Iterative Learning Control with Restricted Input Subspace for Electrode Array-based FES

C. T. Freeman

Abstract—In this paper techniques to identify and apply a restricted input subspace within the iterative learning control framework are developed. This is motivated by the increasing popularity of electrode arrays within rehabilitation and assistive technology communities, which allow functional electrical stimulation (FES) to be applied independently to each array element. This enables more selective muscle activation and improved control of human motion, but increases the input space dimension significantly so that model identification becomes impractical. The approach in this paper embeds past experience and/or structural knowledge in the subspace selection, and derives iterative learning controllers with favorable properties that are independent of the input basis employed. Experimental results using a 40 element surface electrode array confirm accurate tracking of three reference hand postures.

I. INTRODUCTION

Functional electrical stimulation (FES) is a technology used to improve motor control following the paralyzing effects of head trauma, spinal cord injury, multiple sclerosis, stroke and other neurological disorders. The recent emergence of surface electrode array technology has found to increase muscular selectivity and reduce fatigue compared with single electrode pads [1] and is critical in enabling, for example, precise control of hand movement.

Existing control methods embed rule-based selection of suitable sites to produce a desired gesture [2]. Array elements are stimulated sequentially to locate the best single site for drop foot in [3], and simple rule-based array element selection is employed in [4], [5], [6] for finger flexion and extension, wrist stabilization, and hand extension respectively. All such approaches are slow and imprecise since they do not exploit an underlying dynamic model linking FES and resulting motion. The most accurate methods use the principle of superposition: in [7] array electrodes are selected to minimize a cost function based on joint angle data produced during individual activation; and in [8] the same form of data is used to train an artificial neural network. Unfortunately these still suffer from the previous limitations, and their lack of an explicit plant model means that there are no mechanisms to learn from past experience, exploit the physical structures which cause intratrial variation, reduce the input search space in a principled manner, or provide performance and robustness guarantees.

This paper develops the first model-based control approach to finding the optimal stimulation sites in order to address these deficiencies. It utilizes iterative learning control (ILC), a control technique that was developed for industrial systems that are required to repetitively complete the same predefined movement. Over repeated performances, ILC uses previous trial data together with a model of the system to sequentially update the control input signal, reducing error until perfect tracking is achieved. Recent work has successfully used ILC to control FES applied to stroke patients’ arm muscles whilst performing both planar [9] and 3D [10] reaching and grasping tasks in clinical trials [11]. This paper develops a method for combining ILC with a model identification procedure, enabling accurate model-based control without requiring an a priori global dynamic model, which is unavailable in practice. To speed up the time-consuming identification procedure it is assumed that the underlying muscle locations do not change, but that the dynamics of the musculoskeletal system do. Accordingly, the framework embeds a restricted input subspace which specifies which subset of array elements may be stimulated based on physiological data, assumed structure, and past experience. Having reduced the order of the identification problem, optimal subspace selection is addressed, as are the convergence properties of the resulting ILC algorithms.

Section II introduces ILC with restricted input space, and Section III addresses optimal input subspace selection with special relevance to electrode arrays. Section IV expands utility to nonlinear dynamics, using local identification approaches described in Section V. Experimental results in Section VI confirm efficacy, with conclusions in Section VII.

II. ILC WITH RESTRICTED INPUT SUBSPACE

Consider the standard discrete ILC problem in which there are $m$ inputs and $p$ outputs. At time $t$ these are denoted $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ respectively, and the objective is for the latter to track a reference $r(t) \in \mathbb{R}^p$ over the time instants $t = 0, 1, \ldots, N - 1$. Here $N$ is the number of samples in each trial. The system is assumed to undertake the same tracking task over repeated trials, denoted $k \in \mathbb{N}$, with the system reset to identical initial conditions between trials. In the current application $u(t)$ corresponds to the electrical stimulation applied to the $m$ elements of the electrode array at time instant $t$, and $y(t)$ contains the $p$ joint angles of the biomechanical system.

