Modeling of Comprehensive Electric Drive System for a Study of Regenerative Brake System

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Abstract—Regenerative brake system is more complicated compared to a conventional brake system since its design and control goals require not only to achieve a good braking performance but also to recover brake energy as much as possible. In this paper, a comprehensive regenerative brake system model has been developed for a study of brake performance and effective regenerative brake energy recovery. This paper presents a detailed modeling of electric drive system, design of a brake force distribution, and effective brake force allocation between motor brake and friction brake forces. The performance of a proposed model has been evaluated using a commercially available full vehicle model (CARSIM) through simulation.

NOMENCLATURE

- PMSM electric drive system
- \(i_d\) d axis component of stator current (Amps)
- \(i_q\) q axis component of stator current (Amps)
- \(v_d\) d axis component of stator voltage (V)
- \(v_q\) q axis component of stator voltage (V)
- \(R\) stator resistance (Ohms)
- \(L_d\) d axis stator inductance (Henry)
- \(L_q\) q axis stator inductance (Henry)
- \(\lambda\) flux linkage (Weber)
- \(p\) number of pole pairs
- \(w_r\) rotor speed (rad/sec)
- \(J\) combined inertia of rotor and load (kg-m²)
- \(F\) combined viscous friction of rotor and load (N-m-s)
- \(T_e\) electromagnetic torque (N-m)
- \(T_m\) load torque (N-m)
- \(d_d\) continuous duty cycle functions in d axis
- \(d_q\) continuous duty cycle functions in q axis
- \(v_{dc}\) DC voltage (V)
- \(i_{dc}\) DC current (Amps)

Brake system

- \(F_f\) total brake force on front wheels (N)
- \(F_r\) total brake force on rear wheels (N)
- \(a\) distance from C.G. to front axle (m)
- \(b\) distance from C.G. to rear axle (m)
- \(G\) vehicle weight (N)
- \(\phi\) road friction coefficient
- \(h_g\) C.G. height (m)
- \(P_a\) master cylinder pressure (Pa)
- \(P_f\) front wheel cylinder pressure (Pa)
- \(P_r\) rear wheel cylinder pressure (Pa)

INTRODUCTION

Due to the environmental pollution and energy problem of traditional internal combustion engine vehicles (ICEVs), electric vehicles (EVs) and Hybrid EVs (HEVs) are considered as viable solutions to solve the problems of ICEVs and widely investigated. Regenerative braking capability is one of the most important advantages of EVs and HEVs over ICEVs. In recent years research works on regenerative brake system are focused on analyzing the mechanism and brake energy recovery effect of Permanent Magnet Synchronous Motor (PMSM) electric drive system during regenerative braking process [1-3] and designing brake force distribution strategy and cooperative control between regenerative brake and friction brake systems [4-6]. Among these works, there are not many systematic modeling works of regenerative brake system that include both comprehensive PMSM electric drive system and coordinated brake force distribution between friction and regenerative brake system. It is crucial to have a model considering these two aspects in order to develop a realistic regenerative brake system. This paper presents a development of systematic regenerative brake system model including PMSM electric drive, design of brake force distribution to meet the required regulations, and cooperative brake force control between regenerative brake and friction brake.

The rest of a paper is organized as follows. In section II subsystem models of regenerative brake system are described and in section III brake proportioning design of a friction brake system to meet Federal Motor Vehicle Safety Standard (FMVSS) No.135 and pedal simulator design are described. The integration of subsystem models into a whole regenerative brake model is explained in section IV, while in section V the simulation results of regenerative brake model are presented and analyzed. Finally, conclusions are drawn in section VI.

I. MODELING OF REGENERATIVE BRAKE SUBSYSTEMS

A regenerative brake system can be divided into two parts: electrical part and mechanical part. Electrical part includes PMSM, DC/AC inverter, gate drive system, PMSM control, and battery pack. Mechanical part contains hydraulic brake system. The following describes key subsystem models used in the regenerative brake systems.

A. PMSM Electric Drive Subsystem

PMSM electric drive system is widely used in EVs and HEVs due to their advantages over other electric drive...
systems. For instance, DC electric drive system is less reliable and unsuitable for maintenance-free operation due to its commutators and brushes. In addition, compared with other AC motors such as Induction Motor (IM) and Switched Reluctance Motor (SRM), PMSM has higher power density, higher efficiency, higher torque current units, and relatively wider speed operation [7].

