An Experimental Study on Decentralised Backstepping Approaches for a Hydrostatic Drive Train with Unknown Disturbances

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Abstract—A comparison of two decentralised backstepping control strategies – with and without an integral part – is presented in this paper for the tracking control of a hydrostatic drive train, which is commercially used in working machines. An unknown leakage volume flow and a resulting load torque are taken into account as lumped disturbances. These disturbances and two unmeasurable state variables – the normalised swashplate angle and the normalised bent axis angle – are estimated by a nonlinear observer. Thereby, a high tracking accuracy can be achieved for the normalised bent axis angle and the angular velocity of the motor as controlled variables. The efficiency of the proposed controllers is demonstrated by both simulations and experiments.

I. INTRODUCTION

In general, a hydrostatic transmission system as depicted in Fig. 1 represents a hydraulic circuit that consists of a variable-displacement hydraulic pump and a hydraulic motor with either fixed or variable displacement. Compared with pure mechanical transmissions, hydrostatic transmissions provide a continuously variable transmission ratio with high power density and allow for reversing the direction of rotation without changing the gear. Moreover, it is possible to perform wearless braking manoeuvres, cf. [1] and [2]. Nowadays, hydrostatic transmissions represent a characteristic component of drive chains for the application in both working machines and off-road vehicles.

From a control point of view, a hydrostatic transmission is subject to several nonlinearities and characterised by uncertain system parameters as well as unknown disturbances. Even so, current industrial practice for controlling a hydrostatic transmission is still the use of gain-scheduled PID-controllers, see [3]. In order to improve both the energy efficiency and the control performance in mobile applications, a variety of model-based nonlinear control approaches have been proposed in the last two decades, see [4]–[14].

In [4], a flatness-based controller in combination with a nonlinear reduced-order disturbance observer is proposed. The simulation results and the experimental evaluation in [5] show a good tracking accuracy as well as active damping of pressure oscillations. The simplifying assumptions concerning the actuator dynamics and a constant leakage coefficient, however, restrict the applicability of this approach. In subsequent work [6]–[8], several advanced nonlinear approaches like adaptive inverse dynamics, robust inverse dynamics and sliding mode control have been investigated for the tracking control of the hydrostatic transmission system, in which the actuator time constants and leakage volume flow are considered as uncertain parameters or disturbance inputs. These centralised control approaches have been evaluated by simulations only. A decentralised flatness-based controller is presented in [9]. This innovative approach leads to a high tracking accuracy for both controlled outputs, and a singularity due to the vanishing pressure difference, cf. [4], can be avoided. Moreover, the implementation of the decentralised control structure is even simpler than the centralised version, cf. [4]–[8].

Based on the decentralised control structure from [9], nonlinear backstepping control strategies – with and without integral action – are proposed in this paper for the tracking control of the hydrostatic transmission. Aiming at a high tracking accuracy, a nonlinear state and disturbance observer is introduced that estimates both the unmeasurable system states – the normalised swashplate angle $\alpha_P$ and the normalised bent axis angle $\alpha_M$ – and the unknown disturbances like a leakage volume flow as well as disturbance torque acting on the hydraulic motor. This paper is organised as follows: In Section II, the model of the mechatronic drive system is briefly addressed. Based on the derived system model, the decentralised backstepping control approaches with and without integral action are proposed in Section III. In Section IV, a nonlinear state and disturbance observer is designed. The estimated normalised swashplate angle $\alpha_P$ and the normalised bent axis angle $\alpha_M$ as well as the leakage volume flow and the disturbance torque can then be used within the control structure. Using model-based trajectory planning, the limited displacements due to mechanically bounded tilt angles at both pump and motor are taken into account. In Section V, beneath simulation results taking into consideration both measurement noise and quantization error also experimental results from a dedicated test rig are presented. Finally, conclusions and a short outlook on further
work are given in Section VI.

II. MODELLING OF THE HYDROSTATIC TRANSMISSION

A scheme of the considered test rig is depicted in Fig 2. This mechatronic system can be split into a hydraulic subsystem and a mechanical subsystem, which are coupled by the torque generated by the hydraulic motor, cf. [15].