On trial $k$ these signals are arranged in the supervectors

$$u_k = [u_k(0)^T, u_k(1)^T, \ldots, u_k(N - 1)^T]^T \in \mathbb{R}^{mN}$$

$$y_k = [y_k(r_d)^T, y_k(r_d + 1)^T, \ldots, y_k(N - 1 + r_d)^T]^T \in \mathbb{R}^{pN}$$

$$r = [r(r_d)^T, r(r_d + 1)^T, \ldots, r(N - 1 + r_d)^T]^T \in \mathbb{R}^{pN}$$

C. T. Freeman is with Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ, UK. cf@ecs.soton.ac.uk
with \( r_d \geq 1 \) the relative degree of state-space model
\[
x_k(t+1) = Ax_k(t) + Bu_k(t)
y_k(t) = Cx_k(t), \quad x_k(0) = 0, \quad t = 0, \ldots, N - 1
\] (1)
The ‘lifted’ plant is represented by \( y_k = Gu_k \) with \( G := \) (2)
\[
\begin{bmatrix}
CA^{r_d-1}B & 0 & \cdots & 0 \\
CA^{r_d}B & CA^{r_d-1}B & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
CA^{r_d+N-2}B & CA^{r_d+N-3}B & \cdots & CA^{r_d-1}B
\end{bmatrix} \in \mathbb{R}^{pN \times mN}
\]
Let the input space now be restricted such that \( u(t) \in \tilde{S} \) where the set \( \tilde{S} \subset \mathbb{R}^m \). Let this subspace have dimension \( q < m \) and be defined as
\[
\tilde{S} := \{ z \in \mathbb{R}^m : z = \tilde{W} x, \quad x \in \mathbb{R}^q \}
\] (3)
where \( \tilde{W} \) is a \( m \times q \) matrix with full column rank. The column space of \( \tilde{W} \) thus defines the input subspace. The lifted input subspace has dimension \( qN \), and is given by
\[
S := \{ z \in \mathbb{R}^{mn} : z = W x, \quad x \in \mathbb{R}^{qN} \}
\] (4)
where \( W = \text{diag}\{\tilde{W}, \cdots, \tilde{W}\} \). The ILC problem is then
\[
\lim_{k \to \infty} ||u_d - u_k||^2 = 0, \quad u_d = \arg \min_u J_1(u),
J_1(u) = \|r - Gu||^2, \quad u, u_k \in S \quad \forall k
\] (5)
This can be solved through reformulation as an unconstrained ILC problem, using the lower dimension input sequence \( \{v_k\}_{k=0,1,\cdots,\infty} \) such that
\[
\lim_{k \to \infty} \|v_d - v_k\|^2 = 0, \quad v_d = \arg \min_v J_2(v),
J_2(v) = \|r - GWv\|^2
\] (6)
where \( v_d \) is a fixed input signal and \( v_d, v_k \in \mathbb{R}^{qN} \). The input which is applied to the plant \( G \) on trial \( k \) is \( u_k = Wv_k \), and the tracking error is given by \( e_k = r - Gv_k \).

To solve (6) in practice, any globally convergent ILC algorithm may be used. Assuming \( GW \) does not have full row rank, such an algorithm guarantees convergence to the unique minimizer \( v_\infty = v_d = (GW)^\dagger r \) where \( (GW)^\dagger = ((GW)^TGW)^{-1}(GW)^T \). The corresponding non-zero limiting error is
\[
e_\infty = (I - GW(GW)^\dagger) r
\] (7)
If on the other hand \( GW \) has full row rank then the solution to (6) corresponds to \( e_\infty = 0 \) and is not unique if \( p < q \). In this case it is sensible to instead replace (6) by
\[
\lim_{k \to \infty} \|v_d - v_k\|^2 = 0, \quad v_d = \arg \min_v \{ J_3(v), \quad r = GWv \},
J_3(v) = \|v\|_R^2
\] (8)
whose unique minimizer \( v_\infty = (GW)^\dagger r \) is the minimum energy solution to (6) with respect to auxiliary variable \( v \).

Lemma 1: Let \( GW \) have full row rank (implying \( q, m \geq p \)). If a weight is added to the norm definition of the input subspace in (8) such that \( \|v\|_R = v^T R v \) for symmetric positive definite \( R \in \mathbb{R}^{N \times N} \), then \( (GW)^T \) is replaced by \( R^{-1}(GW)^T \) in the solution \( v_\infty = (GW)^\dagger r \). A special case of this is the choice \( R = W^T W \), in which case the unique minimizer of (8) is also the unique minimizer of
\[
\lim_{k \to \infty} \|u_d - u_k\|^2 = 0, \quad u_d = \arg \min_u \{ J_3(u), \quad r = Gu \},
J_3(u) = \|u\|_R^2, \quad u, u_k \in S \quad \forall k
\] (9)
This hence is the minimum input energy solution to (6) with respect to the input \( u \) (i.e., the input applied to the plant \( G \)).

