Close Proximity Dynamics and Control about Asteroids

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Abstract—Small solar system bodies such as asteroids and comets are of significant interest for scientific, robotic, and human exploration missions. However, their dynamical environments are among the most extreme found in the solar system. Uncontrolled orbital trajectories in the vicinity of an asteroid are highly unstable in general and may either impact or escape in timespans of hours to days. Even with active control, the chaotic nature of motion about these bodies can effectively randomize a trajectory within a short time period, creating fundamental difficulties for the navigation of spacecraft in these environments. Dealing with these navigation issues through the estimation and control of asteroid-relative motion and models is a burgeoning area of research that has become higher profile with the recently announced NASA Asteroid Retrieval Mission. The goals of that mission will require significant extensions of capability beyond previously flown asteroid rendezvous missions. Despite this, there has been significant prior research and application of estimation and control techniques for spacecraft in these small body environments that can directly inform and motivate the necessary direction of future research. This tutorial paper will review previous research and applications for asteroid relative proximity operations and indicate areas where additional future research is needed.

I. INTRODUCTION

The scientific exploration of small bodies such as asteroids and comets has become a major topic of sincere interest. This interest has also expanded to include the human exploration of these bodies and the utilization of these bodies through resource extraction. This latest emphasis has been provided a high profile due to the recently announced NASA Asteroid Retrieval Mission (ARM), with its stated goal of discovering an accessible Near Earth Asteroid (NEA), flying a rendezvous trajectory to it, capturing the asteroid (or a boulder from the asteroid), and transporting this back to the Earth-Moon system for further study. The technical challenges of this plan have been a topic of discussion amongst the science and engineering communities, and is motivating significant research into this topic. The control of a spacecraft in close proximity of an asteroid is among the most challenging aspects of these problems.

The current paper is being written to serve as a basic introduction to the problem of close-proximity dynamics and control of a spacecraft close to a small solar system body such as an asteroid or comet. Or main focus will be on the small NEA that are a specific target of the ARM mission. While this is a large and varied topic, with many different interesting aspects for it, our current paper will be more narrowly focused on defining the dynamical environment about small NEA and reviewing past work that has focused on developing controlled trajectories close to these bodies. More general aspects of this problem are covered in detail in the book by Scheeres which we will be used as a summary reference for several different topics not dealt with in detail here.

While there has been much study and analysis of interplanetary trajectories to NEA, the real technical challenges of small body exploration emerge once close proximity operations are considered. The small body environment is perhaps the most strongly perturbed astrodynamical environment found in the solar system. A combination of factors create these challenges, including strongly distended body shapes, a range of spin states and rates, the strength of solar perturbations from gravity and radiation pressure, and non-gravitational forces from the bodies themselves in the case of comets. Due to these effects in isolation or combination it is possible for seemingly stable trajectories to impact or escape from these bodies in short periods of time, causing severe constraints on the remote operation of vehicles in these environments. Despite this complexity, the application of methods from astrodynamics, celestial mechanics and dynamical astronomy can be used to find practical mission operations plans and designs across the spectrum of body sizes, types and locations. A previous review paper has focused on orbital approaches to these issues. While some aspects of orbiting will be covered again here, this paper will also provide a more direct focus on hovering and slow flyby dynamics and control, leveraging from some earlier papers, as these approaches may be more relevant to the mission design goals of the ARM initiative. This paper is also restricted to “solitary” bodies and does not discuss the interesting aspects of mission design about binary asteroids.

II. MOTIVATION

To motivate the specific discussion and review in this paper three examples of the extreme results that can be found for small body orbiters are presented. Figure 1 shows three orbits about the asteroid 433 Eros, accounting only for the gravitational attraction of that strongly distended body. One of these orbits is started at local circular conditions and remains stable and bounded for arbitrary periods of time. Another is shifted by 45 degrees in phase angle from the initial stable orbit and escapes within two orbits. The third is started at local circular conditions a few kilometers closer to the body and impacts within two orbits – each orbit lasting on the order of 16 hours. This range of effects in close proximity to one geometric location is due entirely to interactions with the body’s rotating gravity field, and can be completely

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understood and accounted for using astrodynamics theory.

Fig. 1: Stable, escape and impacting orbits at Eros. The plots are in an inertial frame while the attitude of Eros is shown at the initial conditions.

Figures 2 and 3 show two spacecraft orbits about a small, spherical asteroid with the only perturbation arising from the solar radiation pressure from the sun. Both of these plots are shown in frames rotating with the asteroid’s orbit about the sun, on the order of 1° per day, and thus keeping the solar location fixed along the horizontal axis. Figure 2 presents an example of an initially stable orbit which loses its stability and escapes from the asteroid when the asteroid’s distance from the sun becomes closer than a limit which can be explicitly predicted. Figure 3 shows two orbits, started at locally circular conditions a distance of 100 meters apart from each other. It can be seen that even across this small range of initial conditions, one orbit impacts while the other one escapes. This shows the sensitivity of orbits to modest changes in initial conditions and explicitly shows the challenges that small body orbiters may face. Despite the complexity of these motions, again it is possible to understand them at a deep level and, using astrodynamics theory, develop mission design solutions.

III. DIFFERENCES BETWEEN ASTEROID AND LEO

Before continuing the discussion it is important to describe the differences between asteroid-relative orbital dynamics and relative orbital dynamics about a satellite in Earth orbit. While aspects of these problems are similar, the differences between these problems are significant and must be explained. The motivation for this section is due to the common misperception that similar orbital and control strategies from the field of close-proximity dynamics in Earth orbit can be applied directly to close-proximity operations about asteroids. However, the fundamental physics of these problems are profoundly different.

For relative motion in Earth orbit, the dynamics are described by the Clohessy-Wiltshire equations (or linear Hills equations). These dynamics are essentially linearized motion about a circular orbit (and can be generalized to linearized motion about an elliptic orbit). The central body, usually a target spacecraft, does not exert any measurable gravitational attraction on the satellite. Further, the motion of the satellite about the target has a time period similar to the orbit period of the target about the Earth, generally on the order of a few hours in LEO or 24 hours at GEO. The motion in these cases can be completely solved in terms of simple trigonometric functions and used to predict the relative motion relatively accurately, although perturbations from Earth oblateness are important to incorporate. Finally, there are arbitrary relative equilibria between the two satellites, found by placing the satellite on the “v-bar” direction relative to the target, essentially placing both bodies on the same circular orbit with different phase angles.

For relative motion about an asteroid, the dynamics are described by a significantly more complex set of equations, reviewed in what follows. These dynamics include similar features of the CW equations, but also include the non-trivial attraction of the asteroid, its non-spherical gravity field, and solar radiation pressure (SRP), all of which provide
significant perturbations. First we can review the similarities between relative motion about targets in orbit about the Earth, and targets in orbit about the Sun. When a spacecraft is on the order of several tens or hundreds of kilometers away from the asteroid, the dynamics become similar to those modeled by the CW equations, with the addition of a strong, nearly constant acceleration acting away from the sun due to SRP. The period of motion associated with an asteroid relative spacecraft is on the order of the asteroid’s period about the sun, on the order of a year for an NEA, several orders of magnitude longer than at Earth. Due to these extremely long time scales, the use of Earth-relative orbiter techniques and dynamics are not very relevant for the asteroid case.

