Optimal unit commitment accounting for robust affine reserve policies

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Abstract—We describe a new approach to robust unit commitment for an electricity network, which couples the switching decisions to a set of time-coupled redispatch rules in order to minimize the expected cost of operation over a planning horizon. We assume bounded uncertainties arising from imperfect predictions of loads and intermittent renewable infeeds. We refer to the time-coupled redispatch rules as affine reserve policies, and they apply not only to generators but to other continuously-controllable devices such as energy storage units or demand response installations, which are modelled as generic mixed logical-dynamical systems. We use a lumped-parameter example to demonstrate that unit commitment decisions coupled with affine reserve policies can reduce the number of time periods in which expensive peaking plants need to be employed, and that the effect is present for a wide range of parameter values. An important benefit is that the approach allows the uncertainty of future energy storage levels to be managed more tightly than existing formulations would allow.

I. INTRODUCTION

The uninterrupted, efficient operation of power systems under the uncertainties caused by a large share of intermittent renewables represents a significant challenge. Germany, for example, is expected to have an installed wind power capacity of 46.6 GW by 2017 [8]. The real output of this vast facility is only partially controllable, and extreme ramping events are commonplace. Because other forms of electricity generation have to supply the remaining net load (the load minus the contribution of uncontrolled renewables), this places increasingly arduous switching and ramping requirements on conventional thermal generators [7].

Unit commitment is a long-studied optimization problem, in which the switching actions and operating points of generators are chosen in order to satisfy loads efficiently. Doing this under uncertainty is variously referred to as a robust, stochastic, or security-constrained unit commitment, depending on the problem aim. In robust problems the generators (and perhaps other flexible devices such as curtailable wind farms or energy storage units) must be able to supply the system loads for any uncertainty realization in a given set [3]. In contrast, scenario-based approaches ensure constraints are satisfied either for every modelled contingency [14], [12], or with a certain high probability [11]. A common additional feature is the constraint that all line ratings must be satisfied robustly or with high probability.

This paper studies the minimization of expected cost while providing robustness to a given bounded disturbance set. An approach in the robust unit commitment literature that has gained attention in recent years is to minimize the worst-case cost of accommodating a bounded disturbance [3], [17], [6]. Of particular relevance to our work is the approach of Bertsimas et al., who considered a two-stage adaptive formulation where the unit commitment can be revised during the planning period based on information yet to be revealed [3]. This ability to change the solution after uncertainty information is revealed is known as recourse. It could be argued, though, that since unit commitment decisions are made repeatedly (for example daily), the use of such a min-max objective may not necessarily result in minimum long-run average running costs. In this regard an expected-cost formulation, as employed in some scenario approaches [13] and in this paper, may have advantages. Of course, these advantages come at the cost of needing to estimate some further characteristics of the disturbance, namely moments of its distribution.

We present a unit commitment in which multi-stage recourse can be taken on the continuous variables to compensate for uncertainty, giving each flexible participant what we refer to as an affine reserve policy. This policy consists of a nominal power generation schedule plus a planned, causal, linear modification in response to the errors that will be discovered in the prediction of loads or renewables at each step of the horizon. In previous work [16], it was shown that these reserve policies could reduce redispatch costs relative to a more standard mechanism that did not use time-coupled decision rules. However that approach assumed that all generator switching decisions had been made in advance, so that only continuous variables were accommodated.

To address this, we now extend the approach using the mixed logical-dynamical (MLD) system description [1] to include unit commitment decisions that are coupled to the choice of redispatch policy. In doing this we reveal an additional benefit of such policies, namely the ability to avoid switching expensive generators on in some cases. The number of binary variables in our formulation is no different to that found in a standard unit commitment problem, and our framework is a candidate for the same kinds of solution methods that have been used to solve large-scale unit commitment in the past, such as dual-based methods [15], [2], [12], cutting planes [10], or simulated annealing [18].

In Section II we describe the mixed logical-dynamic system model of switchable power system participants, along with a summary of the more straightforward inelastic and elastic participants and network constraints from [16]. Section III formulates the finite horizon expected-cost minimization problem as a mixed-integer quadratic program. Section IV gives a numerical example illustrating how time-coupled reserve policies can reduce the need for expensive generation.
Section V concludes with a summary of the benefits seen.

