Adaptive Speed Control under Vehicle and Road Uncertainties using Multiple Model Approach*

Feng Gao, Xi-Peng Li, Guang-Qiang Ming

Abstract—To deal with possible uncertainties of vehicle longitudinal dynamics especially in complex road condition, a multiple model switching control system for vehicle speed is designed in this paper. Vehicle dynamics from throttle to vehicle speed is linearized about equilibria and a model set consisting of five multiplicative uncertain models is set up to cover the possible range of vehicle dynamics. Based on this model set, the multiple model switching control system for vehicle speed is accomplished based on robust control theory by designing its estimator, switching performance index and logic, robust performance controller set. Simulation results show that this new controller has better adaptive performance than parameter adaptive method and thus better performances of tracking is achieved.

I. INTRODUCTION

Vehicle speed control is one of the important parts of ADAS (Advanced Driver Assistance System), such as ACC (Adaptive Cruise Control), collision avoidance system, ecologcal driving system, etc. [1] Today almost all vehicle speed control algorithms are designed based on vehicle longitudinal dynamics model. And obviously model uncertainties will affect the speed control performance greatly. As the application range of ADAS was extended to more complex road conditions, the degradation of speed control performance, which results from model uncertainties, has already got much attention.

For distance control, $H_\infty$ method was used to eliminate the effects of unmodeled high order dynamics in [2] and [3]. Unfortunately there is always no satisfying $H_\infty$ controller, when model uncertainties are large. To improve robustness of the speed controller, a dynamic sliding mode controller with wind coefficient estimation was designed in [4] and [5]. It should be noted that if the uncertainties or disturbances are large, there exist high frequency chattering and large hysteresis region when using sliding mode control. Actuators in vehicle can hardly satisfy the optimum switching requirement of sliding mode control. And furthermore, the frequently switching is bad for life of the control system. In [6]-[7], some vehicle and road parameters are estimated online and adaptive speed controllers are designed. Their control performance is barely satisfactory before finishing parameter estimation. Furthermore the online parameter identification method can hardly track sudden parameter changes. So in [8], road slope is predicted by vehicle longitudinal dynamics and a LPV (Linear Parameter Varying) method was used to design the robust speed controller. One shortcoming of LPV is that the controller only can vary with measurable parameters. While in practical, it is still a problem to measure all vehicle and road parameters by onboard sensors today, such as wind velocity, rolling resistance coefficient, etc.

To control vehicle speed precisely with possible uncertainties of vehicle dynamics and external disturbances, the MMSC (Multiple Model Switching Control) approach is used to design the vehicle speed tracking controller in this paper. Firstly, the vehicle dynamics from throttle to speed is linearized about equilibria by discretizing the continuous ranges of parameter. A model set, which consists of five multiplicative uncertain models, is used to cover all the possible range of vehicle dynamics. Then the robust control theory is used to design the MMSC for vehicle speed, including the estimator, switching performance index and logic, robust performance controller set. The robust controller set is solved by the LMI (Linear Matrix Inequality) approach. Simulations have been done and the results have been compared with the parameter adaptive control. It is shown that vehicle speed tracks the desired value precisely and it has better tracking performance than parameter adaptive control method.

II. VEHICLE DYNAMICS AND MODEL SET DESIGN

A. Vehicle Longitudinal Dynamics Model

To analyze vehicle dynamics and validate this new MMC for speed by simulation, a high order nonlinear vehicle model including engine, transmission, drivetrain and vehicle body is depicted in Fig. 1.