When closer than a few tens of kilometers motion is more akin to the Hill 3-Body problem, with the inclusion of SRP and non-spherical central gravity. Thus, motion in this problem is non-integrable, unlike the CW equations, and must be studied using advanced techniques of analysis and numerical integrations. Further, the simple solutions that exist in the CW equations, such as equilibrium locations along the v-bar, do not exist close to an asteroid due to its attraction. The only equilibria which exist are along the sun-line, exist only at a single location (the L1 and L2 equilibria), and are unstable in general. Due to SRP, these points are also strongly displaced from the gravity-only locations, with the sun-side equilibrium point significantly far from the asteroid and the anti-sun side much closer in general. Figure 4 shows a scaled image with the location of these relative equilibria for a particular example (see the more complete discussion in\(^3\)).

![Equilibrium points about an asteroid due to the asteroid gravitational attraction, solar gravitational attraction, and solar radiation pressure. The coordinates are scaled, for the Hayabusa spacecraft at asteroid Itokawa a unit of 1 on the figure is approximately 108 kilometers.](image)

**IV. Orbital Mechanics**

This section briefly reviews the fundamental equations of motion in the small body environment with specific specializations to gravity dominated and solar dominated regimes of motion. The general problem is first stated and then some specific cases are discussed in more detail in following sections.

As in most orbit mechanics problems, the fundamental equations of motion can be most simply stated in an inertial frame. Take a small body-centered frame with an inertially fixed orientation. Assume an attitude matrix \(C\) that takes a vector expressed in the small body-fixed frame and rotates it into the inertial frame. Thus, \(C\) is a function of the small body’s rotational dynamics and defines its attitude. Then the equations of motion for a satellite in its vicinity can be generally described as:

\[
\dot{r} = \frac{\partial U}{\partial r} + \frac{\partial R_S(r,d)}{\partial r} + \frac{\partial R_{SRP}(r,d)}{\partial r}
\]

where \(r\) is the spacecraft position vector relative to the small body center of mass, \(U\) is the gravitational force potential of the body, \(R_S\) represents the gravitational perturbation from the sun, and \(R_{SRP}\) represents the solar radiation pressure perturbation. The vector \(d\) represents the position of the small body relative to the sun (assumed to follow 2-body motion). The gravitational force potential is most generally defined as

\[
U = \frac{\mu_S}{r + \rho}
\]

where \(\mu_s\) is the gravitational constant, \(B\) represents the small body mass distribution, and \(\rho\) is the position of a differential mass element \(dm\) in the small body fixed frame. The representation of this gravity field is usually performed using spherical harmonic expansions or, when in close proximity to the body, with specialized closed-form solutions for a polyhedral-shaped body.\(^1\)

The solar gravitational attraction and the solar radiation pressure are represented using simplified models that capture the main aspect of these forces. Higher-accuracy models can be developed, but the essence of the problem arises from these first-order perturbations:

\[
R_S = \frac{\mu_S}{2d^3} \left[3(d \cdot r)^2 - r^2\right]
\]

\[
R_{SRP} = \frac{\beta}{d^3} d \cdot r
\]

where \(\mu_s\) is the sun’s gravitational parameter and \(\beta = P_0(1 + \rho)(A/m)\) is a combination of the sun’s radiation flux \(P_0 \approx 1 \times 10^8\) kg km\(^3\)/s\(^2\)/m\(^2\), the satellite’s reflectance \(\rho\) and the satellite’s area to mass ratio in units of kg/m\(^2\). This solar radiation pressure model is commonly referred to as the “cannonball” model, and it suffices to capture the main perturbations from SRP, although improved models have been developed in the literature.\(^1\)

To simulate the general motion of a satellite about a small body then requires the specification of that body’s gravitational field, \(U\), its rotational dynamics, \(C\), its orbit
about the sun, \( d \), and the satellite area to mass ratio and its optical properties, contained in \( \beta \). The analytical study of all of these effects in conjunction is difficult, and only limited results are available.\(^{11}\) However, if these effects are viewed independently, splitting gravitational and solar effects, then significant progress has been made in understanding and analyzing the resultant behavior. These effects are only discussed in isolation in the following.\(^{4}\) provides an investigation of what happens when they are combined, using the Rosetta spacecraft at comet 67P/CG as an explicit example.

V. GRAVITY REGIME

First consider the motion of a satellite in the “gravity dominated” regime, defined as one where the perturbations from the solar gravity and radiation are small compared to the gravitational attraction of the central body. This is the regime that was experienced by the NEAR spacecraft at asteroid Eros, and in general will occur for bodies several kilometers or larger at 1 AU, for example. Under this assumption the only force acting on the satellite is from the rotating gravity field of the small body.

A. Body-Fixed Frame Analysis

The dynamical properties of motion in such a system are studied by transforming to a body-fixed frame, which removes the attitude matrix \( C \) from the equation but introduces the angular velocity vector of the small body \( \omega \). A usual assumption is that the body follows torque-free rotation, which is generally accurate over time spans of interest to a space mission. The equations of motion are then

\[
\ddot{q} + \omega \times q + 2\omega \times \dot{q} + \omega \times \omega \times q = \frac{\partial U}{\partial q} \tag{5}
\]

where \( q \) denotes the satellite position in the rotating, body-fixed frame and all time derivatives are taken with respect to this rotating frame (leading to the inclusion of Coriolis and centripetal accelerations). There are two general cases that occur for these systems. More generally is that the small body is in an arbitrary rotation state, tumbling in inertial space and following the torque-free solution. Then the angular velocity vector is time-periodic in the body-frame and the term \( \dot{\omega} \neq 0 \). Dynamics in such a case have been explicitly studied for the tumbling asteroid Toutatis,\(^{14}\) and some representative orbits are shown in Figs. 5 and 6.

The more usual case is for the small body to be uniformly rotating about its maximum moment of inertia. Then the angular velocity vector is a constant, \( \dot{\omega} = 0 \), and the equations of motion in the body-frame become time-invariant. In this case the dynamical system has a Jacobi integral which is conserved, expressed as

\[
J = \frac{1}{2} v^2 + \frac{1}{2} (\omega \times r) \cdot (\omega \times r) - U \tag{6}
\]

There are further implications for the time-invariance of these equations of motion beyond the existence of a Jacobi integral. First, the existence of this integral allows the use of zero-velocity surfaces to be defined and used to establish stability of motion. Second, this implies that equilibrium points can exist and that periodic orbits are dense in the phase space of this problem and can be used to study the phase space structure of this system.

B. Equilibria and Periodic Orbits

The computation and study of special solutions to the equations of motion can form a strong basis for understanding the stability of motion in the small body orbiter problem.
When a periodic orbit or equilibrium point is found, there are well-defined methods for determining the stability of motion in the vicinity of these solutions. The stability of these solutions generically applies to the neighborhood of that point in phase space as well, and thus informs whether motion will remain in the vicinity of an orbit for some time period or whether it will rapidly depart on an unstable trajectory.

For the more general case when the asteroid is tumbling, periodic orbits can only have periods commensurate with the time-periodic angular velocity period in the body-fixed frame. Due to this equilibria are not possible, and periodic orbits in general are "isolated" in phase space, only existing when specific resonances between these orbit period (in the body-fixed frame) and the angular velocity period exist. Figure 6 shows some representative periodic orbits for such a case.

