Observer Backstepping Control for Variable Speed Wind Turbine

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Abstract—This paper presents an observer backstepping controller as feasible solution to variable speed control of wind turbines to maximize wind power capture when operating between cut-in and rated wind speeds. The wind turbine is modeled as a two-mass drive-train system controlled by the generator torque. The nonlinear controller aims at regulating the generator torque such that an optimal tip-speed ratio can be obtained. Simply relying on the measured rotor angular velocity the proposed observer backstepping controller guarantees global asymptotic tracking of the desired trajectory while maintaining a globally uniformly ultimately bounded torsional angle. The proposed controller shows convincing performance when simulated in closed loop within a stochastic environment.

I. INTRODUCTION

In order for wind energy to gain further attention by governments worldwide, the cost of the produced energy must match other competing sources, e.g. coal and gas power. The environmental awareness has only increased, and many countries are showing a large interest in deploying offshore wind power plants. Due to the high risk and capital investment needed to build offshore wind power, the energy produced is supported locally through various regulations, often subsidiaries. When the deployment of offshore wind parks will reach the industrialized state, the subsidiaries will be out phased, but the investment cost will not be reduced. The challenging task of controlling wind turbines for maximum energy output while minimizing drive train stress is therefore of high interest.

The complexity of wind turbine models is a great challenge due to the many degrees of freedom needed to include the most important dynamic effects. The main focus in this article is controlling the drive-train as it is the part which has the most wear, and is a limiting factor in maximizing wind energy capture. The focus of wind turbine control is divided into two regions as a function of the wind speed as shown in Fig. 1. The goal in the region between cut-in and rated wind speeds is to maximize energy capture. In case of wind speeds in the operational range between rated and cut-out, the goal is to keep a constant angular velocity to limit generated noise, stress and vibrations on mechanical and structural components while maintaining the maximum rated power generation. When the power is below rated value, the system should maintain its pitch angle at the optimal value and control the generator torque to achieve optimal tip-speed ratio by opposing the aerodynamic torque. Achieving optimal tip-speed ratio then simplifies to keeping the rotor blades spinning at a rotational frequency related to that of the incoming wind. The choice of operating at the optimal tip-speed ratio is based on the turbulent wake a blade makes when passing through an air stream. Extracting power from turbulent wind is less efficient and will subject the blades to high vibration stress, i.e. the angular speed of the rotor must match the settling time of the wind for optimum power capture.

The modeling of drive-trains varies with regard to assumptions of stiffness in the shafts, damping, inertia assessment and efficiency. The adopted drive-train model is a two-mass system, which compared to the traditional one-mass model includes torsional effects. Further increasing model complexity adds an unnecessary level of detail, and there is a major consensus that a two-mass model is sufficient for representing the important dynamics in power system stability studies. Moreover, it has been shown in [11] that the use of too simple models, as e.g. the one-mass drive train model, could introduce relevant errors in the stability analysis of the dynamic behavior.

In recent years nonlinear control theory has found an interesting test bed in control of wind turbines, which certainly has inherent challenges due to the strong nonlinearities introduced by the aerodynamic torque. The design of nonlinear variable speed controllers has received particular attention, also due to the fact that the nonlinear effect of the aerodynamic torque significantly simplifies when the wind turbine operates in the partial load region. Most of the available literature proposed control strategies, which rely on the lumped one-mass drive train model disregarding completely the mechanical dynamics due to the generator. Interesting examples can be found in [14], where the authors presented a first attempt to apply nonlinear and adaptive control algorithms to the variable speed control problem; in [2] static and dynamic feedback linearizing controllers were introduced to overcome problems of robustness and power losses inherent of the indirect speed control technique; in [9] and [10] robust and adaptive backstepping controllers have been designed, which guarantee global uniform ultimate boundedness and global asymptotic stability of the tracking error, respectively. Also a sliding mode approach has been recently presented in [1] where a sliding mode controller capable of ensuring stability both in the partial load region and in the top region was proposed.
The reviewed literature presents at least two common characteristics: the simplicity of the drive train model and the full knowledge of the system states, also in the few cases where the mechanical model of the rotor is linked to the electrical subsystem driving the generator. Adaptation to system parameters has been taken marginally into account. Moreover due to the low complexity of the chosen models, mechanical stress was completely disregarded. To the best of the authors’ knowledge there is only one publication where a two-mass drive train model has been previously considered in order to design a nonlinear controller. In [8] an output feedback controller has been presented, which resembles a backstepping based control law.

