Coordinated Control of Longitudinal/Lateral/Vertical Tire Forces for Distributed Electric Vehicles*

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Abstract—In this paper, a novel coordinated control method of longitudinal/lateral/vertical tire forces is proposed which overcomes the shortcomings of current studies on global chassis control for electric vehicles. An objective function combining the tire workload and the dynamic ratio of vertical forces is developed, 15 equality and inequality constraints including desired driving demands, tire friction limitations and actuator characteristics are considered, and a non-convex optimization problem with multiple constraints for coordinated control of longitudinal/lateral/vertical tire forces is formulated. An optimization algorithm combining constrained optimization and feasible region planning is proposed to solve this key problem. Finally, simulations based on Matlab/Simulink and CarSim are conducted, demonstrating that the proposed coordinated control method controls vehicle attitude effectively and improves handling stability simultaneously, while providing better stability and robustness over other methods.

I. INTRODUCTION

With development of the automotive technology, various kinds of chassis control systems, including ABS, TCS, DYC, AFS/ARS, SBW, ASS and ABC, have been widely studied and applied. Generally, these systems perform active control of longitudinal, lateral or vertical tire forces respectively to improve vehicle dynamics performance [1]. However, due to complicated coupling characteristics of tire forces and vehicle motions, stand-alone chassis control systems may lead to contradictions of control objectives and no further improvement of dynamics performance. Therefore, global chassis control (GCC) which coordinates two or three directions of tire forces has been widely researched [2].

For coordinated control of longitudinal/vertical tire forces, rule-based methods were commonly used. A. Alleyne et al. discussed coordinated control of braking and active suspension based on the rule that active forces change along with the desired braking torque [3]. C. P. Vassal et al. introduced a slip ratio index and respective rules for coordinated control of braking and active suspension [4]. J. Yoon et al. studied coordinated control of independent braking and active suspension to improve lateral stability and roll safety under specific conditions [5].

Rule-based methods were also adopted in coordinated control of lateral/vertical tire forces. X. Shen et al. proposed a method to increase lateral tire force limitation by controlling the four-wheel-steering system and increasing respective vertical force [6]. C. March et al. introduced fuzzy logic into coordination of the active front steering and the active suspension [7]. R. Rajamani et al. developed a coordinated control method for the steering by wire system and the active suspension system based on a so-called roll index [8].

For coordinated control of longitudinal/lateral tire forces, optimization methods were widely discussed. M. Abe et al. developed the optimal distribution of longitudinal/lateral tire forces based on minimizing weighted sum of tire workload [9]. H. Peng et al. proposed a force distribution method by maximizing remainder tire friction [10]. Y. Dai et al. introduced an optimal distribution method by minimizing weighted sum of workload average and variance [11].

Besides, some studies were performed to optimize three directions of tire forces. J. Yoon et al. and S. Lu et al. designed different control strategies based on driving condition partition [12, 13]. S. Fergani et al. developed a method to enhance tire grip by controlling vertical tire forces, braking forces and steering angle. Yet vertical tire forces control was decoupled from longitudinal and lateral tire force control [14].

As conclusion, optimization methods were mostly applied for coordinated control of longitudinal/lateral tire forces, rule-based methods were often used for coordinated control between vertical and longitudinal/lateral tire forces, and for coordinated control of longitudinal/lateral/vertical tire forces, optimization methods of longitudinal/lateral tire forces and simple rule-based vertical force control were commonly used. To address the limitations of the aforementioned studies, a coordinated control method of longitudinal/lateral/vertical tire forces for a distributed electric vehicle (DEV) equipped with four wheel driving/steering and active suspension systems is proposed in this paper. Simulation results compared with other methods show the effectiveness and superiority of the proposed method.