![Figure 1. Vehicle longitudinal dynamics model](image_url)

The engine is a mean value model consisting of four states and two time delays. The transmission is an automatic transmission with four forward gears. Its torque converter is

*Project No. CDJZR13150011 supported by the Fundamental Research Funds for the Central Universities.  
Feng Gao, Xi-Peng Li, Guang-Qiang Ming are with the Chongqing University, No. 174 Shazhengjie, Shapingba District, Chongqing, 400044, P. R. China (corresponding author is Feng Gao. Phone: 086-18996188196; e-mail: gaofeng1@ cqu.edu.cn ).
modeled by characteristics of capacity and torque ratio. Its gear is determined by both throttle and vehicle speed. This model has already been validated by experiments under different conditions and the detailed information can be found in [9]. The uncertainties considered in this paper are described in Tab. 1.

\[
\mathbf{P} = \{ P : G_i(s)[1+W_i(s)\Delta_{s}], i=1,\ldots,5 \},
\]

Where \( W_i(s) \) is the weighting function of model error and
\[
W_1(s) = \frac{0.0995}{s + 5}, \quad W_2(s) = \frac{0.0475}{s + 5}, \quad W_3(s) = \frac{0.0238}{s + 5},
\]
\[
W_4(s) = \frac{0.1488}{s + 5}, \quad W_5(s) = \frac{0.0585}{s + 5}.
\]

\( G_i(s) \) is the nominal model and \( G_i(s) = \frac{0.0398}{s + 0.0327} \),
\[
G_2(s) = \frac{0.0190}{s + 0.0437}, \quad G_3(s) = \frac{0.0095}{s + 0.0524}, \quad G_4(s) = \frac{0.0595}{s + 0.121},
\]
\[
G_5(s) = \frac{0.0234}{s + 0.118}.
\]

### TABLE I. VEHICLE AND ROAD PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Nominal Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Constant of Engine</td>
<td>0.2–0.4</td>
<td>0.3</td>
<td>s</td>
</tr>
<tr>
<td>Time Delay of Engine</td>
<td>0–0.1</td>
<td>0.05</td>
<td>s</td>
</tr>
<tr>
<td>Vehicle Mass</td>
<td>1100–1500</td>
<td>1300</td>
<td>Kg</td>
</tr>
<tr>
<td>Transmission Efficiency</td>
<td>0.8–0.99</td>
<td>0.895</td>
<td>-</td>
</tr>
<tr>
<td>Transmission Speed Ratio</td>
<td>2.71, 1.44, 1, 0.74</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Rolling Resistance Coefficient</td>
<td>0.01–0.02</td>
<td>0.015</td>
<td>-</td>
</tr>
<tr>
<td>Road Slope</td>
<td>-5–5</td>
<td>0</td>
<td>%</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>-8–8</td>
<td>0</td>
<td>m/s</td>
</tr>
</tbody>
</table>

B. Linearization and Model Set Design

In this section, the transfer function from throttle to vehicle speed is obtained by linearization about equilibria based on the vehicle model depicted in Fig.1.

Firstly, the parameters in Tab. 1 are discretized to many points. Then at each combination of possible values of the parameter, the steady state operating point and transfer function at this equilibrium point are derived by optimal process identification method [10]. An inverse pseudo random exciting signal and a fourth order linear model are used during identification. The identified steady state operating point and bounds of dynamic characteristic under different input and parameter values are shown in Fig. 2 and Fig. 3 respectively.

![Figure 2. Steady state operating point](image)

![Amplitude range of transfer function](image)

(a) Amplitude range of transfer function

![Phase range of transfer function](image)

(b) Phase range of transfer function

**Figure 3. Amplitude and phase bounds of transfer function**

The bold black line in Fig. 2 is the steady operating point at nominal values. Two facts can be found from Fig. 2 and Fig. 3. One is that the steady operating state is greatly affected by the vehicle and road parameters. The other is that the transfer characteristic also varies greatly with the steady operating point because of nonlinearities of vehicle longitudinal dynamics.