![Fig. 2. Structure of the dedicated test rig.](image)

A. Pressure dynamics

With the assumptions of a small swashplate angle $|\alpha_P| \leq 18^\circ$ of the hydraulic pump and a small tilt angle $|\alpha_M| \leq 20^\circ$ of the hydraulic motor, cf. [6], the pump and motor flow can be simplified as follows

$$q_P = \bar{V}_P \bar{\alpha}_P \omega_P, \quad (1)$$
$$q_M = \bar{V}_M \bar{\alpha}_M \omega_M. \quad (2)$$

Here, $\bar{V}_{P,M}$ are constant parameters resulting from the geometric structure of the hydraulic pump and motor. The variable $\bar{\alpha}_P \in (-1,1)$ is the normalised swashplate angle, whereas $\bar{\alpha}_M \in (\varepsilon_M, 1), \varepsilon_M > 0$ denotes the normalised bent axis angle. The angular velocity of the pump and motor is represented by $\omega_P$ and $\omega_M$, respectively.

Neglecting pressure losses in the hydraulic hoses and introducing a reasonable symmetry assumption, an order reduction can be achieved regarding the pressure dynamics. This results in a first order differential equation for the difference pressure $\Delta p$

$$\frac{d\Delta p}{dt} = \frac{2}{C_H} \left( \bar{V}_P \bar{\alpha}_P \omega_P - \bar{V}_M \bar{\alpha}_M \omega_M - \frac{q_U}{2} \right). \quad (3)$$

Here, the hydraulic capacitance is given by $C_H = \frac{V}{\beta}$, where $\beta$ denotes the effective bulk modulus of the fluid, and $V$ is the total volume (hydraulic hose and chamber) of one pressure side. The corresponding leakage volume flow $q_U$ results from internal and external leakage volume flows in the system.

B. Longitudinal dynamics

The longitudinal dynamics of the hydrostatic transmission system is governed by the equation of motion, see Fig. 2, that is given by

$$J_V \omega_M + d_V \omega_M = \bar{V}_M \Delta p \bar{\alpha}_M - \tau_U, \quad (4)$$

with $J_V = J_M + J_E$, where $J_M$ and $J_E$ are the mass moments of inertia of the hydraulic motor and the electric motor on the load side, respectively. The parameter $d_V$ is the damping coefficient at the drive shaft. An unknown disturbance torque acting on the hydraulic motor is denoted by $\tau_U$.

C. Actuator dynamics

The dynamics of the displacement units for both pump and motor are modelled by first-order lag systems, respectively. With $i \in \{P,M\}$, the differential equation for the corresponding normalised tilt angle becomes

$$T_{ai} \ddot{\alpha}_i + \dot{\alpha}_i = k_i u_i. \quad (5)$$

The actuator time constants are denoted by $T_{ai}$, $i \in \{P,M\}$, and the input voltages $u_i$ of the corresponding proportional valves for the displacement units act as physical control inputs. Furthermore, $k_i$ represents the proportional gains of the first-order lag systems. Saturation functions account for the limited outputs of the actuator models

$$\text{sat}_a(\dot{\alpha}_i) = \begin{cases} a & \dot{\alpha}_i \geq a \\ \dot{\alpha}_i & b < \dot{\alpha}_i < a \\ b & \dot{\alpha}_i \leq b \end{cases}, \quad (6)$$

where $a = \dot{\alpha}_{\text{max}}$ and $b = \dot{\alpha}_{\text{min}}$ represent the upper and lower output limits determined by the mechanical design: $\{\varepsilon_M, 1\}, \varepsilon_M > 0$, for the hydraulic motor and $\{-1, 1\}$ for the hydraulic pump. In the simulation model, Eq. (5) is implemented with limited integrators for $\dot{\alpha}_P$ and $\dot{\alpha}_M$, respectively.