This paper proposes an observer backstepping controller for the variable speed wind turbine in order to maximize wind power capture. The output backstepping controller fully exploits the nonlinear two-mass drive-train model of the wind turbine through the combination of a globally exponentially stable (GES) reduced order observer for the estimate of the torsion angle and of the generator shaft speed, and a globally asymptotically stable (GAS) controller that guarantees asymptotic tracking of the desired rotor speed and global uniform ultimate boundedness (GUUB) of the torsion angle. The backstepping controller is designed directly on the dynamics of the rotor/generator angular velocities without the need of previously determining a diffeomorphism to bring the system in normal form as shown in [8]. In fact the dynamics of the rotor/generator angular velocities is already in "quasi" strict-feedback form [7], where the torsional angle can be easily dealt with through a feedforward action inside the backstepping controller thanks to the globally exponentially stable observer, which provides the controller with an estimate of it. Finally, the papers also try to assess the performance of the proposed controller beyond the tracking properties by looking at the drive train stress.

II. MODEL

A. Wind power capture

The efficiency of a wind turbine is described using a power curve which is taken as a function of the blade’s pitch angle $\beta$ and tip-speed ratio $\lambda$. The power coefficient $C_p$ is the ratio between aerodynamic rotor power $P$ and the available wind power $P_w$, and it defines the percentage of power that is possible to capture

$$C_p = \frac{P}{P_w}.$$  \hfill (1)

The available power from the wind is

$$P_w = \frac{1}{2} \rho A v^3,$$  \hfill (2)

where $\rho$ is the air density, $v$ is the wind speed, and $A = \pi R^2$ is the swept area of the rotor with $R$ being the blade tip radius. Inserting (2) into (1) the power captured by the wind turbine can be obtained as

$$P = \frac{1}{2} C_p(\lambda, \beta) \rho A v^3,$$  \hfill (3)

where $\lambda$ is defined as

$$\lambda \triangleq \frac{R_o \omega}{v}$$  \hfill (4)

with $\omega$ being the rotor angular velocity.

Equation (3) shows that operating at a fixed pitch angle makes the power coefficient $C_p$ a function of only $\lambda$ such that an optimal point on the power curve can be obtained by keeping $\lambda$ constant. From (4) it is clear that $\lambda$ is a function of the wind speed $v$, which is an exogenous signal, and of rotor speed $\omega$, which instead is an endogenous variable available for control. In order to sustain maximum power output, then the rotor speed must be adjusted according to wind speed variations. The relationship between captured wind power, rotor speed and aerodynamic torque is derived from (3) and (4), as shown in [14]

$$P = k_w \omega^3$$  \hfill (5)

$$T_a = \frac{P}{\omega} = k_w \omega^2,$$  \hfill (6)

where the constant $k_w$ is found from the optimal values $C_p^*$ and $\lambda^*$

$$k_w^* = \frac{1}{2} C_p^* \rho \pi \frac{R_o^2}{\lambda^3}.$$  \hfill (7)

The aerodynamic torque $T_a$ is applied to the low-speed shaft of the drive-train, and the dynamics of the two-mass drive train system can then be set up.

B. Two-mass drive-train model

The two-mass drive-train model consists of two shafts interconnected by a gearbox. The aerodynamic torque $T_a$ drives the low-speed shaft at the rotor speed $\omega$, while the gearbox increases the angular speed of the high speed shaft to $\omega_g$ while lowering the torque. Thereby the drive-train converts wind energy to mechanical energy and, through the generator, to electrical energy.

The inertia of the rotor and generator is respectively lumped into $J_r$ and $J_g$, and $T_{ls}, T_{hs}$ and $T_g$ denote low speed shaft torque, high speed shaft torque and generator torque. The stiffness of the shafts are modeled through damping and torsional coefficients $B_r, B_g, K_r$ and $B_g$. The inertia of the low speed shaft includes the inertia of the rotor, while the friction component includes bearing frictions. The dynamics of the low speed shaft is given by

$$J_r \dot{\omega} = T_a - T_{ls} - B_r \omega.$$  \hfill (8)
The high speed shaft has similar dynamics, which includes the inertia of the gearbox and generator, and the friction from bearing and gears

\[ J_g \omega_g = T_{hs} - T_g - B_g \omega_g. \tag{9} \]

The drive train torsion is modeled as the sum of a torsional spring force and a frictional force

\[ T_{ls} = K_d \theta_k + B_d \dot{\theta}_k, \tag{10} \]

where \( \theta_k = \theta - \theta_g/N_g \) is the torsional angle, given as the difference between the rotor angular position \( \theta \) and the generator angular position \( \theta_g \) scaled by the drive train gear ratio \( N_g \); \( \dot{\theta}_k \) is the time derivative of the torsional angle. The low speed and high speed shaft are interconnected by the gearbox such that

\[ N_g = \frac{T_{ls}}{T_{hs}} = \frac{\omega_g}{\omega_{ls}}. \tag{11} \]

The two-mass drive train dynamics results from the combination of (8) to (11), as shown in the following

\[
\begin{align*}
\dot{\theta}_k &= \omega - \frac{1}{N_g} \omega_g \\
\dot{\omega} &= \frac{1}{J_r} \left( -K_d \theta_k + k_w \omega^2 - (B_d + B_r) \omega + \frac{B_d}{N_g} \omega_g \right) \\
\dot{\omega}_g &= \frac{1}{J_g} \left( \frac{K_d}{N_g} \theta_k + \frac{B_d}{N_g} \omega - \left( \frac{B_d}{N_g^2} + B_g \right) \omega_g - T_g \right)
\end{align*}
\tag{13-14} \]

III. OBSERVER BACKSTEPPING CONTROL

Given the third order nonlinear system (12)-(14) the control objective is to design a controller capable of dynamically varying the rotor angular velocity \( \omega \) such that a desired speed reference can be tracked. This must be achieved assuming that the only available measurement is \( \omega \).