II. MATHEMATICAL MODEL: NON-CONVEX TIRE FORCE OPTIMIZATION PROBLEM WITH MULTIPLE CONSTRAINTS

In order to coordinate all the tire forces, the key problem of longitudinal/lateral/vertical tire force distribution is discussed in this section. As a preliminary, vehicle state parameters and road information are assumed to be given by measurements or estimators. The tire force distribution problem is formulated into a mathematical optimization model, where all the 12 tire forces are regarded as free variables to be optimized, as given in (1) to (16).
\[
\min J = \text{Var}\left(\frac{F_i^2 + F_j^2}{\mu_i F_i}\right) + w_1 E\left(\frac{F_i^2 + F_j^2}{\mu_i F_i}\right) + w_2 \text{Var}(F_{\text{des}}) / F_i
\]

s.t.
\[
F_{i,\text{des}} = F_{i,\text{fl}} + F_{i,\text{fr}} + F_{i,\text{rl}} + F_{i,\text{rr}}
\]
\[
F_{i,\text{des}} = F_{i,\text{fl}} + F_{i,\text{fr}} + F_{i,\text{rl}} + F_{i,\text{rr}}
\]
\[
F_{i,\text{des}} = F_{i,\text{fl}} + F_{i,\text{fr}} + F_{i,\text{rl}} + F_{i,\text{rr}}
\]
\[
M_{i,\text{des}} = 0.5 t_i (F_{i,\text{fl}} - F_{i,\text{fr}}) + 0.5 t_i (F_{i,\text{rl}} - F_{i,\text{rr}})
\]
\[
M_{i,\text{des}} = -l_i (F_{i,\text{fl}} + F_{i,\text{fr}}) + l_i (F_{i,\text{rl}} + F_{i,\text{rr}})
\]
\[
+ 0.5 t_i (-F_{i,\text{fl}} + F_{i,\text{fr}}) + 0.5 t_i (-F_{i,\text{rl}} + F_{i,\text{rr}})
\]
\[
F_{i,\text{fl}} = 0
\]
\[
F_{i,\text{fr}} = 0
\]
\[
F_{i,\text{rl}} = 0
\]
\[
F_{i,\text{rr}} = 0
\]
\[
\frac{F_{i,\text{max}}}{F_i} - \frac{F_{i,\text{min}}}{F_i} \leq 0
\]
\[
T_{i,\text{max}} / r \leq F_{i,\text{des}} \leq T_{i,\text{max}} / r
\]
\[
F_{i,\text{max}} \leq F_{i,\text{des}} \leq F_{i,\text{max}}
\]
\[
-k_{i,\text{max}} \leq \Delta F_i / \Delta t \leq k_{i,\text{max}}
\]
\[
-k_{i,\text{max}} \leq \Delta F_i / \Delta t \leq k_{i,\text{max}}
\]
\[
F_i \geq 0
\]

where, \(\min\) denotes minimization, \(\text{Var}\) denotes the variance, \(E\) denotes the average value, \(w_1\) and \(w_2\) are both weighting coefficients. The subscript set \(i = \{f, f_r, r\}\) refers to front-left, front-right, rear-left, and rear-right tire respectively. \(F_{i,\text{fl}}, F_{i,\text{fr}}, F_{i,\text{rl}}, F_{i,\text{rr}}\) is the longitudinal, lateral, and vertical force of each tire respectively. \(F_{i,\text{des}}, F_{i,\text{des}}\) and \(F_{i,\text{des}}\) is the desired longitudinal, lateral and vertical force respectively. \(M_{i,\text{des}}, M_{i,\text{des}}\) and \(M_{i,\text{des}}\) is the desired external roll, pitch and yaw moment respectively. \(t_i (t_i)\) is the front (rear) tread respectively. \(l_i (l_i)\) is the distance from central gravity to the front (rear) center point. \(\mu_i\) is the frictional coefficient between each tire and the driving road. \(T_{i,\text{max}}\) and \(F_{i,\text{max}}\) is the maximal motor torque and maximal lateral force respectively. \(r\) is the effective tire radius. \(\Delta t\) is the time step. \(\Delta F_{i,\text{fl}}, \Delta F_{i,\text{fr}}, \Delta F_{i,\text{rl}}\) and \(\Delta F_{i,\text{rr}}\) is the increment of the longitudinal, lateral and vertical force respectively during the time step \(\Delta t\). \(k_{i,\text{max}}, k_{i,\text{max}}\) and \(k_{i,\text{max}}\) is the maximal changing rate of each motor torque, lateral force and active suspension force respectively. \(F_{i,\text{des}}\) is the static vertical load of each tire.