- **Modeling of PMSM:**
  In this paper, the d-q model of PMSM [8] is used:

\[
\frac{d}{dt} i_d = (v_d - R_i d + p w_L q i_d) / L_d \\
\frac{d}{dt} i_q = (v_q - R_i q - p w_L (L_d i_d + \dot{\lambda})) / L_q
\]

(1) (2)

The electromagnetic torque produced by PMSM is:

\[ T_e = 1.5 \pi (\lambda_i q + (L_d - L_q) i_d i_q) \]

(3)

The rotor dynamics of PMSM can be described as:

\[ \frac{d}{dt} w_r = \frac{1}{J} (T_e - F W_r - T_m) \]

(4)

- **Modeling of Three Phase DC to AC Inverter:**
  DC/AC inverter is used to transfer power between DC and AC energy sources. There are two different types of DC/AC inverters: Voltage Source Inverter (VSI) and Current Source Inverter (CSI). In this paper, VSI is used for PMSM electric drive system due to its simple configuration. This inverter model is implemented in EVs/HEVs such as Toyota Prius and GM Volt. VSI is modeled by using switching function concept which is very efficient for simulation study. The schematic diagram of inverter is shown in fig. 3.

![Figure 3. Three-phase DC-AC voltage source inverter](image)

The pole voltages \((u_{ao}, u_{bo}, u_{co})\) can be calculated by the following equation and switching functions as:

\[
\begin{align*}
  u_{ao} &= \frac{u_{dc}}{2} \times (S{F}_A\_TOP - S{F}_A\_BOTTOM) \\
  u_{bo} &= \frac{u_{dc}}{2} \times (S{F}_B\_TOP - S{F}_B\_BOTTOM); SF = \begin{cases} 
    1, & \text{on} \\
    0, & \text{off}
  \end{cases} \\
  u_{co} &= \frac{u_{dc}}{2} \times (S{F}_C\_TOP - S{F}_C\_BOTTOM)
\end{align*}
\]

(5)

In order to calculate the inverter phase voltage \((u_{an}, u_{bn}, u_{cn})\), voltages between neutral point of load and virtual neutral point of dc link \((u_{no})\) is calculated as:

\[ u_{no} = (u_{ao} + u_{bo} + u_{co}) / 3 \]

(6)

At last the phase voltages are obtained as:

\[ u_{am} = u_{ao} - u_{no}, u_{bm} = u_{bo} - u_{no}, u_{cn} = u_{co} - u_{no} \]

(7)

The relationship between dc current and voltage and motor current and voltage in the d-q frame is given by the following equations [8]:

\[
\begin{align*}
  i_{dc} &= 3(d_i d_q + d_i d_d) / 2 \\
  v_d &= d_d v_{dc} \\
  v_q &= d_q v_{dc}
\end{align*}
\]

(8)

- **Modeling of SVPWM Gate Drive System:**
  Gate drive system is used to send gate signals to inverter to realize the control of electric machines. For the gate drive system of PMSM electric drive system, a Space Vector Pulse Width Modulation (SVPWM), which is an efficient PWM method for gate driving system of PMSM is used for this paper. The detailed principle of SVPWM can be found in [9]. Figure 4 shows a schematic diagram of SVPWM. The inputs to SVPWM are \(d\) and \(q\) axis voltage command from former current controller of PMSM control system. The inverse park transform is used to change the \(d\) and \(q\) axis voltages \(U_d\) and \(U_q\) to three phase voltages \(U_a\), \(U_b\), and \(U_c\). The inverse Park Transform is shown in (9). After that, the three phase voltages are transferred into \(\alpha\) and \(\beta\) axis voltages \(U_{\alpha}\) and \(U_{\beta}\) through Clark Transform as shown in (10).

\[
\begin{bmatrix}
  U_a \\
  U_b \\
  U_c
\end{bmatrix} = \begin{bmatrix}
  \cos \theta & \sin \theta \\
  \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) \\
  \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3)
\end{bmatrix} \begin{bmatrix}
  U_d \\
  U_q
\end{bmatrix}
\]

(9)

\[
\begin{bmatrix}
  U_{\alpha} \\
  U_{\beta}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  1 & 1 & 1 \\
  -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
  -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0
\end{bmatrix} \begin{bmatrix}
  U_a \\
  U_b \\
  U_c
\end{bmatrix}
\]