The overall system model comprises four first-order differential equations. Introducing the normalised tilt angles $\bar{\alpha}_i, i \in \{P,M\}$, the difference pressure $\Delta p$, and the motor angular velocity $\omega_M$ as state variables, the state vector results in $\mathbf{x} = [\dot{\alpha}_P, \dot{\alpha}_M, \Delta p, \omega_M]$. The corresponding state-space representation becomes

$$\begin{bmatrix} \dot{\alpha}_P \\ \dot{\alpha}_M \\ \dot{\Delta p} \\ \dot{\omega}_M \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{ap}} \bar{\alpha}_P + \frac{k_P}{T_{ap}} u_P \\ -\frac{1}{T_{am}} \bar{\alpha}_M + \frac{k_M}{T_{am}} u_M \\ \frac{2\bar{V}_p \omega_P \text{sat}_a(\dot{\alpha}_P)}{C_H} - \frac{2\bar{V}_p \omega_P \text{sat}_a(\dot{\alpha}_M)}{C_H} - \frac{q_U}{C_H} \\ \frac{d_V}{C_H} \dot{\omega}_M + \frac{\bar{V}_M}{C_H} \Delta p \text{sat}_a(\dot{\alpha}_M) - \frac{q_U}{C_H} \end{bmatrix}. \quad (7)$$

III. DECENTRALISED NONLINEAR BACKSTEPPING CONTROL

The differential flatness property of the hydrostatic drive train model according to (7) has been discussed in [4] and [9]. As by proper planning of desired trajectories – taking advantage of the flatness property – a saturation of the actuators can be avoided, these saturation effects are not considered explicitly in following.

A. Control of the normalised bent axis angle

The control loop consists of a flatness-based control of the normalised bent axis angle $\dot{\alpha}_M$ of the motor, cf. [9]. For this purpose, the corresponding first-order differential equation is solved for the control input $u_M$, and the first time derivative $\dot{\alpha}_M = \dot{v}_M$ is introduced as stabilising control input

$$u_M = \ddot{\alpha}_M + \frac{\tau_{TM}}{k_M}. \quad (8)$$

The stabilising control law involves a combination of a feedforward and feedback control as follows

$$\dot{v}_M = \ddot{\alpha}_M + k_{a0} \cdot (\dot{\alpha}_M - \bar{\alpha}_M) + k_{aI} \cdot \int_0^t (\dot{\alpha}_M - \bar{\alpha}_M) d\tau, \quad (9)$$
with positive coefficients, i.e., \( k_{\alpha 0} > 0 \) and \( k_{\alpha I} > 0 \). Desired trajectories for the normalised bent axis angle \( \tilde{\alpha}_{Md} \) are specified only within the admissible displacement range \( \{ e_M, 1 \} \).

### B. Backstepping control (BS) of the motor angular velocity

As shown in [9], the motor angular velocity represents a flat output for the system if the normalised bent axis angle \( \tilde{\alpha}_M \) is considered as a gain-scheduling parameter with negligible time derivative. Starting with the flat output candidate \( \tilde{\omega}_M \), the first two time derivatives become

\[
y_f = \omega_M, \tag{10}
\]

\[
y_f = \dot{\omega}_M = -\frac{d_v}{J_v} \omega_M + \frac{\dot{V}_M}{J_v} \Delta p \tilde{\alpha}_M - \frac{\tau_v}{J_v}, \tag{11}
\]

\[
y_f = \ddot{\omega}_M = -2 \frac{\dot{V}_M^2 \alpha_{Mx}}{J_v C_H} \omega_M + \frac{d_v^2}{J_v} \omega_M + \frac{d_v \tau_v}{J_v} + \frac{2 \dot{V}_M \tilde{\omega}_M \omega_M}{J_v C_H} + \frac{d_v \dot{V}_M \Delta p}{J_v^2} \tilde{\alpha}_M. \tag{12}
\]