If only one linear model is used, there exists large model uncertainty. It is difficult to design a satisfactory controller by traditional robust control theory. Thus in this paper, multiple modes are used to describe vehicle dynamics. The model set \( \mathbf{P} \), which consists of five multiplicative uncertainty models, is

\[
\mathbf{P} = \{ P : G_i(s)[1+W_i(s)\Delta_{s}], i=1,\ldots,5 \},
\]

Where \( W_i(s) \) is the weighting function of model error and
\[
W_1(s) = \frac{0.0995}{s + 5}, \quad W_2(s) = \frac{0.0475}{s + 5}, \quad W_3(s) = \frac{0.0238}{s + 5},
\]
\[
W_4(s) = \frac{0.1488}{s + 5}, \quad W_5(s) = \frac{0.0585}{s + 5}.
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\( G_i(s) \) is the nominal model and \( G_i(s) = \frac{0.0398}{s + 0.0327} \),
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\]
\[
G_5(s) = \frac{0.0234}{s + 0.118}.
\]

898
$$\Delta_j \text{ is model uncertainty and satisfies}$$

$$\|\Delta_j\|_\infty^P < 1, \; i = 1, \ldots, 5, \quad (2)$$

Symbol $\|X\|_\infty^P$ represents $H_\infty^P$ norm, which is the induced system norm of $L^\infty$ norm. The definition of $L^\infty$ norm is [11]

$$\|x\|_\infty^P = \sqrt{\int_0^\infty e^{-\delta(t-r)} x^T(r)x(r) \, dr}, \quad (3)$$

Where $x$ is a time domain signal. $\delta$ is the exponentially weighting factor. It affects the attenuation speed of old data. Considering both the performances of adaptive ability and robustness to signal noise, $\delta$ is set to 0.4 in this paper.

III. MMSC FOR VEHICLE SPEED

Based on the model set $P$ derived in the previous part, the structure of the designed MMSC system for vehicle speed is shown in Fig. 4.

![MMSC system for vehicle speed](image)

In Fig. 4, $a$ is throttle angle, $v$ is vehicle speed, $(a_0, v_{des})$ is steady state working point under nominal parameter values. $\Delta v = v - v_{des}$ and $\Delta a = a - a_0$ represents the deviation from the steady state working point. The MMSC system includes a controller set and a supervisory controller. The controller set is designed by common Lyapunov approach and solved using LMI toolbox of Matlab. The supervisory controller evaluates model errors between vehicle and each model $P_i$. It consists of an estimator, switching performance index and switching logic. The estimator computers input signal $z_i$, which is the exciting signal of model uncertainties, and output signal $\Delta v_i$ of uncertainty part. $J_i$ is the switching performance index and it evaluates the $H_\infty^P$ norm of each uncertainty model $P_i$. The switching logic chooses the smallest switching performance index. The controller corresponding to the nearest model will be switched into closed control loop and $\sigma$ is the index of the selected controller. Next, detailed information of each part will be introduced.

A. Estimator

In robust control theory, system gain is used to evaluate model uncertainties. The input and output signal of the uncertainty part is estimated by

$$\Delta v_i = \frac{k_i}{\Lambda(s)} \Delta a + \frac{\Lambda(s) - N_i(s)}{\Lambda(s)} \Delta v,$$

$$z_i = -\frac{k_i}{\Lambda(s)} W_i(s) \Delta a, \; i = 1, \ldots, 5, \quad (4)$$

Where $\frac{k_i}{\Lambda_i(s)} = G_i(s)$ and $\Lambda(s)$ is the characteristic polynomial of the estimator. Considering the frequency bandwidth of vehicle, $\Lambda(s)$ is set to $s^{3/2}$. $\Delta v_i$ is the estimation of $\Delta v$ on model $G_i(s)$. If vehicle dynamics is described by $G_i(s)$, $\Delta v_i$ will converges to $\Delta v$ exponentially. Obviously the estimation error $e_i = \Delta v_i - \Delta v$ contains the information of model uncertainties.