When the body is uniformly rotating it is possible to define equilibrium points and families of periodic orbits. The asteroid Eros, for example, has been extensively analyzed in terms of the periodic orbit structure about that body.\textsuperscript{10} \textsuperscript{15} Indeed, the orbits shown in Fig. 1 can in part be explained with this analysis. The stable orbit presented actually is in the vicinity of the "closest, stable direct periodic orbit" about that body. Thus, by starting an orbit in its vicinity (where the usual circular speed happens to be close to the true periodic orbit speed at that orientation), one can have some confidence that the resulting motion may stay in the vicinity of that periodic orbit. For this case, motion close to a strongly distended gravitational field, the radius and velocity of an orbiter is found to vary significantly as a function of its phase angle with respect to the mass distribution. Thus, by shifting the phase angle by 45\textdegree but keeping the speed constant, the initial conditions are moved away from the relatively narrow stability zone about the stable periodic orbit family members at that distance from the asteroid. Thus, the resultant motion is highly unstable and the orbiter escapes within a few orbits.

The computation of equilibrium points (i.e., 1:1 resonant orbits) also allows for such analyses to be made. It is significant to note, however, that the majority of asteroids have only unstable equilibria in their vicinity.\textsuperscript{16} These relative equilibria generally lie near the body’s equatorial plane (the plane perpendicular to the body’s axis of maximum moment of inertia), with the number of relative equilibria being controlled by the shape of the body. Most small bodies have just four relative equilibria, however cases have been analyzed that have several additional equilibria. As an example of this, Fig. 7 shows a pole-down view of the asteroid Betulia, which has a triangular shape when viewed from this direction.\textsuperscript{17} This body has 6 relative equilibrium points, all near its equatorial plane.

For a given shape and spin rate, the distance of these points relative to the asteroid is a function of the body density (which cannot be measured remotely in most cases). For larger densities the equilibrium points move further from the body and the possibility for at most half of the equilibrium points to become stable occurs. As the density decreases these points all move towards the central body and in general all become unstable.

Generally speaking, the placement of a satellite in or near any of these relative equilibria is not a feasible mission design. First, most of these points are unstable for small bodies of interest. Further, the timescale of their instability tends to be a factor of a few faster than the asteroid’s rotation period. Thus at Eros, for example, which has a 5.27 hour period, the characteristic instability time of its four equilibrium points ranges from 40 to 100 minutes.\textsuperscript{10} Such a rapid divergence of a satellite from its nominal location would be extremely challenging to control from the ground. Similar results are found for almost all small bodies studied to date.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{betulia.png}
\caption{Pole-down view of asteroid Betulia and its 6 equilibrium points.\textsuperscript{17}}
\end{figure}

C. Zero-Velocity Surfaces

For uniformly rotating bodies the existence of the Jacobi integral allows for zero-velocity surfaces to be defined and used to design trajectories that cannot impact with the central body, of significant interest for close-proximity orbit designs. If the z-axis is aligned with the angular velocity vector, then the zero-velocity surfaces can be stated in an implicit form as

$$\frac{1}{2} \omega^2 (x^2 + y^2) + U(x, y, z) \geq -C$$  \hspace{1cm} (7)

where $C = J(x, y, z, \dot{x}, \dot{y}, \dot{z})$ is the value of the Jacobi constant. The Jacobi integral can also be stated in terms of a Tisserand-condition like form in terms of the osculating orbit elements of periapsis radius, eccentricity and inclination

$$\frac{\mu(1-e)}{2r_p} - \omega \sqrt{\mu r_p (1 + e) \cos(i) - U(r_p, \hat{r})} = J$$  \hspace{1cm} (8)

where the gravitational force potential is evaluated at a position vector interpreted as periapsis, and $r_p = a(1 - e)$. This allows the osculating orbit elements to be directly related to the Jacobi constant.
Similar to the restricted 3-body problem, the applicability of the zero-velocity surfaces as sharp constraints for motion degrades as the inclination increases, thus this is most applicable for direct motion (i.e., for inclinations close to 0°). Using these surfaces it is possible to delimit relative locations and speeds that ensure that a spacecraft will not be able to impact with the asteroid surface. This can be very useful, however for small NEA the inclusion of solar radiation pressure effects generally invalidates this conservation, and renders these conclusions not-valid.

D. Analytical Constraints

Finally, it is possible to place sharp analytical limits and predictions for long-term motion in the gravity dominated regime through the application of traditional celestial mechanics techniques of averaging, and other related approaches. Using these techniques at small bodies presents a challenge for these approaches as the perturbation gravity coefficients are generally much larger than have been practically dealt with for precision analytical theories of motion. Thus, inclusion of higher order terms may still not yield a fully convergent solution, taking away some of the motivation for the development of such higher-order theories. Instead, relying on first order estimates of these results can often yield results of sufficient precision to enable the first round of mission design, with iterations relying on the use of detailed simulations. Recall the simple formula for the precession of an orbiter’s longitude of the ascending node and argument of periapsis

\[
\Omega = \frac{3nC_{20}R^2}{2p^2} \cos i
\]

\[
\omega = \frac{3nC_{20}R^2}{2p^2} \left( \frac{5}{2} \sin^2 i - 2 \right)
\]

As a simple way to compare the orbital environment about a small body with an Earth orbiter, compare the magnitude of \(\frac{3n|C_{20}|R^2}{2a^2}\) of the NEAR spacecraft about Eros with those of an Earth orbiter. Making this comparison at a similar distance from each body (as measured in mean radius) shows that the precession rate about Eros is over 200 times faster than an Earth orbiter’s precession rate.

A more serious issue for analytical theories for orbiters is the strong influence that the \(C_{22}\) gravity coefficient has on the orbital dynamics about a strongly-distended small body. For Earth orbiters, this is a very small perturbation and its effect generally averages out over time (except for orbits in a 1:1 resonance). At small bodies this gravity coefficient is extremely important and is what causes most of the observed chaotic motion in these systems. In particular, in Fig. 1 both the impacting and the escaping trajectories occur so rapidly because of the interaction of the satellite with Eros’ \(C_{22}\) gravity coefficient. The challenge is that averaging procedures no longer work for such time-varying components of a gravity field, as the resultant change in an orbit depends sensitively on initial conditions. Taking a different analytic approach to this problem, it is possible to develop explicit predictions on the expected change in an orbit’s state over one interaction with such a rotating gravity field.\(^{18}\)

Ignoring terms that can be demonstrated to be small, it is possible to predict the change in an objects energy and angular momentum over one orbit about a system as

\[
\Delta E = -6\omega C_{22}R^2 \sqrt{\frac{\mu}{p^2}} \cos^4 (i/2) \sin 2(\theta) I_2^2(r_p,e) 
\]

\[
\Delta G = -6C_{22}R^2 \sqrt{\frac{\mu}{p^2}} \cos^4 (i/2) \sin 2(\theta) I_2^2(r_p,e)
\]

\[
I_2^2 = 2 \int_0^\pi (1 + e \cos f) \cos(2f - 2\omega t)df
\]

where \(E\) is the Keplerian energy, \(G\) is the angular momentum, \(\omega\) is the asteroid rotation rate, \(C_{22}R^2\) is the gravity coefficient times the normalizing radius squared, \(\mu\) is the asteroid gravitational parameter, \(p\) is the orbit parameter, \(i\) is the inclination from the equatorial plane, \(\theta\) is the longitude that the periapsis makes in the body-fixed frame, measured from the long-end of the asteroid. The integral \(I_2^2\) is a Hanson Coefficient,\(^{19}\) in the integral \(t\) is time and \(f\) is true anomaly. The integral can be expressed as a function of periapsis radius \(r_p\) and orbit eccentricity \(e\), and is shown in Fig. 8.