Theorem 1: Consider problem (6) where \( W \) is replaced by \( W_{WR} \) with \( W_{WR} \in \mathbb{R}^{q \times qN} \) any full rank matrix. Then:
1. If \( GW \) does not have full row rank, the limiting solutions \( e_\infty \) and \( u_\infty \) resulting from solving problem (6), or equivalently (5), are invariant of \( W_{WR} \).
2. If \( GW \) has full row rank, \( e_\infty = 0 \), and (8) yields a unique minimizer \( u_\infty \) that depends on the choice of \( W_{WR} \). However application of the input subspace weight \( R = W_{WR} W \) in Lemma 1 means that the new solution \( u_\infty \) solves (9) for all \( W_{WR} \), and is hence invariant of \( W_{WR} \).
3. For all \( GW \), any ILC algorithm that minimizes (6) (while converging to the minimum control effort solution of (8) when the minimizer is not unique), using a quadratic cost in which the input space is weighted by \( R = W_{WR} W \), gives rise to the same input sequence \( \{u_k\} \) if \( u_0 = 0 \). Examples of such updates are [12], [13], [14]. Theorem 1 shows that the application of ILC to the problem (6) (or equivalently (5)) does not depend on the choice of underlying basis for the input subspace, as long as the well-motivated norm weighting \( R = W_{WR} W \) is applied.

III. OPTIMAL SUBSPACE SPECIFICATION
The musculo-skeletal system is coupled, time-varying and highly dependent on array location, motivating use of identification procedures that do not assume an underlying structure. The high dimensionality makes lengthy tests impractical, but an input subspace may be chosen to cover those muscles needed to perform a required set of tasks, together with possible variation in array placement. This restricted input subspace reduces the dimension of the model identification task, and can be constructed as follows.

A. Previous Experimental Data
Assume that previous experiments have yielded input and output pairs close to the required reference(s), denoted as \( (u_i, y_i), i = 1, \cdots, c \). Setting \( W = [u_1, u_2, \ldots, u_c] \) means that any reference in the set spanned by a linear combination of \( y_i \) (the column space of \( GW \)), will be tracked with zero error using the ILC algorithms of Section II. If \( r \) does not belong to this set, then the error is the orthogonal projection of \( r \) onto the nullspace of \( GW \). However, \( qN \) cannot be independently prescribed (since \( qN = c \)).

B. Muscle Locations
Knowledge of the underlying physiological can be directly employed by selecting \( q \) muscles that are assumed to generate the required movements, and specifying a stimulation pattern for each one. Then assign elements of \( u_i \) equal to the stimulation levels required to activate the \( i^{th} \) muscle, and insert in \( W = [u_1, u_2, \ldots, u_q] \in \mathbb{R}^{m \times q} \) which then provides \( W = \text{diag}\{\tilde{W}, \cdots, \tilde{W}\} \).
C. Array Positional Variation

The subspace must be expanded to capture possible day-to-day variation in where the electrode array is placed on the patient. This is done by applying suitable translational and rotational transformations to the columns of $W$ and inserting them as additional columns. See [15] for explicit forms.

D. Dimension Optimization

Combining approaches of Sections III-A, III-B and III-C produces a suitable subspace $W \in \mathbb{R}^{mN \times c}$ to solve the tracking problem, but its dimension is likely to exceed, and not optimize, the available identification time. Hence decide on the required dimension, $q$, of the input subspace, denote the $i^{th}$ column of $W$ as $u_i$, and instead compute an alternative $W \in \mathbb{R}^{mN \times qN}$ to minimize the closest distance between each $u_i$ and the column space of $W$, i.e.

$$
\min_{(W,H)} J(W, H), \quad J(W, H) = \sum_{i=1}^{c} \|u_i - WHi\|^2
$$

$$
= \sum_{i=1}^{mN} \sum_{j=1}^{c} |(u_i - WH_i)|^2
$$

$$
= \|u_1, \ldots, u_c\| - WH\|^2_F (11)
$$

where $\| \cdot \|_F$ denotes the Frobenius norm, $H \in \mathbb{R}^{qN \times c}$, and the dimension $qN$ is prescribed prior to the optimization. This matrix factorization procedure is generally non-unique, and there exist many solution methods through incorporation of different constraints (e.g. principal component analysis, singular value decomposition, nonnegative matrix factorization, see [16]). The optimization (10) has the interpretation of calculating a ‘line of best fit’ between $\{u_i\}$ in the input space, as shown in Figure 1. This procedure ensures that inputs close to those previously encountered are achievable, and hence minimizes corresponding tracking error. As the prescribed parameter $qN$ is increased, the set enlarges to include all $\{u_i\}$ (which occurs when $qN$ equals the column rank of $[u_1, \ldots, u_c]$). The approach in this section also extends to output data, see [15] for details.