Considering the following two reasons: (1) Solving five differential equations needs more computing resource. (2) When switching occurs, both state and parameter changes, which increases the difficulty of theoretical analysis. In practical use, state shared format of the estimator is used instead of equation (4). The state shared estimator (5) is derived from its controllable canonical form.

$$\dot{x}_E = A_E x_E + B_{E1} \Delta a + B_{E2} \Delta v,$$

$$\Delta v_i = C_{E1i} x_E, \; i = 1, \ldots, 5, \quad (5)$$

Where $A_E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix}$, $B_{E1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $B_{E2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $x_E$ is the state of the estimator. Matrix $C_{E1i}$ and $C_{E2i}$ satisfy

$$C_{E2i}(sI - A_E)^{-1} B_{E1} = -\frac{k_i}{\Lambda(s)} W_i(s),$$

$$C_{E1i}(sI - A_E)^{-1} B_{E1} = \frac{k_i}{\Lambda(s)},$$

$$C_{E3i}(sI - A_E)^{-1} B_{E2} = \frac{\Lambda(s) - N_i(s)}{\Lambda(s)}. \quad (6)$$

B. Switching Performance Index

From model set (1) and estimator (4), the output estimation error is

$$e_i = \Delta z_i, \; i = 1, \ldots, 5. \quad (7)$$
Then according to the definition of induced norm, the model error between vehicle and models can be evaluated by the following switching performance index.

\[ J_i(t) = \left( \| e_i(t) \|_1 \right)^2 - \left( \| e_i(t) \|_1 \right)^2, \quad i = 1, \ldots, 5 \tag{8} \]

**C. Switching Logic**

The switching performance index \( J_i(t) \) evaluates the model error between real vehicle dynamics and models. It is obvious that the controller designed based on the nearest model has the best control performance and should be switched into the closed control loop as long as stability is preserved. The designed switching logic is

\[ \sigma(t) = \arg \min_{i \in \{1, \ldots, 5\}} J_i(t) \tag{9} \]

In this chapter, the supervisory of MMSC for vehicle speed is designed in detail. In the next part, the controller set will be designed.

**IV. CONTROLLER SET DESIGN**

Before designing the controller set, the MMSC system should be converted into an equivalent switching system first. Then the common Lyapunov method and LMI method is used to solve the controller set.

**A. Equivalent Switching System**

From the state shared estimator (5), the input-output characteristic of a real vehicle can be equivalent to the following switching system.

\[ \dot{x}_E = (A_E + B_E C_{E1n})x_E + B_E \Delta \sigma - B_E e_\sigma, \]

\[ \Delta v = C_{E2n} x_E - e_\sigma, \]

\[ z_\sigma = C_{E2n} x_E, \quad \sigma \in \{1, 2, 3, 4, 5\}. \tag{10} \]

From section II, it is known that when parameter values deviate from nominal values, the steady state working points also vary. To reduce its influence on vehicle speed tracking performance, the selected disturbance attenuation weighting function \( W_{per}(s) \) is

\[ W_{per}(s) = \frac{0.1s + 1.1}{s}. \tag{11} \]

The final equivalent vehicle speed MMSC system is shown in Fig. 5.

![Equivalent vehicle speed MMSC system](image)

**B. **

**Signal d is the disturbance arising from the deviation of the steady state operating point. The switching system \( \Sigma_\sigma \) is described by (10). \( K_\sigma \) is the selected controller at current time. \( \tilde{\alpha} \) is the equivalent model uncertainty of switching system. In the next section, the upper limit of \( \mathbf{H}_{\infty}^d \) norm of \( \tilde{\alpha} \) will be studied.**

**C. Controller Set Design**

The controller set should satisfy the following requirements:

1. Since \( \| \tilde{\alpha} \|_1 < 1 \), the \( \mathbf{H}_{\infty}^d \) norm of the closed loop system, which is made up of \( \Sigma_\sigma \) and \( K_\sigma \), should be smaller than 1 under arbitrary switching. Small gain theorem guarantees the robust stability of the closed loop system.