Notation: The notation \((\cdot)\)' denotes the vector or matrix transpose; \([\cdot]_k\) represents element \(k\) of a vector; \([\cdot]_{1..m}\) column \(m\) of a matrix; \(\otimes\) the Kronecker product; \(I_n\) the identity matrix of size \(n\).

II. POWER SYSTEM MODEL

The power system model used in this paper consists of \(N\) generic participants connected by a lossless transmission network. We associate the following two kinds of power flows (injections or extractions at a node) with each participant:

- Inelastic flows, which cannot be influenced by control signals.
- Elastic flows, which are determined by the result of an optimization over possible control inputs.

We further separate those participants with an elastic flow component into two classes:

- Non-switchable participants, which are always “on.”
- Switchable participants, whose on/off state depends on the value of a binary control input.

This distinction allows for a more realistic model of generators for which switching is costly or slow, or which are not able to operate below a given power threshold when turned on. We now provide a state space description of such participants, and describe how the constraints imposed by the transmission network are modelled.

A. Inelastic power flows

The inelastic power flows attributed to participant \(i\) are represented by the sum \(r_i + G_i\delta\). Neither of these terms can be influenced by the system operator. The vector \(r_i \in \mathbb{R}^T\) is the nominal schedule of power production (using the convention that power injections into the network correspond to a positive value), so that \([r_i]_k\) represents the nominal power flow of the current time step. The second term \(G_i\delta\) maps a random vector \(\delta \in \mathbb{R}^{N_i \times T}\) to the power flows attributed to the participant. The vector \(\delta\) represents the system uncertainties, e.g. from errors in the prediction of a load or renewable infeed, and is of the form \([\delta'_1 \delta'_2 \ldots \delta'_T]'\), where each \(\delta_k \in \mathbb{R}^{N_i}\) is the part of the vector discovered \(k\) steps ahead of the current time. We assume that the uncertainty is restricted to a compact set \(\Delta := \{\delta | S\delta \leq h\}\) containing the origin. In addition, estimates of its first and second moments \(\mathbb{E}[\delta]\) and \(\mathbb{E}[\delta\delta']\) are assumed to be available.

B. Elastic power flows

We model elastic power flows using a standard state space representation (see for example Chapter 2.1 of [9]). Each non-switchable participant \(i\) is modeled using linear time-invariant dynamics, where the continuous state and input of participant \(i\) at time \(k\) are denoted \(x^{c,i}_k\) and \(u^{c,i}_k\) respectively, with

\[
x^{c,i}_{k+1} = \tilde{A}^{c,i}_i x_k^{c,i} + \tilde{B}^{c,i}_i u^{c,i}_k.
\]

For each switchable participant \(i\), we define dynamics incorporating a discrete on/off state \(x^{d,i}_k\):

\[
x^{c,i}_{k+1} = \begin{cases} 
\tilde{A}^{c,i}_i x_k^{c,i} + \tilde{B}^{c,i}_i u^{c,i}_k & \text{if } [x^{d,i}_{k+1}]_1 = 1 \\
\tilde{A}^{d,i}_i x^{d,i}_k & \text{if } [x^{d,i}_{k+1}]_1 = 0
\end{cases}
\]

\[
x^{d,i}_{k+1} = \tilde{A}^{d,i}_i x^{d,i}_k + \tilde{B}^{d,i}_i u^{d,i}_k.
\]

For both switchable and non-switchable participants, the first element of the continuous state \([x^{c,i}_k]_1\) represents the power output at time \(k\) while the remaining elements are used to model internal dynamics or to include prior states.

For switchable participants, we assume a discrete state, whose first element \([x^{d,i}_k]_1\) represents the on/off-state at time \(k\) while other elements of \(x^{d,i}_k\) are used to include prior states (this may be necessary, for example, to constrain or penalize generator switching behaviour over time). The scalar, binary input \(u^{d,i}_k\) determines the on/off-status at step \(k+1\) with a value of \(1\) indicating an on state. The participant can inject or extract power at the next time step \(k+1\) only if it will be on, i.e. \([x^{d,i}_{k+1}]_1 = 1\). Note that \([x^{d,i}_k]_1\) is only affected by binary inputs, thus can be modelled as a continuous variable. The logical constraint (2a) can be written more compactly as a bilinear equality:

\[
x^{c,i}_{k+1} = \tilde{A}^{c,i}_i x_k^{c,i} + \tilde{B}^{c,i}_i x^{d,i}_k u^{c,i}_k, \quad x^{d,i}_{k+1} = \tilde{A}^{d,i}_i x^{d,i}_k + \tilde{B}^{d,i}_i u^{d,i}_k.
\]