In the equations above, the disturbance model \( \tau_v = 0 \) has been considered. This leads directly to the state parametrisation

\[
\Psi_x = \begin{bmatrix}
\alpha_p(\omega_M, \omega_M, \tilde{\alpha}_M, \tilde{\omega}_M, q_U) \\
\Delta p(\omega_M, \omega_M, \tilde{\alpha}_M) \\
\tilde{\omega}_M
\end{bmatrix}
\tag{13}
\]

\[
= \begin{bmatrix}
\frac{d_v}{J_v} \omega_M + \frac{\dot{V}_M}{J_v} \alpha_{Mx} + \frac{\tau_v}{J_v} \\
2 \frac{\dot{V}_M \tilde{\omega}_M \omega_M}{J_v C_H} + \frac{d_v \dot{V}_M \tilde{\alpha}_M}{J_v^2} \\
\frac{\dot{V}_M}{J_v} \tilde{\omega}_M \omega_M + \frac{d_v \dot{V}_M \Delta p}{J_v} \tilde{\alpha}_M
\end{bmatrix}
\tag{13}
\]

A third time differentiation results in

\[
y_f = \dddot{\omega}_M = f(\omega_M, \Delta p, \tilde{\alpha}_M, \tilde{\omega}_M, \omega_M, q_U) + g(\tilde{\alpha}_M) u_p. \tag{14}
\]

Here, \( f(\omega_M, \Delta p, \tilde{\alpha}_M, \tilde{\omega}_M, \omega_M, q_U) \) and \( g(\tilde{\alpha}_M) \) represent state- and disturbance-dependent nonlinear functions. Again, the disturbance models \( \tau_v = 0 \) and \( q_U = 0 \) have been used.

Eq. (14) can be solved for the physical control input \( u_p \), representing the corresponding inverse dynamics.

The backstepping control design starts with a quadratic Lyapunov function defined with the tracking error \( e_1 = \omega_{Md} - \omega_M \) concerning the motor angular velocity

\[
V_1 = \frac{1}{2} e_1^2, \tag{15}
\]

where \( \omega_{Md} \) denotes the desired trajectory of the motor angular velocity. The first time derivative of the positive function results in

\[
\dot{V}_1 = e_1 \dot{e}_1 = e_1 \left( \omega_{Md} - \omega_M \right). \tag{16}
\]

Here, the virtual control input is given by \( \alpha_1 \approx \omega_M \). Introducing the constant parameters \( c_1 > 0 \) and \( c_2 > 0 \), \( \alpha_1 \) is chosen as

\[
\alpha_1 = \omega_{Md} + c_1 e_1 + c_2 e_1^3. \tag{17}
\]

In the second design step, an error variable \( e_2 \) is introduced according to

\[
e_2 = \alpha_1 - \omega_M = \omega_{Md} + c_1 e_1 + c_2 e_1^3. \tag{18}
\]

The corresponding error dynamics can be expressed as

\[
\dot{e}_2 = \omega_{Md} - \omega_M + c_1 \dot{e}_1 + 3 c_2 e_1^2 \dot{e}_1. \tag{19}
\]

Now, the virtual input \( \alpha_2 \approx \omega_M \) is chosen, which is identical in the ideal case to the second time derivative of the motor angular velocity \( \dot{\omega}_M \).

Introducing a Lyapunov function

\[
V_2 = V_1 + \frac{1}{2} \dot{e}_2^2, \tag{20}
\]

its first derivative with respect to time results in

\[
\dot{V}_2 = e_1 \dot{e}_1 + e_2 \left( \dot{\omega}_M - \omega_M + c_1 \dot{e}_1 + 3 c_2 e_1^2 \dot{e}_1 \right). \tag{21}
\]

According to (18), the following relationship is obtained

\[
\dot{e}_1 = e_2 - c_1 e_1 - c_2 e_1^3. \tag{22}
\]

Substituting (22) into (21) results in

\[
\dot{V}_2 = e_1 (e_1 - c_1 e_1 - c_2 e_1^3) + e_2 \left( \dot{\omega}_M - \omega_M + c_1 \dot{e}_1 + 3 c_2 e_1^2 \dot{e}_1 + e_1 \right). \tag{23}
\]

\[
\dot{V}_2 = e_1 (e_1 - c_1 e_1 - c_2 e_1^3) + \dot{\omega}_M - \frac{c_2 e_1^2}{1 - c_2 e_1^3} \frac{\dot{\omega}_M}{\dot{e}_2} \tag{24}
\]

