Convexifying Optimal Power Flow: Recent Advances in OPF Solution Methods

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Abstract—The optimal power flow (OPF) problem is nonconvex and generally hard to solve, see e.g. [1], [2]. In this tutorial, we will provide an overview of two different solution approaches. The first uses the bus injection model, which is the standard model for power flow analysis and optimization. It focuses on nodal variables such as voltages, current and power injections and does not directly deal with power flows on individual branches. A key advantage is the simple linear relationship \( I = Y V \) between the nodal current injections \( I \) and the bus voltages \( V \) through the admittance matrix \( Y \). Recently, it has been observed that this form of OPF can be reformulated as a nonconvex QCQP (quadratic constrained quadratic program), which leads to a standard convex relaxation through semidefinite programming [3]–[5]. For radial networks, different sufficient conditions have been derived under which the semidefinite relaxation turns out to be exact [6]–[8].

The second solution technique employs the branch flow model, which focuses on currents and powers on the branches rather than the nodal variables. The branch flow model has been historically used primarily for modeling distribution circuits, which tend to be radial. It has therefore received far less attention. A branch flow model has recently been proposed for the analysis and optimization of mesh as well as radial networks. The model leads to a new approach to solving OPF that consists of two relaxation steps. The first step eliminates the voltage and current angles and the second step approximates the resulting problem by a conic program that can be solved efficiently. For radial networks, both relaxation steps are always exact, provided there are no upper bounds on loads [9]. For mesh networks, the conic relaxation is always exact and we provide a simple way to determine if a relaxed solution is globally optimal. We describe a simple method to convexify a mesh network using phase shifters so that both relaxation steps are always exact and OPF for the convexified network can always be solved efficiently for a globally optimal solution. We prove that convexification requires phase shifters only outside a spanning tree of the network graph and their placement depends only on network topology, not on power flows, generation, loads, or operating constraints [10].

The tutorial will describe precisely the bus injection model and the semidefinite relaxation of OPF as well as the branch flow model and its associated relaxations. We prove sufficient conditions for exact relaxations and verify our results on simulations of various IEEE test systems. Finally we explain the equivalence between the bus injection model and branch flow model.

REFERENCES


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