However, for optimization purposes this bilinearity is inconvenient. We resolve this by using the standard MLD system representation [1], which converts the model into one involving integers and easier-to-handle linear constraints. This requires an additional auxiliary variable \(z^{c,i}_k := x^{c,i}_k u^{c,i}_k\), to which appropriate extra constraints are applied using the so-called “big-M” reformulation (see [1]). The vector \(z^{c,i}_k\) is zero for times when the participant will be off at step \(k+1\), and equal to \(u^{c,i}_k\) when it will be on.

Substituting the auxiliary variables into the dynamics and concatenating the continuous and discrete state, the system dynamics can be described in the following way:

\[
\begin{bmatrix} x^{c,i} \\ x^{d,i} \end{bmatrix}_{k+1} = \begin{bmatrix} \tilde{A}^{c,i}_i & 0 \\ 0 & \tilde{A}^{d,i}_i \end{bmatrix} \begin{bmatrix} x^{c,i} \\ x^{d,i} \end{bmatrix}_k + \begin{bmatrix} 0 \\ B^{d,i}_i \end{bmatrix} u^{d,i}_k + \begin{bmatrix} \tilde{B}^{c,i}_i \\ B^{d,i}_i \end{bmatrix} z^{c,i}_k,
\]

subject to the aforementioned additional linear constraints enforcing consistent behaviour of the auxiliary variable \(z^{c,i}_k\):

\[
\tilde{E}^{z,i}_k z^{c,i}_k \leq \tilde{E}^{c,i}_k u^{c,i}_k + \tilde{E}^{d,i}_k u^{d,i}_k + \tilde{E}^0_i.
\]

For non-switchable (“always-on”) loads we define \(z^{c,i}_k := u^{c,i}_k\) so that the notation that follows applies equally to non-switchable and switchable participants.

C. Finite-horizon stacked representation

In order to optimize over a finite time horizon, we define a vector for each participant \(i\) containing the stacked future state vectors for steps \(k = 1, \ldots, T\). Defining the state vector
\[ x_k^i \text{ as } [x_k^{c,i}, x_k^{d,i}] \] for switchable loads and as \( x_k^{c,i} \) for non-switchable loads, the state of a switchable participant evolves according to
\[ x_i = A_i x_0^i + B_i^{d,i} u_i^d + B_i^z z_i, \] (6)
and the state of a non-switchable participant according to
\[ x_i = A_i x_0^i + B_i^z z_i, \] (7)
where the following stacked vectors have been defined: \( x_i := [x_1^i, \ldots, x_T^i] \), \( u_i^d := [u_0^{d,i}, \ldots, u_{T-1}^{d,i}] \), and \( z_i := [z_0^i, \ldots, z_{T-1}^i] \). Matrices \( A_i \) and \( B_i^d \) are defined as
\[ A_i = \begin{bmatrix} A_i^1 & \cdots & A_i^{T-1} \end{bmatrix}, \quad B_i^d = \begin{bmatrix} B_i^{d,1} & \cdots & B_i^{d,T} \end{bmatrix}, \]
and \( B_i^z \) is defined analogously to \( B_i^d \).

The network consists of \( N_n \) nodes connected by \( L \) transmission lines, and is modelled using the same standard linear approximation as in [16], in which lines are lossless, bus voltage magnitudes are constant, and phase angle differences along lines are small. Each participant is connected to one of the nodes, and in our model the network constrains the combined actions of the participants in two ways. Firstly, at every time step, the net power injection and extraction from inelastic flows \( r_i + G_i \delta \) and elastic flows \( C_i x_i \) have to balance across all participants:
\[ \sum_{i=1}^{N_n} (r_i + G_i \delta + C_i x_i) = 0. \] (10)

Secondly, the power flows in the lines, which under the linear network approximation become linear functions of the nodal power injections, cannot exceed their respective ratings, in either direction, at any time. This results in a vector constraint with \( 2LT \) elements:
\[ \sum_{i=1}^{N_n} \Gamma_i (r_i + G_i \delta + C_i x_i) \leq \tilde{p}, \] (11)
where the block-diagonal matrix \( \Gamma_i \in \mathbb{R}^{2LT \times T} \) contains participant \( i \)'s contributions to power flows in both directions, in each of the \( L \) lines, for each of the \( T \) time steps, and is determined by which of the \( N_n \) nodes the participant is connected to. The vector \( \tilde{p} \in \mathbb{R}^{2LT} \) contains the line ratings. Each block of \( \Gamma_i \) is derived from the procedure in [4], in which the bus phase angles are eliminated.