E. Subspace Structure

In the previous subsections $W \in \mathbb{R}^{mN \times qN}$ assumed no underlying structure, and hence $GW$ cannot be realized as a causal system operating over $t = 0, 1, \ldots, N - 1$, complicating future identification. To embed this structure requires the form $W = \text{diag}\{\tilde{W}, \ldots, \tilde{W}\}$, and is achieved by changing the dimension of $W$ to be $\mathbb{R}^{m \times q}$ in optimization (11). In addition, each input vector $u_i$ must be ‘unpacked’ and inserted as

$$
u_i = [u_i(0), u_i(1), \ldots, u_i(N - 1)] (12)$$

prior to solving (11) to yield $\tilde{W}$.

Remark 1: A restricted input subspace may also be used to prevent stimulation areas comprising single array elements, which may cause discomfort and muscle fatigue [17]. This is achieved by ensuring that each input comprises a set of two or more adjacent array elements (see [15]).

IV. APPLICATION TO NONLINEAR SYSTEMS

The approaches in Section III have enabled a reduced input subspace $S$ to be constructed, reducing the dimension of the model identification problem. The identification of a suitable linear model is addressed in Section V, and can then be used by the controllers of Section II. First, however, it will be shown how these controllers can be applied locally to control a more general nonlinear system description. This is necessary in the present application since the underlying musculo-skeletal system is nonlinear, but is known to locally satisfy the principle of superposition [7].

Consider the following nonlinear discrete-time system commonly used to represent musculo-skeletal dynamics

$$
x(t + 1) = f(x(t), u(t))
$$

$$
y(t) = h(x(t)), \quad x(0) = x_0, \quad t = 0, \ldots, N - 1 (13)
$$

with the state vector $x(t) \in \mathbb{R}^n$. Here $f(\cdot)$ and $h(\cdot)$ are continuously differentiable with respect to their arguments. Using supervectors yields the relationship $y = g(u)$ where $g(u) = [g_1(u)^T, \ldots, g_N(u)^T]^T$ with $i = 1, \ldots, N$ elements $g_i(x(0), u(0), \ldots, u(i - 1)) = h(x(i))$

$$
= h(f(x(i - 1), u(i - 1))),
$$

$$
= h(f(f(\cdots f(x(0), u(0)), \cdots), u(i - 2)), u(i - 1))),
$$

With this description, tracking problem (5) is replaced by

$$
\lim_{k \to \infty} \|u_d - u_k\|^2 = 0, \quad u_d = \arg \min_u J_1(u),
$$

$$
J_1(u) = \|r - g(u)\|^2, \quad u, u_k \in S \quad \forall k (14)
$$

which, following the approach of (6), is equivalent to

$$
\lim_{k \to \infty} \|v_d - v_k\|^2 = 0, \quad v_d = \arg \min_v J_1(v),
$$

$$
J_1(v) = \|r - \tilde{g}(v)\|^2 (15)
$$

where $\tilde{g}(v) = g(Wv)$. The approach and analysis of Section II can be applied by linearizing the system about an operating point $v_a = [v_a(0)^T, \ldots, v_a(N - 1)^T]^T \in \mathbb{R}^{qN}$. 

4245
(a) Set $k=0$ and set the initial input to $v_0 = 0$.
(b) Apply $u_k = Wv_k$ to the system experimentally and record $y_k$. Calculate postural joint error $i_k = r - y_k$.
(c) Linearize the system about the operating point input $v_k$ (e.g. using the approach of Section V) to obtain the linear model $\tilde{g}'(v_k) = g'(u_k)W$. Denote as $GW$.
(d) Update control input using methods of Theorem 1.
(e) Increment $k$ and goto step (2).