**Fig. 8:** Integral \(I_2^2\) as a function of normalized radius of periapsis and eccentricity. Note that \(|I_2^2| \leq \pi.\)

From these functional relations it can be noted that orbit-to-orbit change in a trajectory depends on where in the body-fixed frame periapsis passage occurs. When it lies in the first and third quadrants (i.e., over the leading edges of the rotating body) the energy and angular momentum are both decreased, while when over the trailing edges (the second and fourth quadrants) the energy and angular momentum are boosted. These changes are strong enough to change an orbit’s energy from negative (bound) to positive (unbound) over one passage (c.f., Fig. 1).

These results also suggest a mission design strategy. As the inclination increases (up to retrograde, or 180° in the limit) these effects become diminished. There are other terms that contribute to changes in the energy and angular momentum as the orbit becomes retrograde, however they tend to have
a much smaller magnitude. Thus, an effective strategy for minimizing the perturbations from these terms is to place spacecraft in retrograde orbits about the body, which is precisely the strategy taken by the NEAR mission at Eros.

VI. SOLAR DOMINATED REGIME

Now consider orbital mechanics about small bodies when the sun is the dominant source of perturbations. There are two primary effects from the sun that influence motion, its gravitational attraction and the solar radiation pressure (SRP) that acts on an orbiter. Except for objects with very low area to mass ratios, SRP is usually dominant over tides for spacecraft orbit dynamics. There are two items of interest when dealing with solar perturbations. First is under what conditions an orbiter will be able to stay bound to a small body in the presence of these additional forces. Second is how their orbital dynamics will evolve over time due to these perturbations. The methodology for answering these questions are quite different, and each is reviewed below. The assumption for this section is that the central body is spherical and modeled as a point mass. In later sections this assumption is relaxed.

A. Escape Limits

Two different approaches have been taken in the literature to establish limits for orbital motion about a spherical body in orbit about the sun and subject to solar radiation pressure (SRP). Using a non-rotating model that does not include the solar gravitational attraction, Dankowicz established a conservative maximum limit for when a satellite would escape from its orbit about a central body. This was expressed in terms of orbit semi-major axis by Scheeres and gives an upper limit beyond which a spacecraft will escape

\[ a_{Max} = \frac{\sqrt{3}}{4} \sqrt{\frac{\mu}{\beta d}} \]  

(14)

Using a different approach, with a more realistic model incorporating the elliptic motion of the small body about the sun and the solar gravitational attraction, Scheeres and Marzari derived an exact necessary condition for escape from a spherical body in orbit about the sun. The full criterion is complex, but it can be simplified under a few assumptions to yield a similar form to the Danckowicz bound. It is a sufficient condition for stability, and thus for a spacecraft to be definitely bound in orbit about a small body the semi-major axis should be less than

\[ a < \frac{1}{4} \sqrt{\frac{\mu}{\beta d}} \]  

(15)

and differs from the other result by a factor of \( \sqrt{3} \). The true limit depends on a number of additional parameters, and this sufficiency condition has been validated with numerical simulations and shown to be sharp for some orbit geometries.

Note that these limits explicitly predict that as a small body moves on an elliptical orbit about the sun that it is possible for a previously stable orbiter to escape from the body, as \( d \) will decrease as perihelion is approached. In Fig. 2 this is explicitly shown, as the orbit is initially bound to the body but abruptly escapes once the small body’s distance from the sun passes a given limit. Similarly, in Fig. 3 the initial orbit semi-major axis is chosen to just barely violate the appropriate stability limit, leading to immediate escape for the larger orbit and to a bound orbit for the closer one (which eventually impacts). These bounds serve as a crucial design tool for developing mission plans about smaller bodies.

B. Secular Orbital Evolution

Using the above limits to ensure bounded orbits about the central body, it becomes possible to perform an averaging analysis to extract the averaged equations of motion for the orbit constants of a satellite. These can then be solved to develop specific predictions on the secular evolution of orbits subject to SRP. The first order average of the SRP potential yields a particularly simple result

\[ \mathcal{R}_{SRP} = \frac{1}{2\pi} \int_{0}^{2\pi} \mathcal{R}_{SRP} d\mathcal{M} \]  

(16)

\[ = \frac{3}{2} \beta \frac{a}{d^2} e \cdot e \]  

(17)

where the average is over one orbit of the satellite about the small body and \( e \) is the eccentricity vector. This problem has been studied in the past, with Hénon and Mignard first noting that the averaged equations can be solved in closed form for a non-rotating SRP force (i.e., a body not moving relative to the sun), and Scheeres developed this solution and generalize it to elliptic motion about the sun. Richter and Keller developed a simpler approach using the Milankovitch orbit elements, and Scheeres explored several aspects of its solution that are relevant for mission design, reviewed here.

The solution procedure adopted by Richter and Keller uses the so-called Milankovitch Orbital elements of eccentricity vector and the scaled angular momentum vector. Since the semi-major axis is conserved for the secular potential defined above, the usual angular momentum vector is scaled by \( \sqrt{\mu a} \) or \( r \times V / \sqrt{\mu a} = h \). Then the magnitude of \( h \) is \( \sqrt{1 - e^2} \) and the scaled angular momentum and eccentricity vectors satisfy the identities \( e \cdot h = 0 \) and \( e \cdot e + h \cdot h = 1 \). With these definitions, the eccentricity and scaled angular momentum vector of a small body orbiter subject to solar radiation pressure varying with the elliptic orbit of the small body about the sun can be solved in closed form. When transformed into a rotating frame with the small body’s heliocentric true anomaly as the independent variable the equations are reduced to a time-invariant linear system, and thus can be solved in closed form. This solution is expressed as

\[ \begin{bmatrix} e \\ h \end{bmatrix} = \Phi(\psi) \begin{bmatrix} e_0 \\ h_0 \end{bmatrix} \]  

(18)

where the “0” subscript denotes an initial condition. The matrix \( \Phi(\psi) \) is an orthonormal \( 6 \times 6 \) matrix and has components

\[ \Phi(\psi) = \cos(\psi) I_{6 \times 6} + (1 - \cos(\psi)) \times \]  

(19)
where $\psi = f / \cos \Lambda$, $\hat{z}$ is the axis about which the asteroid revolves about the sun (perpendicular to $\hat{d}$), two multiplied vectors is a dyad, and $\hat{z}$ signifies the skew-symmetric cross-product tensor. The parameter $\Lambda$ is defined as a function of the asteroid mass, orbit and satellite’s orbit and SRP parameter as

$$\tan \Lambda = \frac{3 \beta}{2} \sqrt{\frac{a}{\mu_{\text{sun}} a_s (1 - e_s^2)}}$$

(20)

where $\mu_{\text{sun}}$ is the sun’s gravitational parameter and $a_s$ and $e_s$ are the asteroid’s heliocentric semi-major axis and eccentricity. The parameter $\Lambda$ is a constant and is well defined for any asteroid and satellite in orbit about it. As the SRP perturbation becomes large $\Lambda \rightarrow \pi/2$, while $\Lambda \rightarrow 0$ for a weak SRP perturbation. The NEAR spacecraft at Eros had a small value for this parameter, while the Hayabusa spacecraft at Itokawa and the Rosetta spacecraft at comet 67P/CG will have large values greater than 45°. Note that the solution is periodic in $\psi$, and that over one asteroid year the SRP solution advances $2\pi / \cos \Lambda$ times. Thus for a strongly perturbed system this solution will repeat frequently, and for a weakly perturbed system it will repeat approximately once per heliocentric orbit. Note that the solution is expressed relative to a frame rotating with the sun about the small body and incorporates the effect of varying SRP strength with distance.