The objective function \(J\) given in (1) is constructed with consideration of the tire workload and the dynamic ratio of vertical tire forces, as shown in (17) and (18).

\[
J_1 = \text{Var}(\gamma_i) + w_1 E(\gamma_i) = \frac{1}{4} \sum_i (\gamma_i - \frac{\sum_j \gamma_j}{4})^2 + w_1 \frac{1}{4} \sum_i \gamma_i
\]
\[
J_2 = w_2 \text{Var}(\varepsilon_i) = w_2 \frac{1}{4} \sum_i (\varepsilon_i - \frac{\sum_j \varepsilon_j}{4})^2
\]

where, \(\gamma_i = \sqrt{F_{i,\text{fl}}^2 + F_{i,\text{fr}}^2 / (\mu_i F_i)}\) denotes the tire workload, \(\varepsilon_i = F_{i,\text{des}} / F_i\) denotes the dynamic ratio of vertical tire forces.

Based on (1), constraints including vehicular driving demands, actuator characteristics and frictional limitations between tires and the road are considered to determine the feasible region of each tire force. First, desired forces and moments determined from vehicular driving demands must be provided by the tire forces together, which lead to constraints (2) to (7). Second, lateral forces of the left/right tires at the same axle have to follow (8) and (9) under the assumptions: (i) the steering angles of front (rear) tires are constrained equally based on the single-track vehicle model; (ii) the lateral tire force can be considered as proportional to the vertical tire force with a relatively small tire sideslip angle [15]. Furthermore, the resultant horizontal force on each tire must not exceed the maximal frictional force, as given in (10). Additionally, the static and dynamic characteristics of motor-driving, steering and suspension systems must be considered in order to guarantee that the distributed forces are executable by the real actuators. Therefore, the maximal capability of each motor torque and lateral tire force should be limited. And the maximal changing rate of each motor torque and lateral/vertical tire force should be also restricted. Moreover, each vertical tire force must be a positive value. These constraints are given in (11) to (16).

In summary, the longitudinal/lateral/vertical tire force distribution problem has to subject to 15 equality or inequality constraints, including 6 non-linear constraints.

III. OPTIMIZATION ALGORITHM FOR NON-CONVEX TIRE FORCE OPTIMIZATION PROBLEM

As the longitudinal/lateral/vertical force distribution problem becomes a non-convex optimization problem for 12 variables with 15 constraints, the current optimization algorithm is no longer effective. For this reason, an optimization algorithm combining constrained optimization and feasible region planning is proposed.

At the first step, the changing rate constraints of tire forces are left aside temporarily. Slack variables are introduced into each inequality constraint and a barrier function is introduced into the objective function respectively. Then the optimization problem is generalized as the following mathematical model.

\[
\min_{x,s} f(x,s) = f(x) - \mu \sum k s_k
\]
\[
\text{s.t. } g_k(x,s) + s_k = 0
\]
\[
h_j(x) = 0
\]

where, \(x=F_{i,\text{fl}}, F_{i,\text{fr}}, F_{i,\text{rl}}, F_{i,\text{rr}}\) denotes a 12×1 vector of each tire force variable. \(f(x)\) is the objective function shown in (1). \(g\) and \(h\) denotes the expressions of equality constraints and inequality constraints respectively. \(k\) and \(j\) refer to the \(k\)th inequality constraint and \(j\)th equality constraint. \(s_k\) is the slack variable of \(k\)th inequality constraint. \(\mu\) is the penalty factor of the barrier function.
Applying the KKT condition and Taylor approximation, the iterative formula for optimal solution is obtained as (20).

\[
\begin{bmatrix} x^{(k+1)} - x^{(k)} \\ g^{(k+1)} - g^{(k)} \end{bmatrix} = \begin{bmatrix} 0 \\ \mu \end{bmatrix} \begin{bmatrix} \nabla \hat{z}(x^{(k)}) & 0 & J_g(x^{(k)}) J_f(x^{(k)}) & \nabla f(x^{(k)}) \end{bmatrix} \begin{bmatrix} x^{(k)} \\ g(x^{(k)}) \\ J_g(x^{(k)}) \\ J_f(x^{(k)}) \end{bmatrix}
\]