Similar to the first design step, \( c_3 \) and \( c_4 \) represent positive constant parameters, and \( \alpha_2 \) is chosen as

\[
\alpha_2 = c_3 e_2 + c_4 e_2^2 + \omega_{Md} + e_1 + c_1 e_1 + 3 c_2 e_1^2 e_1. \tag{25}
\]

Then, a third error variable is defined and the corresponding error dynamics is derived as follows

\[
e_3 = \alpha_2 - \omega_M \tag{26}
\]

\[
= c_3 e_2 + c_4 e_2^2 + e_1 + \left( \omega_{Md} - \omega_M + c_1 e_1 + 3 c_2 e_1^2 \dot{e}_1 \right), \tag{27}
\]

\[
\dot{e}_3 = \dot{\omega}_M - \left( f(\omega_M, \Delta p, \tilde{\alpha}_M, \tilde{\omega}_M, \omega_M, q_U) + g(\tilde{\alpha}_M) u_p \right) \tag{28}
\]

\[
+ \dot{e}_1 + c_1 \dot{e}_1 + 3 c_2 e_1^2 \dot{e}_1 + 3 c_4 e_2^2 \dot{e}_2. \tag{29}
\]

The overall Lyapunov function that is used to stabilise the complete error dynamics is chosen as

\[
V_3 = V_1 + V_2 + \frac{1}{2} e_3^2. \tag{30}
\]

A time differentiation of (27) leads to

\[
\dot{V}_3 = e_1 \dot{e}_1 + e_2 \dot{e}_2 \tag{31}
\]

\[
+ e_3 \left( \dot{\omega}_M - \left( f(\omega_M, \Delta p, \tilde{\alpha}_M, \tilde{\omega}_M, \omega_M, q_U) + g(\tilde{\alpha}_M) u_p \right) \right) \tag{32}
\]

\[
+ \dot{e}_1 + c_1 \dot{e}_1 + 3 c_2 e_1^2 \dot{e}_1 + 6 c_2 e_1^2 \dot{e}_1^2 + c_3 \dot{e}_2 + 3 c_4 e_2^2 \dot{e}_2. \tag{33}
\]

According to (25), \( \dot{e}_2 \) can be represented by

\[
\dot{e}_2 = e_3 - e_1 - c_3 e_2 - c_4 e_2^3. \tag{34}
\]
Substituting (22) and (29) into (28) results in
\[
\dot{V}_3 = \dot{V}_1 + \dot{V}_2 + e_3 \{ \bar{\omega}_{Md} - f - g u_P + f_e + e_2 \} \leq 0
\]
Based on (30), the feedback control law becomes
\[
u_P = g(\bar{\omega}_M)^{-1} \cdot (c_5 e_3 + c_6 e_3^2 - f + f_e + e_2 + \bar{\omega}_{Md})
\]
C. Backstepping control with integral action (BSwI)

To further improve the tracking performance of the closed-loop system, an integral action is introduced in the first design step of the backstepping control as follows

\[
\xi = \int_0^t e_1 \, dt.
\]
By choosing the Lyapunov function
\[
V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} c_2 \xi^2,
\]
its first time derivative becomes
\[
\dot{V}_1 = e_1 (\dot{e}_1 + c_2 \xi) = e_1 \left( \bar{\omega}_{Md} - \bar{\omega}_M + c_2 \xi \right). \tag{34}
\]
The virtual input is chosen as
\[
\alpha_1 = \bar{\omega}_{Md} + c_1 e_1 + c_2 e_3^2 + c_3 \xi \quad \tag{35}
\]
Then, the same procedure as described in Subsection III.B can be applied to derive the feedback control law with integral action $u_P$.