III. Finite-horizon optimization

In this section we state the problem of minimizing expected future costs over control policies, under which participants’ actions at a given time may be any causal function of the information on the uncertainty \( \delta \) available up to that time. We then restrict ourselves to affine policies, under which the future inputs may only consist of a nominal sequence plus a linear mapping of the uncertainty values \( \delta \) known at a given time. We then give an equivalent tractable representation of the same problem, replacing the infinite number of constraints with a finite number. This converts the problem statement into a solver-compatible specification.

A. General problem statement

We wish to minimize the expected short-term running costs \( \sum_{i=1}^{N_n} E [J_i (x_i, u_i^c, z_i, u_i^d)] \) over a horizon of length \( T \), while fulfilling the local constraints (10) and network constraints (11) for all possible realizations of \( \delta \). We do this by using a sequence of continuous control inputs \( u_i^c \) which can vary with \( \delta \). We call this dependence on \( \delta \) a policy and denote the most general causal policy \( \pi_i(\delta) \) for participant \( i \). We assume that \( \delta_{k+1} \) is discovered just in time to apply the inputs \( u_i^d \) in order to reflect the way generators track a varying load. Our definition of a causal policy is therefore one where \( [\pi_i(\delta)]_{k+1} \) is only a function of \( \delta_1, \ldots, \delta_{k+1} \). The binary control inputs \( u_i^d \) are chosen open loop, meaning the...
timing of generator switching actions will not be varied based on the value of δ discovered at a later time.

Substituting the input \( u_i^d = \pi_i(\delta) \) and the state equation into (8), (9), (10), and (11) to eliminate the state, we obtain the following optimization problem.

\[
\begin{align*}
\min_{z_i, u_i^d, \pi_i, \delta} & \quad \sum_{i=1}^{N_p} \mathbb{E}[J_i(x_0^i, \pi_i(\delta), z_i, u_i^d)] \\
\text{s.t.} & \quad \sum_{i=1}^{N_p} [r_i + G_i \delta + C_i(A_i x_0^i + B_i^d u_i^d + B_i^e z_i)] = 0, \quad \forall \delta \in \Delta \\
& \quad \sum_{i=1}^{N_p} \Gamma_i [r_i + G_i \delta + C_i(A_i x_0^i + B_i^d u_i^d + B_i^e z_i)] \leq \bar{p}, \quad \forall \delta \in \Delta \\
& \quad \left[ A_i x_0^i + B_i^d u_i^d + B_i^e z_i \right] \\
& \quad \pi_i(\delta) \quad \in \mathbb{Z}_i, \quad \forall i, \forall \delta \in \Delta,
\end{align*}
\]

(12a)

(12b)

(12c)

(12d)

(12e)

\[ \pi_i \text{ causal, } \forall i \].

B. Restriction to affine policies

The main issue with problem (12) is that optimizing over general causal functions \( \pi_i \) is not tractable in general. Therefore we consider from now on, as in [16], only affine policies of the form

\[
u_i^d = e_i + D_i \delta,
\]

where \( e_i \in \mathbb{R}^T \) is a nominal input schedule for the next \( T \) time steps, while \( D_i \in \mathbb{R}^{T \times N_p} \) determines how the nominal input will vary linearly with respect to the prediction errors \( \delta \) measured along the way. In order to obtain a causal feedback law over disturbances, \( D_i \) is restricted to be a lower triangular matrix, i.e. \( [D_i]_{l,m} = 0 \) for \( m > l \). Correspondingly, for \( z_i \), we introduce the terms

\[
z_i = z_i^e + Z_i^D \delta,
\]

Such a policy must satisfy the constraints (5) and (9) for all values of the prediction error \( \delta \). We denote the set of feasible policies for participant \( i \) given current state \( x_0^i \) as:

\[
\mathcal{F}_i(x_0^i) := \begin{cases} 
D_i, & e_i, \quad z_i^e, \\
& L_i^D, & u_i^d
\end{cases}
\]

(15)

The matrices \( E_i^{c,e} \) and \( E_i^{c,d} \) are built from individual stage-wise constraints of the form (5).