The strategy chosen is an output feedback controller based on the observer backstepping control [7]. First a reduced order observer is designed for the subsystem \((\theta_k, \omega_g)\), which has an exponentially stable estimation error dynamics. Then, a new system is considered where the unmeasured dynamics is replaced with the observer dynamics. Hence backstepping is applied using the state estimates as virtual control inputs and considering the estimation errors as disturbances whose behavior must be dominated.

Since the control objective is reference tracking the system (12)-(14) is rewritten in terms of the tracking error \( e_\omega \triangleq \omega - \omega_d \) where \( \omega_d(t) \in C^2 \) is a bounded reference trajectory with bounded derivatives. By inserting \( e_\omega \) into Eqs. (12)-(14) the following dynamics is obtained

\[
\begin{align*}
\dot{\theta}_k &= e_\omega + \dot{\omega}_d - \frac{1}{N_g} \omega_g \\
\dot{\omega}_d &= \frac{1}{J_r} \left( -K_d \theta_k + k_w (\omega + \omega_d)^2 + \frac{B_d}{N_g} \omega_g \\
&- (B_d + B_r) (\omega + \omega_d) \right) \\
\dot{\omega}_g &= \frac{1}{J_g} \left( \frac{K_d}{N_g} \theta_k + \frac{B_d}{N_g} (\omega + \omega_d) \\
&- \left( \frac{B_d}{N_g^2} + B_g \right) \omega_g - T_g \right)
\end{align*}
\tag{15-17} \]

A. Observer Design

A reduced order observer of the \((\theta_k, \omega_g)\) dynamics is built as

\[
\begin{align*}
\dot{x}_2 &= \xi + Ly \\
\xi &= M \xi + NT_g + Ry
\end{align*}
\tag{18-19} \]

where \( x_2 = [\dot{\theta}_k, \dot{\omega}_g]^T \), \( y = x_1 = \omega \) is the measured output, \( \xi = [\xi_1, \xi_2]^T \) is an auxiliary state vector, the matrices \( M \), \( N \), and \( R \) are design parameters, and \( L = [l_1, l_2]^T \) is the observer gain matrix.

Let \( e_o = [\dot{\theta}_k, \dot{\omega}_g]^T \) be the estimation error. Then the estimation error dynamics is given by

\[
\begin{align*}
e_o &= \dot{x}_2 - \dot{x}_2 \\
&= (A_{21} + Lc_1A_{11}(x_1)) + MLc_1 - Rc_1 \right) x_1 \\
&+ (A_{22} - Lc_1A_{12} - M) \xi \\
&+ (b_2 - Lc_1b_1)T_g + Me_o \\
&= Me_o
\end{align*}
\tag{20} \]

with

\[
M \triangleq A_{22} - Lc_1A_{12} \
\tag{21} \]

\[
N \triangleq b_2 - Lc_1b_1 = \begin{bmatrix} 0 & -\frac{1}{J_g} \end{bmatrix}^T \tag{22} \]

\[
R \triangleq \frac{1}{c_1} (A_{21} + Lc_1A_{11}(x_1)) + ML \tag{23} \]

\[
\begin{bmatrix} r_1(x_1) \\
r_2(x_1) \end{bmatrix} = \begin{bmatrix} l_1K_d/J_r + l_2K_d/J_r - \frac{1}{J_g} \left( \frac{B_d}{N_g} + B_g \right) + \frac{1}{J_g} \frac{B_d}{N_g} \frac{K_d}{N_g} x_1 \\
\right) \frac{1}{J_r} \left( \frac{B_d}{N_g} + B_g \right) + \frac{1}{J_r} K_d/J_g + \frac{B_d}{J_g} - \frac{1}{J_r} x_1 \right) + \frac{l_2}{J_r} K_d/J_g + \frac{B_d}{J_g} - \frac{l_2}{J_r} \left( \frac{B_d}{N_g^2} + B_g \right) \tag{24} \]

\[
\begin{align*}
r_1(x_1) &= 1 - \frac{l_2}{N_g} + \frac{l_2^2 K_d}{J_r} + \frac{l_1}{J_r} (B_d + B_r) \\
&- \frac{l_2 l_1 B_d}{J_r N_g} - \frac{l_2 k_w}{J_r} x_1 \\
&- \frac{l_2}{J_r} K_d + \frac{l_2 (B_d + B_r)}{J_r} - \frac{l_2^2 B_d}{J_r N_g} - \frac{l_2 k_w}{J_r} x_1.
\end{align*}
\tag{25} \]
The matrices $A_{ij}$ in (21)-(23) are defined as follows