IV. Nonlinear State and Disturbance Observer

For the implementation of the proposed control approaches a nonlinear state and disturbance observer, cf. [16], is introduced in this section. It estimates both unmeasurable system states – the normalised swashplate angle $\bar{\alpha}_p$ and the normalised bent axis angle $\bar{\omega}_M$ – and unknown system disturbances – the leakage volume flow $q_U$ and a disturbance torque $\tau_U$ acting on the hydraulic motor. Therefore, the state equations (7) are extended with four integrators as state and disturbance models
\[
\dot{y}_m = f(y_m, \tau_d, x_e), \quad \tau_d = 0,
\]
where $y_m = [\Delta p, \omega_M]^T$ denotes the measurable difference pressure and motor angular velocity. Moreover, $x_e = [\bar{\alpha}_p, \bar{\omega}_M]^T$ is the vector of the normalised actuator angles and $\tau_d = [q_U, \tau_U]^T$ represents the vector of system disturbances. The estimated states $x_e$ and the disturbances $\tau_d$ follow from
\[
\begin{bmatrix}
x_e \\
\tau_d
\end{bmatrix} =
\begin{bmatrix}
H_1 & H_2 \\
H_3 & H_4
\end{bmatrix}
\begin{bmatrix}
y_m \\
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} \tag{37}
\]
where $H$ represents the observer gain matrix. The state equations for the observer state vector $z$ are chosen as
\[
\dot{z} = \Phi(y_m, \tau, u_P, u_M)
\]
The observer gain matrix with four unknowns
\[
H = \begin{bmatrix}
11 & 0 & 0 \\
0 & 11 & 0 \\
0 & 0 & 11
\end{bmatrix}
\]
and the vector of nonlinear functions $\Phi$ are determined in such a way that the steady-state observer error $\dot{\xi} = \tau - \dot{\tau}$ converges to zero. Thus, $\Phi$ results from the demand for a vanishing steady-state estimation error according to
\[
\dot{\xi} = 0 = \dot{\tau} - H \cdot \dot{y}_m - \Phi(y_m, \tau, u_P, u_M).
\]
Considering $\tau_d = 0$ and using (5) for $x_e$, (40) yields
\[
\Phi(y_m, \tau, u_P, u_M) =
-\frac{\bar{\tau}_p + k_p u_P}{H_3}
+ \begin{bmatrix}
\frac{-2 q_p}{c_{in}} + \frac{2 q_m}{c_{in}} + \frac{q_r}{c_{in}} \\
-\frac{d_Y q_m}{c_{in} Y} + \frac{q_m}{c_{in} Y} - \frac{5 q_r}{c_{in} Y}
\end{bmatrix}
\]
The linearised error dynamics $\dot{\xi}$ has to be asymptotically stable. Therefore, all eigenvalues of the Jacobian are placed in the left complex half-plane according to
\[
\det (s I - \partial \Phi(y_m, \tau, u_P, u_M)/\partial \tau) = \prod_{i=1}^{4} (s + s_B i).
\]
With positive values $s_B i > 0$, $i \in \{1, 2, 3, 4\}$, the observer gains follow directly from Eq. (42).

V. SIMULATION AND EXPERIMENTAL RESULTS

In this section, both tracking performance and steady-state accuracy w.r.t. the normalised bent axis angle $\bar{\alpha}_M$ and the motor angular velocity $\omega_M$ are investigated by simulation and experimental studies. The synchronised desired trajectories comprise a sequence of motions, which are shown in the upper parts of Fig. 6 and Fig. 7, respectively.

A. Evaluation of the observer by simulation

Fig. 3 depicts the simulation results of the unmeasurable system states $\bar{\alpha}_p$ and $\bar{\omega}_M$ estimated by the combined state and disturbance observer. It can be seen that the estimated state variables match the simulated ones with high accuracy. As a consequence, they can be directly employed in the
control law. The same holds for both system disturbances (Fig. 4): the leakage volume flow $q_U$, which is assumed to be proportional to the difference pressure, and the disturbance torque $\tau_U$, which is modelled by

$$
\tau_U = 0.1 J_M \dot{\omega}_M + 5 \tanh \left( \frac{\omega_M d}{0.1} \right).
$$

The tracking errors between the simulated and desired values can be found in Fig. 5. It becomes obvious that the desired trajectories are tracked with negligible errors for both controlled variables. The tracking performance is further improved by introducing an integral action in the backstepping control.

