlinear control laws are $\tau = 1.2$ sec and $\tau = 0.95$ sec, respectively. The process noise covariance and the measurement noise covariance in this case are $Q = R = 1 \times 10^{-5}I$ and the initial error covariance is $P_0 = 1 \times 10^{-3}I$. The results for the estimated delay using the linear controller given in Eq. (5) and the nonlinear controller given in Eq. (13) are demonstrated in Figs. 5 and 6, respectively. Note that the control gains for the linear control law given in Eq. (5) in this case are $K_1 = 0.2451I$, $K_2 = 0.9834I$, and the control gains for the nonlinear control law given in Eq. (13) are the same as in the first case.

The control torque for these observer-based controllers for the first case are plotted in Fig. 7. It can be seen that the control torque of the linear controller given in Eq. (5) is less than that of the nonlinear controller given in Eq. (13) for the first couple of seconds.

VI. Conclusions

In this paper, two observer-based controllers were designed to address the stabilization problem of attitude dynamics in the presence of time delay in the measurements or the actuators, where the TRIAD algorithm was successfully employed to process the measurements. The two proposed control laws were based on the inverse dynamics approach and Lyapunov-Krasovskii functional. The observer-based control laws were shown to be capable of accurate control of spacecraft attitude from noise-corrupted attitude measurements. For future work, the QUEST algorithm can be used with quaternions as the attitude parameterization.

References


Fig. 3: The attitude states $\sigma(t)$ and the angular velocity components $\omega(t)$ of the observer-based controller using the linear control law given in Eq. (5) along with the corresponding estimated states, where the TRIAD-based measurements is used for $\sigma$ and $\omega$ is measured directly (Eq. (19)).

Fig. 4: The attitude states $\sigma(t)$ and the angular velocity components $\omega(t)$ of the observer-based controller using the nonlinear control law given in Eq. (13) along with the corresponding estimated states, where the TRIAD-based measurements is used for $\sigma$ and $\omega$ is measured directly (Eq. (19)).

Fig. 5: The estimated delay using the linear control law given in Eq. (5), where both states $\sigma$ and $\omega$ are measured directly.

Fig. 6: The estimated delay using the nonlinear control law given in Eq. (13), where both states $\sigma$ and $\omega$ are measured directly.

Fig. 7: The control torque for the first set of the simulations for the linear controller given in Eq. (5) (solid line) vs. the control torque for the nonlinear controller given in Eq. (13) (dashed line).