$$A_{11}(x_1) = \frac{k_w x_1 - (B_d + B_r) J_r}{J_r}, \quad A_{12} = \left[ \begin{array}{c} -\frac{K_d}{J_r} \\ B_d \end{array} \right],$$

$$A_{21} = \left[ \begin{array}{c} 1 \\ \frac{B_d}{J_r N_g} \end{array} \right]^T, \quad A_{22} = \left[ \begin{array}{c} -\frac{1}{N_g} \\ -\frac{B_g}{J_r N_g} - B_g \end{array} \right].$$

The input vector $b = [b_1, b_2]^T = [0, 0, -1/J_g]$, and the output gain $c_1 = 1$.

Due to linearity of (20), exponential stability of the estimation error can be easily assessed by checking if for a given choice of observer gain $L$ the matrix $M$ is Hurwitz; however the Lyapunov based analysis is here preferred.

**Proposition 1:** The origin estimation error dynamics (20) is GES for $l_1 < 0$ and $l_2 > l_{2,\min}$, with $l_{2,\min} \in \mathbb{R}_+$. 

**Proof:** Consider the Lyapunov function candidate

$$V_o(e_o) = e_o^T P_o e_o$$

where

$$P_o = \left[ \begin{array}{c} -\frac{J_r}{2 \beta_2} \\ 0 \\ \beta_2 \left( \frac{J_r}{B_d + \frac{B_d}{N_g} + 2l_2 B_d}{J_r N_g} \right) \end{array} \right],$$

$P_o = P_o^T > 0$, is the solution of the Lyapunov equation $P_o M + M^T P_o = -Q$, with $Q = \text{diag}\{1/\beta_1, 1/\beta_2\}$ and $\beta_i > 0$. The derivative along the trajectories of (20) results in

$$\dot{V}_o(e_o) = e_o^T (P_o M + M^T P_o) e_o = -e_o^T Q e_o \leq -\lambda_{\min}(Q) ||e_o||^2 < 0.$$  

Inequality (25) is satisfied if $P_o$ is a positive definite matrix, which is true if $l_1 < 0$ and $l_2 > l_{2,\min}$, where $l_{2,\min}$ is the root of the denominator of the entry $P_o(2,2)$. Hence the origin $e_o = (0, 0)^T$ is GES [6, Theorem 4.10], and the estimation error is always bounded

$$||e_o(t)|| \leq \kappa_e ||e_o(t_0)|| e^{\lambda_{\max}(M)(t-t_0)}, \quad \forall t \geq t_0$$

where $\kappa_e > 0$ and $\Re\{\lambda_{\max}(M)\} < 0$. 

**B. Output Feedback Backstepping Controller**

An output feedback backstepping controller is now designed for the system

$$\dot{e}_\omega = \frac{1}{J_r} \left( -K_d \left( \hat{\theta}_d + \hat{\varphi}_d \right) + k_w (e_\omega + \omega_d)^2 
- (B_d + B_r) (e_\omega + \omega_d) + B_d \frac{N_g \left( \bar{\omega}_g + \bar{\omega}_d \right)}{N_g} - \omega_d \right)$$

$$\dot{\bar{\omega}}_g = \frac{1}{J_g} \left( K_d \hat{\theta}_d + \frac{B_d}{N_g} (e_\omega + \omega_d) - \frac{B_d}{N_g} + B_g \right) \bar{\omega}_g - T_g$$

where the estimate $\hat{\varphi}_d$ is used as a virtual control to stabilize (27), and $T_g$ is the physical control input. Further, during the controller design the torsional angle $\theta_k = \hat{\theta}_k + \hat{\varphi}_d$ is considered as an exogenous disturbance, which can be compensated with a feed-forward action since it is estimated.

**Proposition 2:** Consider the system (27)-(28), and the reference vector $\Omega_d = [\omega_d(t), \hat{\omega}_d(t), \bar{\omega}_d(t)]^T$. The output feedback backstepping control law

$$T_g = T_g \left( \hat{\theta}_k, e_\omega, z, \Omega_d \right)$$

with

$$\kappa_1 > \frac{\gamma_1^2}{4 \delta_1 B \lambda_{\min}(Q)}, \quad 0 < \delta_1 < 1$$

$$\kappa_2 > \frac{\gamma_2}{1 - \delta_2}, \quad 0 < \delta_2 < 1$$

$$\kappa_3 > 1 \lambda_{\min}(Q) \left( \frac{\gamma_2^2}{4 \delta_2 \kappa_2} + \frac{\gamma_1^2}{4} \right)$$

renders GAS the origin of the $(e_\omega, z, e_o)$ system, where $T_g \left( \hat{\theta}_k, e_\omega, z, \Omega_d \right)$ is shown in (32),

$$z \triangleq \hat{\varphi}_d - \alpha \left( \hat{\theta}_k, e_\omega, \Omega_d \right)$$

and $\alpha \left( \hat{\theta}_k, e_\omega, \Omega_d \right)$ is shown in (33). The coefficients in (32) are given in Appendix V-A.