(20)

where, \( \varepsilon = [x, s, y, \lambda, z, \lambda_e, \lambda_f, y_j] \) are Lagrangian multipliers of \( g_i(x) \) and \( h_i(x) \) respectively, \( J_g \) is the Jacobian matrix of \( g(x) = [g_1(x), ..., g_e(x)]^T \), \( J_f \) is the Jacobian matrix of \( h(x) = [h_1(x), ..., h_j(x)]^T \), \( e = [1, ..., 1]^T \), \( \Delta = \text{diag}[\lambda_e, ..., \lambda_f] \), \( \mu = \text{diag}(s^{(e)}_{i_1}^{-1}, ..., s^{(e)}_{i_k}^{-1}) \).

Furthermore, as it is almost impossible to get the global optimum of a non-convex optimization problem within limited iterations, the optimum obtained from (20) may not be the global minima. Besides, as mentioned before (19), the changing rate constraints of tire forces are not included yet. To solve these problems, a feasible region planning method is introduced to locate the optimum properly as shown in (21).

\[
x^{(k)} - \xi(E_{\Delta \varepsilon}) \leq x^{(k+1)} - x^{(k)} + \xi(E_{\Delta \varepsilon})
\]

(21)

where, \( E = [F_{x, \text{des}}, F_{y, \text{des}}, M_{x, \text{des}}, M_{y, \text{des}}, M_{z, \text{des}}]^T \), \( E_{\Delta \varepsilon} \) is the maximal increment of \( E \) during the time step, \( \xi(E_{\Delta \varepsilon}) \) is the neighborhood radius.

In combining (20) with (21), the search space for the optimum can be limited inside a neighborhood determined by the previous optimum. In this way, the feasible region is narrowed, changing rate constraints of tire forces are included, and optimal tire forces for vehicle driving are finally obtained.

IV. DESIRED FORCES/MOMENTS AND ACTUATOR CONTROL

The determination of desired forces/moments and control of driving/steering/suspension actuator systems are discussed in this section.

A. Determination of Desired Forces/Moments

As deviation between actual vehicle position and the ideal trajectory always exists and has to be eliminated, the desired turning radius and desired yaw rate is determined as follows.

\[
R = \frac{R_0}{1 - \left( \frac{d_f - d_r}{l} \right)^2 + \frac{d_f^2 + d_r^2}{l^2}}
\]

(22)

\[
\gamma_{\text{des}} = \frac{f_{\gamma, \text{opt}}}{R} + \gamma_{\text{des}} - \gamma_{\text{act}} \frac{V_s}{l}
\]

(23)

where, \( R \) is the desired turning radius, \( R_0 \) is the turning radius from the ideal trajectory, \( \gamma \) is the actual yaw rate, \( \gamma_{\text{des}} \) is the desired yaw rate, \( \gamma_{\text{act}} \) is the desired front (rear) tire slip angle feedback from (32), \( L \) is the previewed distance, \( d_f, d_r \) is the distance from front (rear) axle midpoint to the ideal trajectory, \( l \) is the wheelbase.

The desired longitudinal/lateral/vertical forces and desired roll/pitch/yaw moments are defined by (24) to (29).

\[
F_{x, \text{des}} = \max(F_{x, \text{des}})
\]

(24)

\[
F_{y, \text{des}} = MV_s^2 / R
\]

(25)

\[
F_{z, \text{des}} = mg
\]

(26)

\[
M_{x, \text{des}} = (K_{p1} + K_{s1} \cdot s - K_{d1} \cdot s)(\rho_{\text{des}} - \rho)
\]

(27)

\[
M_{y, \text{des}} = (K_{p2} + K_{s2} \cdot s + K_{d2} \cdot s)(\theta_{\text{des}} - \theta)
\]

(28)

\[
M_{z, \text{des}} = (K_{p3} + K_{s3} \cdot s + K_{d3} \cdot s)(\gamma_{\text{des}} - \gamma)
\]