**TABLE I**

| ILC PROCEDURE USING REDUCED INPUT SUBSPACE $S$. |

| $u_a = Wv_a$, and each |
| $\frac{\partial \tilde{g}_a}{\partial v}|_{v = v_a} = \frac{\partial g_a}{\partial u}|_{u = u_a} \tilde{W} \in \mathbb{R}^{p \times q}$ |

so that (16) can be realized in state-space form

$$
\dot{x}(t + 1) = A(t)\dot{x}(t) + B(t)v(t)
$$

$$
y(t) = C(t)\dot{x}(t)
$$

with input $v$, output $y$, and the matrices

$$
A(t) = \left( \frac{\partial f}{\partial v} \right)_{u_a(t)}, \quad B(t) = \left( \frac{\partial f}{\partial u} \right)_{u_a(t)} \tilde{W},
$$

$$
C(t) = \left( \frac{\partial h}{\partial v} \right)_{u_a(t)}
$$

where $\tilde{f}(\cdot, v) = f(\cdot, Wv)$. The ILC approaches of Section II can be locally applied using the matrix $GW$ with form (2) (trivially extended to the time-varying case) with elements (19).

To ensure the linearized system continues to accurately capture the plant dynamics, the process may be repeated by applying the updated input $v_{k+1}$ and then linearizing the system about the new operating point ($v_a = v_{k+1}$). To produce each input update, any of the ILC schemes highlighted in Section II may be used. The resulting update procedure is given in Table I. Within step (d) it is also possible to incorporate constraints on the system input imposed by the hardware, with full details given in [15].

**V. MODEL IDENTIFICATION FOR POSTURAL CONTROL**

The problem of identifying the linearized system $\tilde{g}'(v_a)$ in step (c) of Table I can be stated as

$$
\tilde{g}'(v_a) = \arg \min_X J(X), \quad J(X) = \| \Delta y - X \Delta v \|^2
$$

where $X \in \mathbb{R}^{pN_r \times qN_r}$ is required to have the Toeplitz form corresponding to the state-space dynamics (18) expressed in lifted form, and $N_r$ is the number of samples.

Here $(\Delta v, \Delta y)$ are an experimental signal pair chosen to sufficiently excite the system dynamics about $v_a$. Note that $\Delta v$ and $\Delta y$ are taken relative to the operating point, hence $v = v_a + \Delta v$, $y = \tilde{g}(v_a) + \Delta y$ are the experimental input and output signals. Many methods are available to solve (20), each of which benefits from the reduced input subspace $S$ constructed in Section III. In the current application the selected method must be fast, sufficiently exciting, and avoid injecting large or rapidly varying signals which may be uncomfortable for the patient (see [18] for details).

Simplification is immediately possible by using tests in which only one channel of $v$ is varied at a time. This requires $q$ tests each of duration $N_r$ samples, where in the $i^{th}$ test a signal $v^i$ is applied, whose $i^{th}$ input component is $v^i_i = v_{a,i} + \Delta v^i_i$, while the remaining input components are fixed at $v_{a,j}, j \neq i$. To guarantee excitation of dynamics, a range of stimulation, $u_{i,\text{width}}$, is specified for the $i^{th}$ test to affect a compromise between accuracy and proximity to the operating point, which is then translated to the signal $v$ using $v_{i,\text{width}} = \|W\|^{-1} u_{i,\text{width}}$. In [15] a ramp signal is employed for each $\Delta v^i_i$, and full details are given of the subsequent identification procedures used to yield $\tilde{g}'(v_a)$.

**VI. EXPERIMENTAL RESULTS**

The subspace selection, identification and ILC update procedures of previous sections are now tested in a clinically relevant setting to confirm utility. The hardware consists of a $5 \times 8$ element electrode array (Fatronik-Tecnalia, Spain), comprising a flexible PCB with a conductive hydrogel layer. Each of the 40 array elements can be routed to one of four FES channels. Routing is achieved using custom made RS232 controlled multiplexor hardware, comprising an Arduino board and shift register array. The control system produces a 5V 40Hz square pulse train with variable pulsewidth for each FES channel. These pulses are then amplified by a modified commercial stimulator (Odstock, UK), resulting in bi-phasic voltage-amplified stimulation. The pulse amplitude for each of the four channels is determined manually, while the pulsewidth is the controlled variable. The stimulation signal pulsewidth is limited to between 0 to 300 µs and the amplitude is set to be within a comfortable level for each participant. The stimulator has been used in several clinical trials [9], [10], and the range of pulsewidth, amplitude and frequency employed are selected to produce a smooth muscle contraction. The stimulator, routing hardware, and array are shown in Figure 2. A data glove (5DT 14 Ultra, 5DT, USA) measures hand position and incorporates 14 fiber-optic bend sensors that are positioned over the metacarpophalangeal and proximal interphalangeal joints to measure flexion and between each finger to measure abduction. A twin axis electrotoregioniometer (SG75, Biometrics, UK) is used to measure wrist flexion and radial deviation.