**Proof:** The first step is to stabilize the tracking error dynamics through the virtual control $\hat{\varphi}_d$. Consider the control Lyapunov function (CLF) candidate

$$V_1(e_\omega, e_o) = J_r e_\omega^2 + V_o(e_o)$$

whose derivative along the trajectories is given by

$$\dot{V}_1 = e_\omega \left( -K_d \left( \hat{\theta}_d + \hat{\varphi}_d \right) - (B_d + B_r) (e_\omega + \omega_d) \right)$$

$$+ k_w (e_\omega + \omega_d)^2 - \frac{B_d}{N_g} \left( \bar{\omega}_g + \bar{\omega}_d \right) - \frac{1}{\beta_1} \hat{\theta}_d^2 / \beta_2 \bar{\omega}_g^2.$$  

The virtual control $\bar{\omega}_g = \alpha \left( \hat{\theta}_k, e_\omega, \Omega_d \right)$ given in (33) renders GAS the origin of the $(e_\omega, e_o)$ system. In fact by inserting (33) into (36) $V_1(e_\omega, e_o)$ reads

$$\dot{V}_1 \leq -(B_d + B_r) e_\omega^2 + \left( -K_d \theta_k + \frac{B_d}{N_g} \bar{\omega}_g \right) e_\omega$$

$$- \kappa_1 B_d e_\omega^2 - \lambda_{\min}(Q) \left\| \frac{\gamma_1}{2 \delta_1 \lambda_{\min}(Q)} \right\| \| \xi \|_2^2$$

$$\leq -(B_d + B_r) e_\omega^2$$

$$+ \max \left\{K_d, \frac{B_d}{N_g} \right\} \left\| \frac{\gamma_1}{2 \delta_1 \lambda_{\min}(Q)} \right\| \| \xi \|_1$$

$$\leq \left( 1 - \delta_1 \right) \kappa_1 B_d \left( e_\omega^2 + \left( 1 - \frac{\gamma_1}{2 \delta_1 \lambda_{\min}(Q)} \right) \| \xi \|_1^2 + \left( \lambda_{\min}(Q) - \frac{\gamma_1}{4 \delta_1 \lambda_{\min}(Q)} \right) \right)^2.$$  


\[
T_g(\hat{\theta}_k, e_\omega, z, \Omega) \triangleq \frac{1}{\bar{\rho}_1} \left[ -(\bar{\rho}_1 + \rho_1 c (e_\omega + \omega_d)) \dot{\hat{\theta}}_k - \bar{\rho}_2 \alpha (e_\omega, \dot{\hat{\theta}}_k, \omega_d, \dot{\omega}_d) - \rho_2 e (e_\omega + \omega_d) \left( z + \alpha \left( e_\omega, \dot{\hat{\theta}}_k, \omega_d, \dot{\omega}_d \right) \right) - \bar{\rho}_3 \omega \right. \\
- \left( \bar{\rho}_4 + \bar{\rho}_5 (e_\omega + \omega_d) + \bar{\rho}_6 (e_\omega + \omega_d)^2 \right) (e_\omega + \omega_d) - \bar{\rho}_7 \dot{\omega}_d - \bar{\rho}_8 \ddot{\omega}_d - \kappa_2 z - \kappa_3 (e_\omega + \omega_d)^2 z \right]
\]

(32)

\[
\alpha (\hat{\theta}_k, e_\omega, \Omega_d) \triangleq \frac{N_g}{B_d} \left( -k_w (e_\omega + \omega_d)^2 + (B_d + B_r) \omega_d + K_d \dot{\theta}_k + J_r \ddot{\omega}_d \right) - N_g \kappa_1 e_\omega
\]

(33)

which is negative definite for \( \kappa_1 \) as in (29), and where

\[ \lambda_{1,\min}(Q) = \min \left\{ \frac{1}{\beta_1}, \frac{1}{\beta_2} \right\}; \quad \gamma_1 = \sqrt{2} \max \left\{ K_d, \frac{B_d}{\kappa_3} \right\}. \]

Introducing the error variable \( z \) as in (34) the design of the real control input \( T_g \) is undertaken. The \( z \)-dynamics reads

\[ \dot{z} = \dot{\hat{\theta}}_g - \frac{\partial \alpha}{\partial \hat{\theta}_k} \dot{\hat{\theta}}_k - \frac{\partial \alpha}{\partial e_\omega} \dot{e}_\omega - \frac{\partial \alpha}{\partial \Omega_d} \dot{\Omega}_d \]