(29)

where, \( m \) is the vehicle mass, \( a_{\text{des}} \) is the desired longitudinal acceleration, \( V_s \) is the longitudinal velocity, \( g \) is the gravitational constant, \( K_{p1}, K_{p2} \) and \( K_{p3} \) are proportion parameters, \( K_{s1}, K_{s2} \) and \( K_{s3} \) are integration parameters, \( K_{d1}, K_{d2} \) and \( K_{d3} \) are differential parameters, \( s \) is the Laplace operator, \( \rho \) is the roll angle and the desired roll angle \( \rho_{\text{des}} = 0 \), \( \theta \) is the pitch angle and the desired pitch angle \( \theta_{\text{des}} = 0 \).

B. Actuator Control

The desired driving torque of each wheel \( T_{\text{ui}} \) and the desired active suspension force at each corner \( F_{\text{ui}, \text{des}} \) are determined by (30) and (31) respectively.

\[
T_{\text{ui}} = F_{\text{ui}, \text{opt}} \cdot r
\]

(30)

\[
F_{\text{ui}, \text{des}} = (F_{\text{ui}, \text{opt}} - \hat{F}_{\text{ui}}) / i_s
\]

(31)

where, \( r \) is the tire radius, \( i_s \) is the leverage factor between the suspension and tire, \( \hat{F}_{\text{ui}} \) is the estimated vertical tire force considering longitudinal and lateral load transfer.

For lateral tire force control, an analytical inverse tire model given in (32) is introduced to calculate desired tire sideslip angles from desired lateral forces [16].

\[
\alpha_{\text{des}} = \begin{cases} \tan^{-1} \frac{F_{\text{ui}, \text{opt}} C_{\alpha}}{C_{\gamma}(C_{\alpha} - F_{\text{ui}, \text{opt}}))} & (D \geq 1) \\ \tan^{-1} \frac{C_{\alpha}^2 F_{\text{ui}, \text{opt}} F_{\text{ui}, \text{opt}}^2}{4 C_{\gamma} C_{\alpha} (F_{\text{ui}, \text{opt}}^2 + F_{\text{ui}, \text{opt}}^2) (\mu F_{\text{ui}, \text{opt}} - \sqrt{F_{\text{ui}, \text{opt}}^2 + F_{\text{ui}, \text{opt}}^2}) - C_{\alpha}^2} & (D < 1) 
\end{cases}
\]

(32)

where, \( C_{\gamma} \) is the longitudinal tire slip stiffness, \( C_{\alpha} \) is the lateral tire slip stiffness, \( D = 0.5 \mu F_{\text{ui}, \text{opt}} (F_{\text{ui}, \text{opt}}^2 + F_{\text{ui}, \text{opt}}^2) \).

Based on the single-track vehicle model with \( \delta_{\text{des}} = 0 \), the desired tire steering angle \( \delta_{\text{des}} \) is determined by (33).

\[
\begin{bmatrix} \delta_{\beta, \text{des}} \\ \delta_{\theta, \text{des}} \\ \delta_{\gamma, \text{des}} \\ \delta_{\alpha, \text{des}} \end{bmatrix} = \begin{bmatrix} \alpha_{\text{fl,des}} \\ \alpha_{\text{fr,des}} \\ \alpha_{\text{rl,des}} \\ \alpha_{\text{rr,des}} \end{bmatrix} = \begin{bmatrix} \alpha_{\text{fl,des}} \\ \alpha_{\text{fr,des}} \\ \alpha_{\text{rl,des}} \\ \alpha_{\text{rr,des}} \end{bmatrix} = \begin{bmatrix} l_f \\ l_f \\ l_f \\ l_f \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}
\]

(33)
V. SIMULATION RESULTS

Simulations based on Matlab/Simulink and CarSim are carried out in the straight accelerating (SA), the uniform circling (UC), and the accelerating double lane change (A-DLC) conditions to validate the proposed coordinated control method.