Despite its simple form, the solutions for eccentricity, inclination, longitude of the ascending node and argument of perihelion are quite complex and change drastically as a function of their initial conditions and parameter $\Lambda$. Two general cases are discussed. First, if the angular momentum vector of the satellite is parallel to the vector $\hat{z}$ (i.e., the orbit is in the ecliptic) and the orbit is initially circular the evolution of eccentricity will follow the equation

$$e(\psi) = 2 \sin \Lambda |\sin(\psi/2)| \sqrt{1 - \sin^2 \Lambda \sin^2(\psi/2)}$$

(21)

The evolution of the eccentricity as a function of $\psi$ is shown in Fig. 9 for a range of $\Lambda$. Note the complex behavior and that the maximum value of eccentricity goes to unity when $\Lambda \geq 45^\circ$. This result precisely explains the bound orbit that impacts shown in Fig. 3. Even though an orbit’s eccentricity goes through unity (i.e., its perihelion radius goes to zero) it does not mean that it will immediately impact, as perihelion may increase above the asteroid surface by the time the satellite passes through periapsis again. However, if the eccentricity repeated goes through unity (as in the example) it becomes likely that it will eventually impact. The most important aspect of this solution, however, is that it admits constant orbital elements for special initial conditions. Specifically, if the angular momentum is initially directed towards or away from the sun, the eccentricity vector directed above or below the ecliptic plane (respectively), and the eccentricity chosen to equal $\cos \Lambda$, then the orbit remains constant on average. As described, these orbits lie in the terminator plane of the orbit, although they are slightly displaced away from the sun. Thus, these orbits automatically track the sun, due to SRP torques acting on the orbit angular momentum. Also significant, the frozen eccentricity becomes more circular as the orbit perturbation becomes stronger, and thus these orbits are well defined for very small bodies, so long as the orbit remains bounded. Finally, the orbit remains frozen even as the asteroid travels through perihelion and aphelion, due to the balance between true anomaly rate of change and variation in SRP, as both vary as $1/d^2$. Figure 10 shows a frozen terminator orbit about asteroid 1989ML numerically simulated over a full asteroid year, incorporating the full elliptic motion of that body about the sun. These orbits serve as the nominal choice for any orbital mission to a small asteroid, as almost all other orbits

Fig. 9: Time histories of eccentricity for a range of perturbation strengths.\textsuperscript{11}

Fig. 10: Frozen terminator orbit propagated in a numerical simulation over a full asteroid year.\textsuperscript{11}
about these bodies will suffer large variations in eccentricity and the other orbit elements. In a recent paper, Shupe and Scheeres probe the minimum asteroid size for when such orbits remain feasible. For an Orion-class spacecraft they were able to find a feasible range of orbits about a body as small as ten meters across, which begins to encroach on the target sizes identified for the ARM mission.

**VI. Mixed Results**

The above analyses are each idealized in that they neglect the effect of the other perturbation. For real systems, however, both gravity and SRP perturbations are present and can provide real limitations on the mission design results discussed above. Analyzing both gravity and SRP perturbations jointly is difficult, and only limited analytical results have been found. In one set of analyses it was shown that despite the joint effects of gravity and SRP it would have been feasible for the Japanese Hayabusa spacecraft to orbit about the asteroid Itokawa. Thus, the mission design principles outlined here can still be applied and used for the initial design of a close proximity orbit mission at a small body. In such an analysis is performed for the Rosetta spacecraft at Comet 67P/Churyumov-Gerasimenko, using previously published shape models of that body.

**VIII. Controlled Hovering Motion at an Asteroid**

Having reviewed the dynamics of uncontrolled motion about small asteroids, we now review previous work that has been focused on the controlled motion about these bodies, specifically the application of control maneuvers to enact hovering about a small body. This technique was applied during the Hayabusa mission to asteroid Itokawa, where the spacecraft maintained its location on the sun-side of the asteroid by performing occasional thrusting maneuvers. For a small body this is a feasible set of operations as the necessary propellant to maintain a given distance can be quite small. This chapter draws from a number of different analyses of such motion. Very recently, additional work has been performed on these topics by Profs. Eric Butcher and Amit Sanyal, both in collaboration and in isolation. These recent results are also not discussed here, but are referenced in the following.

**A. Motivation**

Performing scientific explorations of small bodies such as asteroids and comets can be simplified in many cases by abandoning an “orbital” approach in favor of a “hovering” approach in which the spacecraft thruts continuously, near-continuously, or sporadically to null out gravitational and rotational accelerations, either fixing its position in a body-fixed frame or in a heliocentric orbit-fixed frame. Such approaches to small body exploration make it possible to obtain high resolution measurements, and even samples, from multiple sites over the body surface without having to make complicated transitions from orbital to body-fixed trajectories between each near-surface observation period.

The Hayabusa mission to Itokawa implemented a mixture of these approaches, maintaining an orbit-fixed location when far from the asteroid and transitioning into a body-fixed frame during its descents to the asteroid surface.

A variant of such hovering is to perform a sequence of slow hyperbolic flybys of the target body, with \( V_{\infty} \) on the order of centimeters to tens of centimeters per seconds, ending each flyby with a small maneuver to turn the trajectory around for the next flyby. Such an approach can also enable the gravity field of the body to be determined, which may be more difficult if the spacecraft is in a fixed location and frequently thrusting (which was the case for the Hayabusa spacecraft).

The application of such approaches are an attractive approach for asteroid capture or boulder extraction missions, as at a terminal phase of these activities it will always be necessary to assume a fixed position and orientation with respect to the NEA. Thus, building in a hovering approach for all close-proximity trajectories can provide a single, unified mission plan for carrying out the full range of operations associated with such missions. The following discussion is borrowed, in part, from.

**B. Hovering Approaches**

The main issue that arises with a hovering approach is that the artificial equilibrium point that the spacecraft creates in the relevant frame of reference is almost always unstable, and thus must always be implemented with the navigation of this hovering solution in mind. In the following we review some simple results that can be developed for a hovering analysis, and discuss procedures for the stabilization of such hovering trajectories.

There are two general approaches to controlled motion: near-inertial hovering and body-fixed hovering. In near-inertial hovering the spacecraft is stationed at a fixed location relative to the asteroid in the sun-asteroid frame, the asteroid rotating beneath the spacecraft. This is the hovering mode implemented by Hayabusa during most of its mission. In body-fixed hovering the spacecraft is stationed at a fixed location relative to the rotating asteroid, implying that the spacecraft is rotating with the asteroid in inertial space. This mode is essential for sampling a small-body surface, as at some point the spacecraft must control its motion in the asteroid-fixed frame. Again, the Hayabusa mission implemented body-fixed hovering in a dynamic sense during its sampling runs close to the surface. Both of these ideas, and their generalizations, are discussed in more detail below.