(38)

and the detailed expression can be found in Appendix V-A. Then consider the following CLF

\[ V_2 (z, e_\omega, e_\omega) = V_1 (e_\omega, e_\omega) + \frac{1}{2} z^2 + V_0 (e_\omega) \]

(39)

whose derivative along the trajectories reads as

\[ \dot{V}_2 \leq \dot{V}_1 + z \left[ -(\kappa_2 - \bar{\rho}_2) + (\bar{\rho}_9 + \rho_{10c} (e_\omega + \omega_d)) \bar{\rho}_2 \right. \\
+ (\bar{\rho}_{10} + \rho_{10c} (e_\omega + \omega_d)) \bar{\rho}_9 - \kappa_3 (e_\omega + \omega_d)^2 z \]

- \lambda_{2,\min}(Q) \left\| \dot{\xi} \right\|^2_2 \\
\leq \dot{V}_1 - \left( (1 - \delta_2) \kappa_2 - \bar{\rho}_2 \right) z^2 - \delta_2 \kappa_2 z^2 \\
+ \max \left\{ \| \bar{\rho}_9 |, \| \bar{\rho}_10 \right\} \left\| \dot{\xi} \right\|^2_1 \left| e_\omega + \omega_d \right| z \\
- \kappa_3 (e_\omega + \omega_d)^2 z^2 - \lambda_{2,\min}(Q) \left\| \dot{\xi} \right\|^2_2 \\
\leq \dot{V}_1 - \left( (1 - \delta_2) \kappa_2 - \bar{\rho}_2 \right) z^2 \\
- \left( \lambda_{2,\min}(Q) - \frac{\gamma_2^2}{4 \delta_2 \kappa_2} - \frac{\gamma_3^2}{4 \kappa_3} \right) \left\| \dot{\xi} \right\|^2_2 \\
- \delta_2 \kappa_2 \left( \| z \| + \frac{\gamma_2}{2 \delta_2} \right) \left\| \dot{\xi} \right\|^2_2 \left| e_\omega + \omega_d \right| z \left( \| \bar{\rho}_9 |, \| \bar{\rho}_{10c} \right\} \left\| \dot{\xi} \right\|^2_1 \\
- \kappa_3 \left( \| e_\omega + \omega_d \| | z \| + \frac{\gamma_3}{2 \kappa_3} \right) \left\| \dot{\xi} \right\|^2_2 \]

(40)

which is negative definite for \((\kappa_2, \kappa_3)\) as in (30)-(31), and where

\[ \lambda_{2,\min}(Q) = \min \left\{ \frac{1}{\beta_3}, \frac{1}{\beta_2} \right\}; \quad \gamma_2 = \sqrt{2} \max \left\{ \| \bar{\rho}_9 |, \| \bar{\rho}_{10c} \right\} \quad \gamma_3 = \sqrt{2} \max \left\{ \| \bar{\rho}_{10c} |, \| \bar{\rho}_{10c} \right\}. \]

C. Boundedness of the \( \theta_k \) Dynamics

As last step in the design of the output tracking controller it is important to prove that the dynamics of the torsion angle \( \theta_k \) remains bounded. Towards this end consider the candidate Lyapunov function

\[ V_3 (\theta_k) = \frac{1}{2} \theta_k^2 \]

(42)

whose derivative along the trajectory of (12) is

\[ \dot{V}_3 = \theta_k \left( e_\omega + \omega_d - \frac{1}{B_d} \left( -k_w (e_\omega + \omega_d)^2 \\
+ (B_d + B_r) \omega_d + K_d \dot{\theta}_k + J_r \ddot{\omega}_d \right) \right) \]

\[ \leq - \frac{K_d}{B_d} \theta_k^2 + \left( \| k_w \| \| e_\omega + \omega_d \| | \theta_k | + \frac{K_d}{B_d} \| e_\omega + \omega_d \| | \theta_k | \\
+ \frac{B_d + B_r}{B_d} \| \omega_d \| | \theta_k | + \frac{K_d}{B_d} \| \dot{\theta}_k \| \| \theta_k | \right) \]

\[ + \frac{B_d + B_r}{B_d} \| \omega_d \| | \theta_k | + \kappa_1 \| e_\omega \| | \theta_k |. \]

(44)

The desired trajectory \( \omega_d (t) \) is a class \( C^2 \) bounded function with bounded first derivative, that is

\[ \| \omega_d (t) \| \leq \omega_{d,\text{max}} < \infty \]

\[ \| \dot{\omega}_d (t) \| \leq \dot{\omega}_{d,\text{max}} < \infty \]

(45)

(46)

Moreover the dynamics of the tracking error \( e_\omega (t) \) was shown to be asymptotically stable hence

\[ \| e_\omega (t) \| \leq \gamma (| e_\omega (t_0) |, t - t_0), \quad \forall t \geq t_0 \]