We redefine the cost function for participant \( i \) to take its dependence on the optimization variables \( u_i^d, e_i, D_i, z_i^e, Z_i^D \) and the current state \( x_0^i \) into account:

\[
\bar{J}_i(x_0^i, D_i, e_i, z_i^e, Z_i^D, u_i^d)
\]

This can be expanded to give a function that is quadratic in the optimization variables and linear in the error statistics \( \mathbb{E}[\delta] \) and \( \mathbb{E}[\delta^2] \). In contrast to [16], this function now depends on the discrete input \( u_i^d \) and the auxiliary variables \( (z_i^e, Z_i^D) \).

C. Equivalent tractable reformulation

Constraints (12b) and (12c) are semi-infinite, since each \( \delta \in \Delta \) defines a separate constraint; consequently the problem cannot, even after the restriction of the functions \( \pi_i(\delta) \) to affine policies, be passed directly to a solver. However it can be shown (using the argument from Section III-B of [16]) that the problem can be rewritten equivalently as:

\[
\min_{W \geq 0, (D_i, e_i, z_i^e, Z_i^D, u_i^d) \in \mathcal{F}_i(x_0^i)} \sum_{i=1}^{N_p} \bar{J}_i(x_0^i, D_i, e_i, z_i^e, Z_i^D, u_i^d)
\]

(16a)

(16b)

(16c)

(16d)

(16e)

where constraints (16b)-(16e) replace (12b) and (12c), and \( S \) and \( h \) define the uncertainty bounds (see Section II-A). Auxiliary variable \( W \) is constrained to have non-negative elements (see [16], where it is called \( Z \), for details). The problem is now a conventional mixed-integer quadratic program (MIQP). Only the coupling constraints have been written out explicitly; the sets \( \mathcal{F}_i(x_0^i) \) and constraint \( W \geq 0 \) are left implicit because they do not couple across participants.

IV. NUMERICAL EXAMPLE

We now demonstrate the use of the method described above for a case study in which energy storage and generation units must satisfy an uncertain net load over a finite planning horizon. We compare it to a method that does not use recourse, meaning that the reserve action in response to the discovery of prediction errors is not allowed to be time-coupled across the planning horizon. We show that time-coupled affine reserve policies introduce extra degrees of freedom that reduce the need to switch generators, and therefore reduce costs.
A. Problem description

We consider how slow-ramping cheap generation, expensive but fast-acting “peaking” generation, and energy storage devices should collectively satisfy an uncertain net load. All three categories of device take part in the reserve mechanism that counteracts the uncertainty. Although the net load is poorly predicted, it is known that it will increase suddenly within the planning horizon, within the bounds shown in Fig. 1. Such a situation often arises, for example, when a passing weather front causes a sharp drop in wind power availability, meaning that conventional generators must make up the resulting power shortfall.

We model the slow-ramping cheap generation as a single lumped unit that stays switched on throughout, but which cannot ramp up its power output fast enough to compensate for the sudden drop in wind power. The difference is tracked by a lumped energy storage capability that represents, for example, the flexibility provided by demand response in the system, or batteries, or hydro-storage. However, since the energy storage has only limited capacity, it may not be able to track the difference alone. When this is the case, we activate expensive peaking generation as a last resort (or rather, the unit commitment routine switches a lumped unit representing expensive generation on). The model parameters, from which the participant models described in Section II were built, are given in Table I. Note that the cost of peaking generation is such that it will only be activated when no other solution exists; the fuel costs are chosen to reflect typical daily electricity price fluctuations.

We measure how often the peaking generation needs to be activated during the planning period for various ramping limits on the cheap generation, and for various energy storage capacities. Two cases are studied:

1) Matrices $D_i$ restricted to a diagonal structure, which represent the best achievable performance where the reserve response is proportional to the instantaneous load mismatch, but cannot be time-coupled.