For the implementation of either approach to be feasible some minimal level of sensing capability is needed on-board the spacecraft. First is the ability to directly sense altitude, either using a laser altimeter or by the efficient processing of stereoscopic optical measurements. This measurement type forms the backbone of an automatic control system to maintain altitude and position relative to an asteroid. In addition to this, it is ideal for the vehicle to have the ability to sense its location relative to the asteroid surface. This can be implemented by optical sensors or scanning.
lasers, both of these technologies are in different stages of development. These are not the only types of measurements available or useful, however they are the most essential. The efficient measurement of altitude allows for the implementation of automatic control algorithms that stabilize the spacecraft hovering position, while measurements of body-relative location allows for an expanded capability for the control and motion of the spacecraft. For the latter case, the spacecraft must be able to correlate measured features with a global topography map in order to locate its current location. For some specific applications it may only be necessary to measure and detect lateral motion in addition to vertical motion, however for the most general applications the ability to determine its global location on the asteroid is necessary. This implies that a global map of the asteroid surface has been created at some earlier point, ideally using the same instruments to be used for the relative navigation. The development and implementation of such sensor systems is a technology that is currently being developed, and is available for use in the future.

In addition to the above sensing and estimation capability, the spacecraft will also require precise 6-DOF control capability. This implies a full set of thrusters for executing arbitrary control moves, perhaps augmented by momentum wheels for fine attitude control. It may be feasible to use more restrictive thruster configurations for the control of the spacecraft, although these would have to be carefully designed for specific implementation approaches. Finally, some, but not all, of these active control approaches imply that the vehicle may be out of sun-light for considerable periods of time. Thus, such power considerations should be factored into the development and design of space vehicles for these advanced approaches.

C. Near-Inertial Hovering

In this approach the spacecraft fixes its location relative to the body in the rotating body-sun frame, creating an artificial libration point in this frame. A useful way to think about this approach is to first consider the sun-asteroid libration point. A spacecraft placed in this location will, ideally, remain fixed in its position. If, however, the spacecraft adds a constant thrust acceleration away from the asteroid, it would have to move its location closer to the asteroid in order for the forces to balance again. If a sufficiently large acceleration is added, it could conceivably hold its position relatively close to the asteroid. If close to the asteroid, it would have to supply an acceleration of $\sim \mu/r^2$ to “hover” at a radius of $r$ from the attracting asteroid.

From this relationship an approximate measure of the propellant cost to hover can be developed. Assuming a spherical asteroid of density $\rho$ and radius $R$ the control acceleration needed to null the gravity at a given distance $r$ is

$$u \sim \frac{4\pi}{3} G \rho R \left( \frac{R}{r} \right)^2$$

$$\sim 24 \rho \left( \frac{R}{r} \right)^2 \text{ m/s/day}$$

where the units have been converted to meters per second of $\Delta V$ per day and $R$ is in km. Thus, assuming a density of 2 g/cm$^3$, hovering at the surface of a 0.1 km body costs 4.8 m/s/day while hovering at the surface of a 10 km body costs 480 m/s/day – the former being reasonable and the latter being unreasonable for extended hovering.

Considering the more general case, it is possible to specify the necessary control acceleration to maintain position at an arbitrary location in the asteroid-sun rotating frame. Due to the relatively slow motion of the body about the sun (on the order of degrees per day at fastest), this position can be considered to be nearly inertial over relatively short periods of time. Thus this discussion will assume that the spacecraft wishes to fix itself in an inertially oriented frame. Assuming that the spacecraft is attracted to the rotating asteroid and also has some thrusting capability, represented as a control acceleration $u$, the full equations of motion are

$$\ddot{r} = \frac{\partial U}{\partial r} + u$$

where $r$ is the position of the spacecraft relative to the asteroid and is specified relative to an inertially fixed frame, $U(C^I \cdot r) = \mu/r^2 + R(C_0^I \cdot r)$ is the gravitational field, $C^I$ is the transformation dyad that takes the asteroid-fixed frame into the inertial frame, and $R$ is the perturbing, non-spherical component of the gravity field. Simply put, open-loop inertial hovering is enacted by choosing the control acceleration to balance against the local attraction at the desired hovering location $r^*$. For hovering above a uniformly rotating non-spherical asteroid the control law is time-varying and periodic with the rotation of the asteroid. If the central body is tumbling then the control acceleration will not repeat in general, as a tumbling body will generally not present the same orientation in inertial space twice.

Although this hovering specification is simplistic, as it is not necessary to continuously throttle the propulsion system to hover, it is a useful point from which to consider the stability of this approach. Specifically, consider the effect of a small error in positioning of the spacecraft – this could either be due to navigation uncertainties in the precise placement of the spacecraft or could be due to an error in the propulsion system, meaning that the current control law is actually the hovering law for a neighboring spacecraft location. Linearizing about $r = r^* + \delta r$ and assuming that the control is exactly canceling the gravitational attraction at

$$u = -\frac{\partial U}{\partial r^*}$$
r* yields
\[ \delta \ddot{r} = \left. \frac{\partial^2 U}{\partial r^2} \right|_{\star} \cdot \delta r \] (26)
\[ \left. \frac{\partial^2 U}{\partial r^2} \right|_{\star} = -\frac{\mu}{r^3} \left[ U - 3r^* \dot{r}^* \right] + \left. \frac{\partial^2 R}{\partial r^2} \right|_{\star} \] (27)

where U is the unity dyadic, defined such that \( a \cdot U = U \cdot a = a \) for all vectors \( a \). The stability of this system for a uniformly rotating body can be evaluated using Floquet’s Theorem, and for complex rotation would require the computation of Lyapunov characteristic exponents. These more detailed issues are considered in,\(^{30}\) while for current purposes It is more instructive to ignore the gravitational perturbation component \( R \) and instead just focus on the point-mass attraction term. This is often justified as inertial hovering will generally occur relatively far from the central body where the gravitational perturbations are small and could be ignored.

When considering the point mass term the stability is tractable as the dynamics matrix \( U_{\text{ff}} \) is time invariant, and its eigenvalues and eigenvectors can be easily identified. There exists one eigenvector along the \( \dot{r}^\perp \) vector, and dotting this unit vector with the dynamics matrix yields
\[ \frac{2\mu}{r^3} \dot{r}^\perp = -\frac{\mu}{r^3} \left[ U - 3r^* \dot{r}^* \right] \cdot \dot{r}^\perp \] (28)

Note that the eigenvalue is \( \pm \sqrt{2\mu} \), and thus has an exponentially stable and unstable component. The other eigenvector is found by dotting the dynamics matrix with any vector perpendicular to \( \dot{r}^* \), denoted as \( \dot{r}^\perp \), yielding
\[ -\frac{\mu}{r^3} \dot{r}^\perp = -\frac{\mu}{r^3} \left[ U - 3r^* \dot{r}^* \right] \cdot \dot{r}^\perp \] (29)