To establish feasibility, tests were conducted on two unimpaired participants (denoted P1, P2) who were instructed to provide no voluntary effort. The array was positioned as shown in Figure 3, to cover wrist and finger extensor
muscles including: extensor carpi radialis longus, extensor carpi radialis brevis, extensor digitorum, extensor pollicis longus, extensor pollicis brevis, extensor indicis, and flexor digitorum profundus. At the beginning of each test session, the stimulator amplitudes were set by routing one channel to two elements of the array, outputting a 300 μs signal and slowly increasing the amplitude until a maximum comfortable level was reached. The amplitudes of the remaining four channels were set to identical levels.

A. Tests using Unrestricted Input Subspace

The iterative procedure given in Table I was first tested using a full subspace to establish maximum accuracy, setting $\hat{W} = I_m$. To identify a model, a ramp input was sequentially applied to the $m = 40$ array elements in turn, while the $p = 16$ angular outputs were recorded. The duration of each test was set at 4 seconds, a sampling time of $T_s = 0.5$ was used, however data was collected at 40 Hz and filtered to reduce effects of noise. Using $N = 1$, the LTI state-space form of (18) was then identified off-line using computations given in [15]. Three reference postures were selected to verify the optimization procedure; “pointing” with the index finger, a “pinch” hand posture and an “open” hand posture. These postures were performed at a wrist angle of approximately 35° extension, with each test starting from an initial angle of approximately 10° flexion. Each task incorporated specific finger movements as well as extension of the fingers and wrist, which comprise challenging movements for stroke patients to perform. This led to 3 reference vectors, $r \in \mathbb{R}^p$, with examples shown in Figure 4.

Within the ILC update of step (d), the Inverse ILC update (21) was applied to solve (6) in a single trial, using

$$v_{k+1} = v_k + (GW)^{+}e_k \quad (21)$$

To quantify the accuracy attained by successive input updates, percentage error was calculated across all joints for each posture using $100 \times \|e_k\| / \|e_0\|$, where $e_0 = r - y_0$, and $y_0$ is the initial posture prior to stimulation.

Results are shown in Table II for the first 3 trials of each task. Each ILC trial reduces the error by approximately 30%, yielding results with a mean joint angle error of typically less than 5°. Figure 4 shows the stimulation sites corresponding to $u_k$ across each task for participant P1. The use of an iterative model-based update leads to significantly reduced error compared with the most accurate existing approaches in the literature, but incurs an identification test duration of 160 seconds per iteration.

<table>
<thead>
<tr>
<th></th>
<th>Pointing</th>
<th>Pinching</th>
<th>Open</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 \times</td>
<td>|e_k|/|e_0|</td>
<td>P1</td>
<td>29.7221</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P2</td>
<td>28.6643</td>
</tr>
<tr>
<td>$100 \times</td>
<td>|e_2|/|e_0|</td>
<td>P1</td>
<td>11.1865</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P2</td>
<td>13.6153</td>
</tr>
<tr>
<td>$100 \times</td>
<td>|e_3|/|e_0|</td>
<td>P1</td>
<td>3.5815</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P2</td>
<td>1.4588</td>
</tr>
</tbody>
</table>

TABLE II

Unrestricted input space: Percentage error across all joints for trials $k = 1, 2, \text{and} 3$.