(47)

where \( \gamma (r, s) \) is a class \( KL \) function; whereas the dynamics of the estimation error \( e_\omega \) was shown to be exponentially stable therefore

\[ \| \dot{\theta}_k (t) \| \leq \kappa_e \| \dot{\theta}_k (t_0) \| e^{\lambda_{\text{max}}(M)(t-t_0)}, \quad \forall t \geq t_0. \]

(48)
By replacing these upper bounds into (44) we obtain
\[
\dot{\theta}_k \leq -(1 - \delta_d) \frac{K_d}{B_d} \dot{\theta}_k^2 - \delta_d \frac{K_d}{B_d} \theta_k^2 \\
+ \left(1 + \kappa_1 \right) \gamma \left| \left| \epsilon_\omega (t_0) \right| \right|, 0 \\
+ \left( 1 + \frac{B_d + B_v}{B_d} \right) \omega_{d,\text{max}} \left| \theta_k \right| \\
+ \frac{k_w}{B_d} \left( \gamma \left| \left| \epsilon_\omega (t_0) \right| \right|, 0 + \omega_{d,\text{max}} \right)^2 \\
+ \frac{K_d}{B_d} \lambda_k \left| \dot{\theta}_k (t_0) \right| + J_r \dot{\omega}_{d,\text{max}} \left| \theta_k \right| \\
\leq -(1 - \delta_d) \frac{K_d}{B_d} \dot{\theta}_k^2
\] (49)
for all \(|\theta_k| > \mu\)

\[
\mu = \frac{B_d}{\delta_d B_d} \left(1 + \kappa_1 \right) \gamma \left| \left| \epsilon_\omega (t_0) \right| \right|, 0 \\
+ \left( 1 + \frac{B_d + B_v}{B_d} \right) \omega_{d,\text{max}} \\
+ \frac{k_w}{B_d} \left( \gamma \left| \left| \epsilon_\omega (t_0) \right| \right|, 0 + \omega_{d,\text{max}} \right)^2 \\
+ \frac{K_d}{B_d} \lambda_k \left| \dot{\theta}_k (t_0) \right| + J_r \dot{\omega}_{d,\text{max}} \right| \theta_k \right|
\] (50)
whit \(0 < \delta_d < 1\). Hence \(\theta_k (t)\) is GUUB.

IV. CONTROL STRATEGY TESTING

A. Operating point

The system is designed to operate in the partial load region, i.e. the interval of wind speeds ranging from 5 m/s to 12.3 m/s for this particular wind turbine. A wind speed \(\bar{v}\) and a tip-speed ratio \(\lambda\) are selected and the state of the system is calculated. Using (4) the angular velocity \(\omega\) of the rotor can be calculated and through (11) the angular velocity of the generator shaft \(\omega_g\) can be found. Inserting (4) into (6) yields
\[
\bar{T}_a = \frac{1}{2 \bar{\omega}} \rho A \bar{\omega}^3 C_p(\lambda, \tilde{\beta}),
\] (51)
and utilizing (11) gives
\[
\bar{T}_g = \frac{1}{N_g} \bar{T}_a.
\] (52)
The steady state torsion angle \(\bar{\theta}_k\) can be found by inserting (10) into (8), which gives
\[
\bar{\theta}_k = \frac{\bar{T}_a - B_v \bar{\omega}}{K_d}.
\] (53)

Equations (51)-(53) enables the nonlinear simulation model to be initiated in an operating point. The tip-speed ratio value selected for simulations reflects an initial condition related to a configuration for maximum power generation. The values used in simulation are shown in Table I.

<table>
<thead>
<tr>
<th>[\bar{v}\text{[m/s]}]</th>
<th>[\lambda]</th>
<th>[\tilde{\beta}\text{[deg]}]</th>
<th>[T_g\text{[MNm]}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.33</td>
<td>8.00</td>
<td>0.00</td>
<td>2.17</td>
</tr>
<tr>
<td>22.86</td>
<td>80.42 \cdot 10^{-4}</td>
<td>0.46</td>
<td></td>
</tr>
</tbody>
</table>

B. Testing

First the designed observer backstepping controller is tested in a deterministic environment, i.e. while tracking a square wave, in order to assess the asymptotic tracking capabilities. Subsequently, the controller is also tested in a stochastic environment in order to evaluate its performance in a more realistic operational condition.

The rotor speed tracking of a square wave with a frequency of 0.06 rad/s and an amplitude of 10 percent of the operation point is shown in Fig. 2 and 3. It is evident that the proposed control scheme achieves smooth and precise asymptotic speed tracking, while the observer estimates the unmeasured states perfectly. It is also worth noticing that the control objective is fulfilled with a control authority completely in line with the system capabilities, as shown in the bottom plot of Fig. 3.