In this case the eigenvalues along this direction are imaginary and equal to \( \pm i \sqrt{2\mu} \). Thus the 3-DOF system can be reduced to a 1-DOF system along \( r^* \) which has a stable and unstable component and a 2-DOF system consisting of two uncoupled harmonic oscillators perpendicular to the hovering position vector. The oscillation period of the motion in the perpendicular direction has period equal to the orbit period of a circular orbit with semi-major axis equal to the hovering radius, \( 2\pi r^{*+2/3}/\sqrt{\mu} \). The characteristic time of the hyperbolic manifolds equals \( r^{*+2/3}/\sqrt{\mu} \). In the radial direction this leads to motion of the form
\[ \delta r = \alpha_+ e^{\sqrt{2\mu} t} + \alpha_- e^{-\sqrt{2\mu} t} \] (30)

Thus any small disturbance in this direction will grow exponentially with a characteristic time that is only a function of body density and \( r^*/R \). As this is the dominant term in the stability analysis it implies that hovering must always be carried out with control along the radial, hovering direction. Thus, practical implementation requires the addition of a closed-loop feedback control that senses the altitude or distance deviation of the spacecraft from its ideal hovering point. The necessary control loop to stabilize this motion is actually quite simple, and can be implemented in an automatic way using minimal spacecraft resources.\(^{6}\) There are limits to this approach, however. A spacecraft cannot inertially hover within the maximum radius of the asteroid at its hovering latitude, due to obvious physical constraints. Additionally, as the radius of hovering becomes closer to the body, the automatic control approach described here can become unstable due to the neglected higher order terms, potentially leading to difficulties in implementation.

It is not necessary, however, to force the spacecraft to be fixed precisely at one location. A generalization of this idea places the spacecraft in an elliptic or hyperbolic orbit relative to the asteroid, and has its velocity vector “reflected” whenever it gets within a certain distance to the asteroid (for an elliptic orbit) or gets a certain distance away from the asteroid (for a hyperbolic orbit), forcing the spacecraft to travel back on, or close to, its original path but in the opposite direction. Figure 11 shows this graphically, while Fig. 12 presents the trajectory of a spacecraft hovering relative to an asteroid in an asteroid-sun rotating frame (this hovering point is away from the sub-solar point and below the equatorial plane, as seen in the graphics, and includes the attraction of the asteroid, the attraction of the sun, and the solar radiation pressure). Despite the simplicity of this control law – providing a fixed \( \Delta V \) every time a distance to the asteroid is crossed, it can maintain the spacecraft within a fixed control volume over an arbitrarily long time-span. This approach can be thought of as hovering with a relatively large dead-band control about the nominal hovering point, and requires essentially the same control and sensing capability on-board the spacecraft. This is essentially the approach used by the Hayabusa spacecraft during its mission at Itokawa.\(^{37}\) In this approach the time between control maneuvers can be made arbitrarily long by increasing the size of the dead-band box away from the asteroid. The Lyapunov stability of this approach to hovering was investigated in detail in\(^{30}\) and found to be robust. The use of hyperbolic-flyby only interactions with a small body has been proposed for a number of small body missions, although it has yet to be fully implemented.\(^{7}\)

Inertial hovering, or its above variations, has several attractive attributes which may make it a mainstay approach for future exploration. There are also a number of drawbacks and limitations, however. On the positive side, this approach can be applied to any small body, and the cost of inertial hovering can theoretically always be driven to zero by hovering at a high enough altitude (not accounting for the statistical control to stabilize the hovering point). However, the position where hovering is feasible may be far from the body, and may not afford the optimal viewing geometry. For example, if the NEAR-Shoemaker spacecraft had taken a hovering approach to its mission to Eros and implemented inertial hovering at a distance of 50 km from the asteroid (which was the nominal orbit radius for most of the mission), it would have required over 15 m/s per day to maintain this position, or for its prime 9 month mission would have required a total \( \Delta V \) on the order of 4 km/s. Contrasted with the actual fuel usage (on the order of a few tens of m/s), hovering was clearly not a reasonable approach for that body. Thus, to
Fig. 11: Simple inertial hovering scheme, where a constant $\Delta V$ vector is applied (relative to the sun-asteroid frame) whenever a threshold distance is crossed.

gain high resolution scientific measurements this approach is largely limited to smaller bodies with their associated smaller hovering cost.

A related drawback pertains to the ability of a spacecraft to accurately measure the mass and gravity field of the asteroid, and hence to compile an accurate understanding of the body's internal structure. When using such a controlled hovering mode, errors in the spacecraft thrusters and solar radiation pressure parameters will compete with the signature of the asteroid gravity field acting on the spacecraft. While it might be possible to extract some averaged results on the total mass of the asteroid, these results would be corrupted by many different uncertainties that will not be uniquely separated from the gravity signature. Additionally, higher order gravity fields will be nearly impossible to extract. While it might be possible to extract some averaged results on the total mass of the asteroid, these results would be corrupted by many different uncertainties that will not be uniquely separated from the gravity signature. Additionally, higher order gravity fields will be nearly impossible to extract. Again, this was the situation for the Hayabusa mission as it was only able to estimate a relatively imprecise estimate of the asteroid’s mass and was not able to determine any of its higher-order gravity field coefficients. This is a serious limitation, as it deprives the mission of essential scientific data and may make it difficult to subsequently transition to a body-fixed hovering exploration of the asteroid. Recent research has investigated the ability of a spacecraft to obtain mass and higher-order gravity field coefficients for a small body by carrying out several slow hyperbolic flybys of the body. Using this approach it is feasible to gain precise information on the gravity field up to 2nd degree and order (assuming that the shape has also been estimated) and to detect up to 4th degree and order gravity coefficients for an asteroid the size of Itokawa (with a mean radius of $\sim$ 160 meters).

D. Body-fixed Hovering

The counterpart to inertial or near-inertial hovering, with its range of implementation options, is hovering in the small body-fixed frame. In this approach the spacecraft fixes its position relative to the rotating body. A natural way to visualize this approach is to imagine using a “jet-pack” to levitate off of the surface of a rotating body, such as the Earth or an asteroid. Since the gravitational attraction is relatively weak at asteroids, it is possible to implement such hovering trajectories for extended periods of time (hours) with total costs that can be relatively modest, on the order of meters per second. This approach to controlling motion in close proximity to small bodies has been analyzed in detail and a detailed simulation of this approach has been developed for analysis of hovering over arbitrary models of asteroids. During its final descent, the Hayabusa spacecraft also enacted a form of body-fixed hovering in its sampling mission.

The implementation of this approach has similarities to inertial hovering, but now everything must be done relative to the rotating body’s coordinate system. The spacecraft must control both its position and orientation with respect to the rotating body. This requires precise measurements of the body’s rotation and the ability to apply appropriate control moments. The spacecraft must also account for the effects of the asteroid’s gravity and radiation pressure on its motion. Despite these challenges, the body-fixed hovering approach offers some potential advantages over inertial hovering.

Fig. 12: Dynamics of a fixed $\Delta V$ hovering scheme for a spacecraft situated off the sub-solar point and below the asteroid orbital plane. Projection of the trajectory into the orbital $X - Y$ plane (left) and into the $X - Z$ plane (right). The trajectory remains bounded and non-linearly stable for arbitrary periods of time.
to the asteroid-fixed frame, which generally has a rotation period on the order of hours to days at most. Thus, the spacecraft acceleration must accommodate both the gravitational and centrifugal accelerations, although there are locations where the hovering cost is zero (at the synchronous orbits). Additionally, the spacecraft must reorient its attitude in inertial space in order to maintain the same orientation relative to the asteroid surface. This body-fixed hovering approach also suffers from the same basic instability noted for the inertial hovering case, although there are regions where this approach yields completely stabilized motion.29

A similar control strategy, using altimetry to maintain a fixed altitude, can stabilize a hovering point so long as it is located within the synchronous radius of the body. This result holds approximately true over the entire body and places an altitude “ceiling” on hovering for a simple control law to be able to stabilize its location.