B. Tests using Restricted Input Subspace

A subspace was constructed for each reference using the data $u_1$, $u_2$, $u_3$, $u_4$ for each of the tests undertaken using the unrestricted subspace in Section VI-A. These were directly inserted as columns in $\hat{W}$ in the manner outlined in Section III-A. Each participant took a rest period of 30 minutes following the previous tests. Results using this input subspace are shown in Table III. The results show only a small reduction in accuracy, while the identification test time is reduced to 16 seconds per iteration. Note that similar convergence rate are observed to the previous experiments.
since $\tilde{W}$ has been chosen such that the reference belongs to the column space of $GW$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Pointing & Pinching & Open \\
\hline
$100 \times \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$ & $\begin{bmatrix} 40.7579 \\ 42.7579 \\ 15.9019 \end{bmatrix}$ & $\begin{bmatrix} 31.271 \\ 41.4948 \\ 19.7373 \end{bmatrix}$ & $\begin{bmatrix} 30.4679 \\ 26.8857 \\ 19.8704 \end{bmatrix}$ \\
$100 \times \begin{bmatrix} e_4 \\ e_5 \\ e_6 \end{bmatrix}$ & $\begin{bmatrix} 5.832 \\ 3.511 \\ 5.311 \end{bmatrix}$ & $\begin{bmatrix} 5.0617 \\ 4.6333 \\ 5.2064 \end{bmatrix}$ & $\begin{bmatrix} 5.7917 \end{bmatrix}$ \\
\hline
\end{tabular}
\caption{TABLE III \hspace{1cm} TASK AND PARTICIPANT-SPECIFIC RESTRICTED INPUT SPACE, $q = 4$: PERCENTAGE ERROR ACROSS ALL JOINTS FOR TRIALS $k = 1, 2, \text{ AND } 3$.}
\end{table}

The previous results require a separate subspace for each task. To generate a single subspace for all three tasks, the procedure of Section III-D was then applied to produce a single input space of dimension $q = 6$, with results shown in Table IV. The optimization (11) was performed using the Matlab function \texttt{nmmf}. With only a small drop in accuracy this subspace hence covers a wide range of clinically relevant tasks, and corresponds to an identification test time of 24 seconds.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Pointing & Pinching & Open \\
\hline
$100 \times \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$ & $\begin{bmatrix} 45.7363 \\ 24.0869 \\ 7.1121 \end{bmatrix}$ & $\begin{bmatrix} 36.5454 \\ 23.6647 \\ 6.1126 \end{bmatrix}$ & $\begin{bmatrix} 31.756 \\ 22.527 \\ 6.0978 \end{bmatrix}$ \\
$100 \times \begin{bmatrix} e_4 \\ e_5 \\ e_6 \end{bmatrix}$ & $\begin{bmatrix} 50.7903 \\ 27.4855 \\ 5.511 \end{bmatrix}$ & $\begin{bmatrix} 45.6745 \\ 24.9782 \\ 5.0096 \end{bmatrix}$ & $\begin{bmatrix} 34.8906 \\ 23.4431 \\ 6.7765 \end{bmatrix}$ \\
\hline
\end{tabular}
\caption{TABLE IV \hspace{1cm} PARTICIPANT-SPECIFIC RESTRICTED INPUT SPACE $q = 6$: PERCENTAGE ERROR ACROSS ALL JOINTS FOR TRIALS $k = 1, 2, \text{ AND } 3$.}
\end{table}

These results indicate that an identification time of less than 90 seconds (comprising three iterations and application of each updated $u_k$) is sufficient to produce an input subspace that covers the range of hand and wrist movements required within a clinical treatment session.

\section{VII. CONCLUSIONS}

This paper has developed restricted input subspace ILC methods, motivated by the problem of finding optimal electrode array stimulation sites for FES-based wrist and hand control. A suitable framework has been developed, along with approaches to determine the subspace using experimental data, structural properties and physiological knowledge. The optimization procedure has been verified experimentally, with results indicating significant potential for use in FES assistive and rehabilitation technologies.

\begin{table}[h]
\centering
\begin{tabular}{|c|}
\hline
\end{tabular}
\caption{TABLE III \hspace{1cm} TASK AND PARTICIPANT-SPECIFIC RESTRICTED INPUT SPACE, $q = 4$: PERCENTAGE ERROR ACROSS ALL JOINTS FOR TRIALS $k = 1, 2, \text{ AND } 3$.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|}
\hline
\end{tabular}
\caption{TABLE IV \hspace{1cm} PARTICIPANT-SPECIFIC RESTRICTED INPUT SPACE $q = 6$: PERCENTAGE ERROR ACROSS ALL JOINTS FOR TRIALS $k = 1, 2, \text{ AND } 3$.}
\end{table}

\begin{thebibliography}{10}
\end{thebibliography}