2) Matrices $D_i$ that can have any lower-triangular structure (recalling that elements above the diagonal would produce non-causal behaviour). These are “full” affine reserve policies, whose response can be a function of any load mismatch measured up to that point.

B. Results

A time horizon of $T = 10$ hours was used, and transmission line constraints were assumed not to be reached (i.e., we assumed a “copper plate” network). The MIQPs were solved using CPLEX [5], and the mean computation time was 223 ms with s.d. 144 ms in case 1, and 755 ms with s.d. 725 ms in case 2. Both cases contained 30 binary and 4260 continuous variables, with structural constraints trivially restricting some of these variables to particular values.

Fig. 2 shows the resulting number of times steps during the planning horizon in which the peaking plant had to be used, for both cases. Case 2 shows a reduced reliance on the peaking plant for a rather large range of energy storage sizes and ramp rate limits. This comes from the choice of more intelligent rules governing how the storage reacts to the uncertainty as its values are revealed over the planning horizon, and the beneficial effect is particularly prominent for intermediate storage sizes and ramp rates. For very large storage units and ramp rates (the top-right region of Fig. 2 (c)), the peaking plants are not required to ramp up the generation total in either case, and for very small storage or slow ramping limits (the bottom-left region) their use is in both cases unavoidable. Fig. 3 shows the corresponding solution costs.

We illustrate the operational changes brought about by time-coupled reserves in Fig. 5, which shows the optimal reserve policies for cases 1 and 2 for an energy storage size of 40.0 MWh and a ramp rate limit of 8.0 MW h$^{-1}$.

In case 1, the energy storage unit must be operated relatively conservatively (the nominal energy level must be kept relatively near the middle of its capacity), because its reserve rule, shown in Fig. 5, can only act on the prediction error revealed at that time step. Because of the integrating effect of energy storage dynamics, the uncertainty range of the storage energy level can only broaden over the planning period. In contrast, case 2 allows the storage unit to take time-coupled actions so that it makes better use of the available storage capacity under uncertainty. This ultimately helps to reduce the use of the expensive generation.

Fig. 4 shows the nominal plans and range of possible
Fig. 2. Number of time steps in which expensive peaking plants were activated. The first plot shows case 1, where matrices $D_i$ are restricted to a diagonal structure; the second plot shows case 2, where lower-triangular $D_i$ matrices are allowed. The third plot shows the difference between the two, illustrating the reduced number of steps in case 2 during which the peaking plant was forced to run. The number of peaking periods saved was between 0 (black) and 5 (white) across the parameter range.

Fig. 3. Expected finite horizon cost (fuel plus switching costs). As with Fig. 2, the first plot shows case 1, the second shows case 2, and the third shows the difference between the two. The cost reduction was in the range $[0, 0.960 \times 10^4]$.

Fig. 4. Nominal plans and deviations, for the parameters $E = 40$ MWh, $R = 8$ MW/h. The first plot shows case 1, the second shows case 2.
power outputs implied by the reserve policies chosen, for cases 1 and 2 respectively. Note that in case 2 the energy storage may provide a larger range of output thanks to the extra degree of operating freedom the time-coupled decision rule has introduced. It should be emphasised that in case 2, there is in general no scenario in which a device’s output follows the upper (or lower) boundary of the shaded range; rather, the upper (lower) boundary at each time step is the highest (lowest) value that any sequence $\delta \in \Delta$ can cause at that particular time under the time-coupled decision rule.

V. Conclusions

This paper demonstrated a unit commitment optimization that accounts for multi-stage affine reserve policies, in a setting that can model a wide range of (possibly switched) linear systems such as energy storage units, flexible loads, or curtable renewables. The policies plan how device operating points should be adjusted as uncertainties are progressively revealed during the planning horizon, and it was shown that coupling them into a unit commitment optimization could reduce the activation of expensive generators. The extra degree of freedom that planned recourse provides appears to be particularly interesting for reserve provision by energy storage units. As the case study showed, it was possible to control the uncertainty range of future energy storage levels much more tightly than would otherwise have been possible.

The formulation we outlined does not introduce any more integer variables than existing unit commitment problems, and therefore the scaling issues encountered by our approach and others are similar. A useful and obvious future step would be to apply some of the existing scaling methods to solve real-sized problems less conservatively.

References