![Graph showing comparison of simulated and estimated values for the leakage volume flow $q_U$ and the disturbance torque $\tau_U$.](image1)

![Graph showing tracking errors of the normalised bent axis angle $\tilde{\alpha}_M$ and motor angular velocity $\omega_M$ as well as their tracking errors in the case of a vanishing disturbance.](image2)

### B. Experimental validation of the control approach

The proposed nonlinear backstepping control in combination with the state and disturbance observer has been implemented on the test-rig depicted in Fig. 1 at the Chair of Mechatronics, University of Rostock. It consists of two electric motors with current converters: one replaces a diesel engine to drive the hydraulic pump (right hand side in Fig. 2), and the other one serves as a load torque generator, providing a specified torque acting on the hydraulic motor (left hand side in Fig. 2). The two main hydraulic components are connected in a closed-circuit configuration with volume flow sensors at both pressure sides. Two pressure sensors are mounted on the top of hydraulic pump measuring the pressure in both the low pressure and the high pressure hose. Fig. 6 and Fig. 7 indicate the tracking performance of the bent axis angle $\tilde{\alpha}_M$ and motor angular velocity $\omega_M$ as well as their tracking errors in the case of a vanishing disturbance.

![Graph showing tracking performance w.r.t. the normalised bent axis angle $\tilde{\alpha}_M$ with a vanishing disturbance torque: experimental results for backstepping control and backstepping control with integral action.](image3)

![Graph showing tracking performance w.r.t. the motor angular velocity $\omega_M$ with a vanishing disturbance torque: experimental results for backstepping control and backstepping control with integral action.](image4)
torque. It can be concluded that the proposed decentralised nonlinear control strategy guarantees an excellent tracking performance for both controlled outputs. Furthermore, the comparison depicted in Fig. 7 points out that the implemented control involving an integral action of the tracking error of \( \dot{\theta}_M \) leads to a superior tracking accuracy. As Fig. 8 indicates, similar results can be achieved when a disturbance torque of 30 Nm is applied to the hydraulic motor. In Fig. 9, the control inputs for both controllers are shown.

![Graph showing control inputs for both controllers](image)

Fig. 8. Tracking errors w.r.t. the normalised bent axis angle \( e_\alpha \) and the motor angular velocity \( e_\omega_M \) with a load torque of 30 Nm: experimental results for backstepping control and backstepping control with integral action.

![Graph showing tracking errors](image)

Fig. 9. Control input \( u_\tau \) for the motor angular velocity \( \dot{\theta}_M \) with a load torque of 30 Nm: experimental results for backstepping control and backstepping control with integral action.

VI. CONCLUSIONS

In this paper, a model-based decentralised trajectory control for a hydrostatic drive train is presented. It consists of a flatness-based control of the normalised bent axis angle \( \dot{\alpha}_M \) and two alternative backstepping techniques – with and without an integral part – used for the control of the motor angular velocity \( \dot{\theta}_M \). Due to the presence of the unmeasurable states and unknown system disturbances, a nonlinear state and disturbance observer is proposed. First, the decentralised control structure in combination with the reduced-order observer is evaluated by simulations, which show a high accuracy regarding both the tracking performance and the combined state and disturbance estimation. In an experimental validation, the implemented control strategy leads to an excellent closed-loop performance. To further enhance the control performance, an integrator is introduced in the backstepping approach. A comparison with the case without an integrator shows a significant reduction of the tracking error w.r.t. the motor angular velocity. In the near future, more sophisticated actuator models will be introduced, e.g. with dead zone and time delay, and other nonlinear control strategies will be investigated, e.g. extended linearisation and nonlinear model predictive control.

REFERENCES