(10)

![Figure 4. Schematic diagram of SVPWM](image)

The SVPWM gate drive signals are derived based on \(\alpha\) and \(\beta\) axis. Firstly the sector of desired stator voltage is determined using (11), which is used to calculate the sector angle and then sector number is decided based on sector angle [9]. After the sector is decided, the operating time of three basic voltage vectors is calculated. The turn on time of three switching devices \(T_{aon}, T_{bon}\), and \(T_{con}\) can be calculated according to (12). The turn-on time signals of three switching devices are SVPWM modulation signals. They are compared with SVPWM triangular carrier signals to generate SVPWM gate signals for three phase VSI. This is shown in figure 6. The SVPWM carrier signal magnitude is \(T_s / 2\). \(T_s\) is the period of the carrier which is \(5e^{-4}s\). So the switching frequency of the SVPWM is 2kHz.
\[ \theta_{\text{sector}} = \arctan \left( \frac{U_d}{U_q} \right) \]  
(11)

\[

t_{\text{con}} = \frac{T_{\text{bon}} + T_{k}}{2} \quad , k = \text{sector}\#
\]

(12)

- **Control of PMSM:**

  There have been various control methods for PMSM introduced in recent literatures ([1], [10-11]). Among these control methods, the following methods were used for this study: 1) Maximum Torque Per Ampere (MTPA) control for a low speed range operation 2) Flux Weakening (FW) control for a high speed range operation. Figure 7 shows a torque control strategy for PMSM employed from [10] where \(i_{ds} \) and \(i_{qs}\) are q and d axis current of PMSM, \(I_{s\text{max}}\) is maximum stator current of PMSM which is the current limit circle. Due to the limitation of maximum stator voltage of PMSM, there are also voltage limit ellipses for different motor speeds. \(w_{c1}\) and \(w_{c2}\) represent the base speed and the maximum speed of PMSM, respectively. When a motor speed is less than the base speed, \(i_{qs}\) and \(i_{ds}\) follows a MTPA trajectory to get maximum torque per unit stator current. When the motor speed becomes larger than the base speed, \(i_{qs}\) and \(i_{ds}\) moves along the current limit circle due to the flux weakening effect. Thus, in figure 7, OB is the MTPA control and BC is the FW control of PMSM. Figure 8 shows a schematic diagram of overall torque control.

The performance of a PMSM control system is evaluated through simulation by comparing its responses with published data [11]. Figure 9 shows motor responses with MTPA control for full load simulation. The motor speed well follows the reference speed (1500 rpm) and motor electromagnetic torque is controlled to the full load torque (40 Nm) at about 0.1s. And motor d, q axis and three phase currents are also controlled to steady-state values which are related to motor torque and parameters based on MTPA control. These responses are similar to the published results in [11].

- **Modeling of Battery Pack:**

  The battery is modeled with a non-linear empirical method from [15]. This model has the benefit of having a pre-determined open circuit voltage based on the current State of Charge (SOC) of the battery. For simulation, the GM Volt battery pack parameters [12] have been used with some parameters estimated.

**Figure 6.** SVPWM gate signal generating principle [9]

**Figure 7.** PMSM control strategy [10]

**Figure 8.** Schematic diagram of control of PMSM

**Figure 9.** PMSM control validation

### B. Hydraulic Brake System

For a friction brake system, a hydraulic dynamics in the brake system is included as a first order delay function:

\[ G_{bf}(s) = \frac{1}{0.06s + 1} \]  
(13)

**II. BRAKE PROPORTIONING AND BRAKE PEDAL SIMULATOR**

It is important for regenerative brake system to have the same brake performance compared to traditional friction only
brake system. Figure 10 shows a brief description of brake proportioning design of a conventional friction brake system. A. Brake Proportioning Design

The ideal brake proportioning curve satisfies the following equations [13]:

\[
F_f + F_r = \varphi G
\]

\[
F_f = b + \varphi h_g
\]

\[
F_r = a - \varphi h_g
\]  

(14)

By solving (14), the rear brake force becomes:

\[
F_r = \frac{1}{2} \left[ \frac{G}{h_g} \sqrt{h_g^2 + \frac{4h_gL}{G}F_f - \left( \frac{Gb}{h_g} + 2F_f \right)} \right]  
\]

(15)

The brake proportioning design that meets the requirements of Federal Motor Vehicle Safety Standards (FMVSS) No. 135 can be determined from the required longitudinal deceleration (for the stopping distance requirement) and front and rear wheel lockup curves. The cold effectiveness brake test and requirement is shown in Table 1. Figure 11 shows a design of brake force proportioning line.