A. Simulation Setup

The setup for each driving condition is given in table I. The circle radius in the UC condition is $R_0=100$ m and the double lane in the A-DLC condition is shown in Fig.1.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Initial speed and longitudinal acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>36 km/h; trapezoidal acceleration with maximum of 7 m/s²</td>
</tr>
<tr>
<td>UC</td>
<td>72 km/h; no longitudinal acceleration</td>
</tr>
<tr>
<td>A-DLC</td>
<td>36 km/h; trapezoidal acceleration with maximum of 5 m/s²</td>
</tr>
</tbody>
</table>

![Figure 1. Route setup for A-DLC condition](image)

In order to validate the effectiveness of the proposed method, three other methods introduced in current studies are chosen for comparison. In method 1 (M1), longitudinal forces are distributed equally and steering angles are determined by (33) but excluding tire sideslip angles. In method 2 (M2), longitudinal/lateral forces are optimally distributed based on minimizing weighted sum of workload average and variance [16]. In method 3 (M3), optimal longitudinal/lateral force distribution and rule-based vertical force distribution are combined, which applies active suspension forces due to desired roll (pitch) moments equally to left and right (front and rear) tires [17]. Method 4 (M4) is the coordinated control method proposed in this paper.

B. Results and Analysis

The comparison results, including pitch/roll angle, sideslip angle, yaw rate, tire workload and the vertical force, are given in table II to table VI. In the SA and UC conditions, it makes no sense to take method 2 into comparison as only longitudinal or lateral acceleration exists. While in the A-DLC condition, all the methods are compared.

From table II to table VI, it can be concluded that the proposed method successfully controls vehicle body pitch/roll angle due to longitudinal and lateral acceleration, improves vehicle sideslip and yaw rate, reduces tire workload deterioration caused by unreasonable vertical force distribution, reduces fluctuation of vertical tire forces and improves stability of vertical force control. Due to limited space of the paper, only the detailed results and analysis of A-DLC scenario are given as follows.

![Figure 2. Pitch angle and roll angle in A-DLC condition](image)

Besides of improvement of vehicle pitch/roll attitude, the effectiveness for vehicle handling stability improvement as
well are shown in Fig.3 to Fig.6. From Fig.3, it can be told that the peak value of sideslip angle is significantly reduced from 2.4 deg, 0.9 deg and 1.4 deg to 0.5 deg in method 4. Fig.3 also shows that the yaw rate in method 4 is steadier and much smaller after 6s. At 6.5s, the peak value of yaw rate in method 4 is 12.2 deg/s, while that in other methods are 14.0 deg/s, 14.5 deg/s and 13.9 deg/s. Therefore, both vehicle sideslip angle and yaw rate are improved in method 4.

As shown in Fig.5, in method 1, the vehicle produces the most serious tire workload with a maximal value of 84% and an average value of 29%. In method 2, the tire workload is reduced to a maximal value of 68% and an average value of 27%. In method 3, as simple vertical force distribution is overlaid, the tire workload turns worse with a maximal value of 79% and an average value of 28%. While in method 4, longitudinal/lateral/vertical forces are optimally distributed and the tire workload deterioration in method 3 is reduced to a maximal value of 71% and an average value of 24%.

The longitudinal acceleration and vehicle trajectory are shown in Fig.4. It is clear that all the methods achieve the desired longitudinal accelerations and follow the designed path. However, method 4 produces the smallest deviation of 0.02 m, while those values of other methods are 0.37 m, 0.1 m and 0.04 m. Therefore, in method 4, the vehicle follows the desired path most smoothly and closely, and achieves the best path-following performance in the A-DLC condition.

As shown in Fig.6, in method 1 and method 2, the maximal vertical forces are 5173 N and 5130 N, and the minimal values are 1015 N and 1087 N, respectively. In method 3, vertical forces fluctuate even more seriously. The maximal vertical force is 5908 N and the minimal value falls to only 292 N. Besides, the vertical forces turn to be divergent after 7s, which may lead to control failure and vehicle instability. In method 4, the maximal vertical force keeps below 4846 N and the minimal value stays above 1746 N. Therefore, the fluctuation of vertical forces is significantly reduced and each vertical force is more close to the respective static vertical load, which is favorable for smaller suspension displacement and vehicle stability.
In spite of the achievements of this paper, better real-time performance and robustness to parameter variation will be the next focus in the future work.

VII. ACKNOWLEDGEMENT

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