In the stochastic environment the wind is simulated using RISO DTU SB-2 wind model [4] with a mean wind of 10.33 m/s and 12% turbulence. Given an optimal tip-speed ratio and the measured wind speed, the optimal rotor speed with respect to wind power capture can be calculated from (4). The simulated wind speed and the tracking of the optimal rotor angular velocity are shown in Fig. 4. The closed loop system shows good tracking capabilities of the desired rotor speed with a maximum tracking error of \(\|\epsilon_\omega\|_\infty \leq 324 \mu\text{rad/s}\). The generated power \(P_g\) and the control effort \(T_g\) are shown in Fig. 5.

A more stringent evaluation of the performance of the proposed controller can be obtained by calculating a figure of merit as the stress of the drive train. The drive train stress is measured as the magnitude of the change in torsion angle
\[
S = \int_{t_0}^{t_f} \dot{\theta}_k(t)^2 dt.
\] (54)
where \(t_0\) and \(t_f\) are the initial time and the final time of the simulation. The stress affecting the system during the stochastic simulation shown in Figs. 4-5 is 2.08-10^{-3} \text{rad}^2/\text{s}, which is comparable to similar systems.

V. CONCLUSIONS

Maximizing wind power capture in wind turbines is a major challenge given the constant evolution of the technologies involved and measurements available. In this work an output feedback backstepping approach has been proposed for the variable speed control of the wind turbine. Due to the challenges in measuring the torsion angle and generator speed a linear observer was designed and the estimation error dynamics was shown to be globally exponentially stable.
Then an output feedback backstepping controller was designed exploiting the measured and estimated states and the closed-loop system was shown to be globally asymptotically stable. Finally, it was also proven that the dynamics of the torsion angle remains bounded under the action of the controller; in particular it was shown that it is globally uniformly ultimately bounded. Simulation results have confirmed the effectiveness of the proposed approach, both in the deterministic and stochastic environment. The paper also provided an assessment of the performance of the controller looking at the drive-train stress, which appears to be within reasonable values.

**APPENDIX**

**A. Dynamics of the Error Variable $z$**

The error variable dynamics is given by

$$
\dot{z} = \dot{\xi}_2 - l_2 \omega - \frac{\partial \alpha}{\partial \theta_k} (\dot{\xi}_1 - l_1 \dot{\omega}) - \frac{\partial \alpha}{\partial e_\omega} \dot{e}_\omega - \frac{\partial \alpha}{\partial \Omega_d} \dot{\Omega}_d
$$

$$
= (\tilde{\rho}_1 + \rho_{1e}) \dot{\theta}_k + (\tilde{\rho}_2 + \rho_{2e}) \left( z + \alpha (e_\omega, \dot{\theta}_k, \omega_d, \dot{\omega}_d) \right)
+ \tilde{\rho}_3 \omega + \tilde{\rho}_4 (e_\omega + \omega_d) + \tilde{\rho}_5 (e_\omega + \omega_d)^2 + \tilde{\rho}_6 (e_\omega + \omega_d)^3
+ \rho_{7e} \dot{\omega}_d + \rho_{8e} \dot{\omega}_d + (\tilde{\rho}_9 + \rho_{9e}) \dot{\theta}_k + (\tilde{\rho}_{10} + \rho_{10e}) \dot{\omega}_y
+ \rho_{11} T_g
$$

where the coefficients $\tilde{\rho}_1$ and $\rho_{1e}$ are given by

$$
\tilde{\rho}_1 \triangleq \frac{K_d}{J_g N_g} - \frac{K_d N_g k_1}{J_r}
$$

$$
\rho_{1e} \triangleq -\frac{2K_d N_g k_w}{J_r B_d}
$$

$$
\tilde{\rho}_2 \triangleq \frac{1}{J_g} \left( \frac{B_d}{N_g^2} + B_g \right) + \frac{K_d}{B_d} + \frac{B_d k_1}{J_r}
$$

$$
\rho_{2e} \triangleq \frac{2k_w}{J_r}
$$

$$
\tilde{\rho}_3 \triangleq -\frac{K_d N_g}{B_d} + \frac{B_d}{J_g N_g}
$$
\[ \bar{\rho}_4 \equiv -\frac{\kappa_1 N_g}{J_r} (B_d + B_r) \]
\[ \bar{\rho}_5 \equiv \frac{1}{J_r} \left( N_g \kappa_1 k_w - \frac{2 N_g k_w B_d}{B_d} (B_d + B_r) \right) \]
\[ \bar{\rho}_6 \equiv \frac{2 N_g k^2_w}{J_r B_d} \]
\[ \bar{\rho}_7 \equiv -N_g - \kappa_1 N_g - \frac{B_r N_g}{B_d} \]
\[ \bar{\rho}_8 \equiv \frac{J_g N_g}{B_d} \]
\[ \bar{\rho}_9 \equiv -\frac{K_d}{J_r} \left( l_2 + N_g \kappa_1 - \frac{N_g K_d l_1}{B_d} \right) \]
\[ \bar{\rho}_{10} \equiv -\frac{K_d}{J_r} \left( 2 N_g k_w \right) \]
\[ \bar{\rho}_{11} \equiv \frac{2 k_w}{J_g} \]

References