<table>
<thead>
<tr>
<th>Test name</th>
<th>Test Speed (km/h)</th>
<th>Road Surface</th>
<th>Stopping Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold Effectiveness</td>
<td>100</td>
<td>0.9</td>
<td>( S \leq 70 )</td>
</tr>
</tbody>
</table>

B. Brake Pedal Simulator Design

Brake pedal feel is an important issue in brake system design of traditional vehicle and EVs/HEVs. Here a brake pedal simulator is designed based on curve fitting of testing data in [14], which has the testing data of the relationship between pedal travel and master cylinder pressure for a medium sized passenger car. After curve fitting of the testing data in [14], the relationship between normalized pedal travel and master and wheel cylinder pressure is shown in figure 12.

From this master cylinder pressure, the desired front and rear brake torques are determined using a brake proportioning block. Then these brake torques are sent to PMSM electric drive system to get the actual regenerative brake torque. If the desired brake torque on a front axle is larger than the maximum regenerative brake torque from PMSM, the rest of front brake torque is supplied by friction brake system. For the
rear axle, all brake torque on a rear axle is applied by the friction brake system. Actual brake torque supplied by both regenerative brake system and hydraulic brake system are sent to CarSim vehicle model. The vehicle responses including wheel speed are sent to PMSM as feedback signals. The solid line connections in figure 13 are mechanical signals and the dot line connections are electrical signals.

IV. SIMULATION RESULTS

In order to assess the effectiveness of the proposed regenerative brake system, a CarSim vehicle model incorporating regenerative brake system has been simulated.

As shown in figure 14, the brake performances of a friction only brake system and new regenerative brake system are very similar. When a normalized brake pedal input of 0.9 is applied, the average vehicle deceleration is about 0.8 g and total brake distance is about 60 m. This brake distance meets the requirement of cold effectiveness test in FMVSS 135, where the stopping distance should be no larger than 70 m. Figure 15 shows a brake pedal input applied to both friction only brake and regenerative brake systems.

Figure 16 shows PMSM torque-speed and torque-time responses during regenerative braking. As shown in figure 16, the actual motor torque well follows the reference motor torque. Since the motor speed during the braking is less than the base speed (2839 rpm), the reference motor torque is the maximum motor torque (370 Nm). The motor three-phase current is shown in figure 17. It can be seen that the three-phase motor current is sinusoidal and maximum motor current is limited to 295.4 A.

Figure 18 shows battery pack responses including current, SOC, and terminal voltage. At the beginning there is no braking input, thus the battery current is zero and battery terminal voltage and SOC are kept constant. When regenerative braking starts working, battery current becomes negative and battery is charging with SOC increasing. Since the motor speed decreases, the battery charging current also decreases. When the vehicle is stopped, the battery current becomes zero.

Figure 19 compares the transient responses between friction only brake system and regenerative brake system. As shown in figure 19, the applied total brake torque follows the reference total brake torque in both friction only brake system and regenerative brake system. However, the regenerative brake system has a faster response compared to friction only brake system. This is due to the faster response of PMSM than that of friction brake system. This explains why the regenerative brake system has a shorter stopping distance (59.9 m) than that (60.4 m) of friction only brake system in the bottom part of former figure 14.
In this paper, a comprehensive regenerative brake system model has been developed for a study of brake performance and energy recovery of EVs/HEVs. This model includes PMSM electric drive system, friction brake system, brake pedal simulator, and brake proportioning system. For a design of brake force proportioning between front and rear axles, the requirements of FMVSS 135 has been used. In addition, detailed PMSM electric drive system model containing PMSM, three-phase voltage source inverter (VSI), SVPWM gate drive system, PMSM control system, and battery pack system is developed. Then, the complete regenerative brake system model is connected with a commercial vehicle dynamics model (CarSim) to analyze the brake performance and brake energy recovery of proposed regenerative brake system model. Simulation results indicate that the developed regenerative brake system performs well compared to the friction only brake system. The overall vehicle model used in this study can be applicable for the study of safety of EVs/ and HEVs.

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REFERENCES