2. \( \mathbf{H}_{\infty}^d \) norm from disturbance \( d \) to signal \( q \) should be smaller than 1. This requirement ensures the robust performance.

By using the common Lyapunov method, the problem of designing the controller set can be transferred to a LMI problem [12]. The LMIs are solved by LMI toolbox of Matlab and producing
\[ K_i(s) = 65146 \frac{(s + 0.115)(s + 2.45)}{s(s + 134)(s + 59.8)}, \]
\[ K_2(s) = 42266 \frac{(s + 0.119)(s + 2.17)}{s(s + 50.8)(s + 49.0)}, \]
\[ K_3(s) = 6010 \frac{(s + 0.0397)(s + 2.41)}{s(s + 56.5)(s + 31.2)}, \]
\[ K_4(s) = 24395 \frac{(s + 0.121)(s + 2.36)}{s(s + 95.7)(s + 47.1)}, \]
\[ K_5(s) = 81663 \frac{(s + 0.0502)(s + 3.12)}{s(s + 104)(s + 57.3)}. \]

(16)

If switching between above controllers directly, the control input will change suddenly when switching occurs. In practice, the following observable canonical form of multiple controllers is used to avoid sudden changes.

\[ K_i(s) = \begin{bmatrix} x_C = A_{ci} x_c + B_{ci} \Delta v, i = 1, \ldots, 5 \\ \Delta a = C_c x_c \end{bmatrix}. \]

(17)

Where \( C_c = [1 \ 0 \ 0], \) \( C_1(sI - A_{ci})^{-1} B_{ci} = K_i(s). \) When the controller switches, state \( x_c \) is continuous. Since different controllers have the same output matrix \( C_c, \) the control input is also continuous.

V. SIMULATION STUDIES

In the previous parts, vehicle speed MMSC has been designed in detail. In this part, simulations will be done to test performance. Furthermore, the control results of MMSC will be compared with that of a parameter adaptive speed controller, which is introduced in [6], to validate that MMSC can adapt to changes of vehicle dynamics more quickly than parameter adaptive control method.

The simulation conditions are listed in Tab. II. The resistance force is big in condition 1 and small in condition 2.

<table>
<thead>
<tr>
<th>Index</th>
<th>Wind Speed</th>
<th>Vehicle Mass</th>
<th>Rolling Resistance Coefficient</th>
<th>Road Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4 m/s</td>
<td>1500 Kg</td>
<td>0.02</td>
<td>Uphill, 2% after 70s</td>
</tr>
<tr>
<td>2</td>
<td>4 m/s</td>
<td>1100 Kg</td>
<td>0.01</td>
<td>Downhill, 1.5%</td>
</tr>
</tbody>
</table>

The profile of the desired vehicle speed is depicted in Fig. 6. It includes two acceleration processes and one deceleration process. The simulation results of condition 1 and 2 are shown in Fig. 7 and Fig. 8 respectively.

In condition 1, driving resistance is big. Larger control input is needed to track the desired speed. The parameter adaptive method only can adjust control parameters continuously. It can’t compensate for the increased resistance force in good time, which results in bad tracking performance, especially when meeting with an uphill slope of 2% after 70s. The maximum tracking error reaches 1.6 m/s, while that of
that arise from model uncertainties in good time. Under two simulation conditions the max tracking error is smaller than 0.2m/s.

From the above results, it can be found that MMSC improves control performance with uncertainties over the parameter adaptive control method, because the controller is adjusted by switching in MMSC. The MMSC approach allows more large uncertainties than traditional robust control, such as $H_\infty$ control, because multiple models are used.

Today most vehicle control systems are designed based on models. The complexity of vehicle systems, variable road and traffic conditions, and more strict requirements of vehicles require more robust and high performance controllers.

VI. CONCLUSION

To extend ADAS to more complex road condition, an adaptive vehicle speed control system is designed by using MMSC. This new controller can make the vehicle speed track the desired value precisely. It can compensate disturbances