There are a number of drawbacks related to body-fixed hovering as well. First, it is essential that a fairly accurate model of the asteroid spin, topography and gravity field be available. The gravity field must be defined down to the surface of the body as well, something which is not always easy to do (see the detailed discussion of this in12, 41). Thus, body-fixed hovering should be proceeded by a period of characterization at a relatively high level of accuracy. In the future, it may be possible to dispense with this requirement, but that would only be after the basic technology and approach has been proven. Most importantly, however, is that a body-fixed hovering vehicle could experience periods of solar occultation, making the presence of batteries or non-solar power generation essential for long-term operations at the surface of a body. Additional operations issues also exist, such as communications, attitude determination, and the mechanical interface of its control thrust plumes with the surface.

In the following the stability of hovering in a uniformly rotating asteroid-fixed frame is considered. An analytical consideration of hovering is only investigated above a point mass in the following, although some examples of the more complex cases of hovering over non-spherical mass distributions are presented. The equations of motion for a point mass in a uniformly rotating frame can be restated from Eqn. 5 with the addition of a control acceleration

\[ \dot{q} + 2\omega \cdot \dot{q} + \omega \cdot \omega \cdot q = -\frac{\mu}{q^3} q + u \]  

To generalize this to a uniformly rotating arbitrary mass distribution replace the gravitational acceleration \(-\frac{\mu}{q^3} q\) with \(U q\) and have the angular velocity vector be aligned with the maximum moment of inertia and equal in magnitude to its rotation rate. Hovering at a specified point in the asteroid-fixed frame, \(q^*\), requires a control acceleration of \(u^* = \frac{\mu}{q^3} q^* + \omega \cdot \omega \cdot q^*\). For simplicity, assuming that hovering occurs in the equatorial plane, this simplifies to \(u^* = \left(\frac{\mu}{q^3} - \omega^2\right) q^*\). Thus the nominal hovering cost can go to zero if hovering radius equals the synchronous orbit radius. For a non-spherical mass distribution this is equivalent to placing the spacecraft at one of the libration points. These are often unstable, and have a more complex instability structure than found for the inertially hovering equilibrium points, and thus the stability properties of body-fixed hovering orbits are expected to be different than the simple structure found for inertial hovering points.

For this point mass hovering analysis denote the hovering location vector as \(q^* = q^* [\cos \theta \hat{x} + \sin \theta \hat{z}]\), where \(\theta\) is the latitude of hovering, the asteroid rotates about the \(\hat{z}\) axis, and \(q^*\) is the distance from the body center of mass. Assuming \(q = q^* + \delta q\) one can linearize the equations of motion about this point to find

\[ \delta \ddot{q} + 2\omega \hat{z} \cdot \delta \dot{q} = \left\{ -\omega^2 \hat{z} \cdot \hat{z} - \frac{\mu}{q^3} [U - 3q^* q^*] \right\} \cdot \delta q \]  

Writing out the characteristic matrix for this case, with \(\lambda\) as the test eigenvalue, yields

\[ \left[ \begin{array}{ccc} \lambda^2 - \omega^2 + \omega^2 (1 - 3 \cos^2 \theta) & -2\omega + \frac{\mu}{q^3} & -3\omega^2 \sin \theta \cos \theta \\ -2\omega \lambda & \lambda^2 - \omega^2 + \omega^2 & -3\omega^2 \sin \theta \cos \theta \\ -3\omega^2 \sin \theta \cos \theta & -3\omega^2 \sin \theta \cos \theta & \lambda^2 + \omega^2 (1 - 3 \sin^2 \theta) \end{array} \right] \]

where \(\omega^2_q = \mu/q^3\) and the explicit hovering position has been substituted. The characteristic equation for the system is a fully coupled polynomial of degree 3 in \(\lambda^2\). In Fig. 13 the different stability results are given as a function of scaled radius and \(\theta\), with different colors indicating different stability types. There are four types of stable and unstable motion found for hovering in the body-fixed frame. For all latitudes of hovering with \(\omega_q > \omega\) (i.e., hovering within the resonance radius of the asteroid) there is unstable motion with characteristics similar to inertial hovering: one stable and unstable manifold and 2 center manifolds. For this case the stable and unstable manifolds no longer line up with the radius vector in general. Transitioning to hovering locations above the resonance radius there is a larger set of hovering stability types. The red regions correspond to spiral stable and unstable manifolds and one oscillatory manifold, the white regions are fully stable, and the dark regions have two pairs of stable and unstable manifolds and one stable oscillation.

The inclusion of non-spherical mass distributions can change the distribution of stability types markedly. This is analyzed in30 where the hovering stability type is mapped over the surface of a number of different asteroids. Some specific examples of this are shown in Figs. 14 - 16, which plot the hovering cost, the stability type, and the characteristic time for a spacecraft hovering above the Eros surface at an altitude of 100 meters. A further generalization of body-fixed hovering is to model the translational motion relative to the asteroid surface. This is analyzed in5 where it is shown that motion against the rotation of the asteroid is in general more stable than motion in the same sense over the asteroid surface.

The implementation of body-fixed hovering can also be accomplished using a simple dead-band controller acting on the altitude of the spacecraft, in conjunction with a single thrust direction properly aligned relative to the asteroid gravity
A specific focus has been given on the use of controlled hovering motion to enact close-proximity operations. Such motions will require that the spacecraft have the capability to descend and ascend from the surface, additional sensing and control technology will have to be used. First, the spacecraft must maintain its attitude in the body-fixed frame – it should note that the spacecraft will not do so naturally, as its attitude will remain fixed in inertial space and will want to spin in the asteroid-fixed space. Second, for it to perform translational motions will require that the spacecraft have the capability to locate itself relative to the surface and perform some higher-level control to null out transverse oscillations about the hovering point.

The development of this surface relative motion capability is perhaps the most advanced concept tendered here. This idea also solves the problem of rover locomotion over an asteroid surface, as instead of relying on natural trajectories induced by mechanical “jumpers” it provides controlled motion from one location to another. There are a number of interesting peculiarities associated with such surface relative motion, such as the fact that there is a preferred direction of motion about an asteroid in this mode. Motion in the same direction as asteroid rotation can actually destabilize the dynamics of the spacecraft control, while translational motion in the opposite direction will tend to stabilize the control system. Other than this observation, which can be easily proven, there is little known about the stability and control of surface relative motion at small bodies, making it an essential topic for future research.

IX. Conclusion

The dynamical environment about small solar system bodies such as asteroids and comets has been reviewed. A specific focus has been given on the use of controlled hovering motion to enact close-proximity operations. Such
approaches are likely to find use for the recently proposed NASA ARM initiative. Close-proximity dynamics about these bodies serves as a challenging problem for astrodynamics, and presents real opportunities for the continued advancement of the field in pursuit of better understanding motion in these extreme environments.

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