Parameter Matching using Adaptive Synchronization of Two Chua’s Oscillators

Valentin Siderskiy\(^1\) and Vikram Kapila\(^1\)

**Abstract**—In this paper, we develop an adaptive synchronization technique for parameter matching with chaotic persistent excitation (PE). Two Chua’s oscillators, identical in every parameter except for one, are set up in a master/slave configuration where the slave’s mismatched parameter is adaptable. Using Lyapunov functions and incorporating the presence of PE, adaptive control laws are designed to ensure exact parameter matching. One of the derived adaptive controllers is experimentally validated by using an adaptive inductor-gyrator composed of current feedback op-amps (CFOAs).

I. INTRODUCTION

Leon Chua is credited with developing one of the simplest to construct chaotic circuits [17], which is now termed the Chua’s circuit. The basic properties of chaotic systems, specifically, sensitivity to initial conditions and dense topology, intuitively suggest that in the physical world a chaotic system is too sensitive and its behavior is too complex to predict or match. In non-chaotic systems such problems are frequently solved by using an array of tools from the stability and control theory. Moreover, the experiment of [21] showed that we can not only build chaotic circuits but also go beyond intuition and work with chaos, e.g., to cause one chaotic system to mimic another chaotic system by using stability and control theory. Since [21], synchronization of chaotic systems has received considerable attention from the research community. Naturally, the works of [17] and [21] led to synchronization of two Chua’s circuits [3].

Synchronization of Chua’s oscillators along with parameter adaptation has been considered since 1996 [4, 15, 20, 26]. For example, [26] used a Lyapunov-based approach to adapt parameters relating to the Chua’s diode, while [4] used an adaptive control technique introduced by [10, 12] to adapt mismatched parameters for the rest of the circuit. The theoretical results of [26] showed asymptotic stability for the error dynamics of the states of the Chua’s oscillator but not for the adaptive parameters. Despite this, the adaptive parameters were shown to asymptotically converge in simulation results and it was conjectured that “parameters match when the dynamics are very rich” [26]. These rich dynamics are commonly referred to as PE [16] and [7] has shown that adaptive synchronization of Chua’s circuit yields convergence of mismatched parameters under PE. Note that [26] and [4] gave simple controllers that were later compared in [27].

Numerous other papers have considered adaptive synchronization for Chua’s oscillators [5, 6, 8, 14, 29]. While some of these works are theoretical in nature and are difficult to realize experimentally, others may not yield exact parameter matching [1]. To render adaptive synchronization of chaotic circuits closer to physical realization, [28] has provided SPICE simulations and [11] has suggested circuit schematics.

Adaptive synchronization of Chua’s oscillators can be categorized in two parts: adapting the control coupling between the two circuits and adapting one or more parameters of a Chua’s oscillator [4]. Both adaptive synchronization approaches have been digitally implemented for secure communication applications. The first approach is used to account for changes in signal strength [30], while the second approach introduces deliberate changes in the parameters as a way to send binary messages as a “key” [4]. Ref. [4] has provided an experimental validation of synchronization with adaptive control coupling where the entire system is realized using analog circuitry. To the best of our knowledge, the present paper is the first one to provide an experimental validation of synchronization with adaptive parameter tuning for a Chua’s oscillator where the entire system is realized using analog circuitry.

The focus of this paper is to experimentally demonstrate adaptive synchronization with parameter matching. We use the Lyapunov function technique of [26] to create parameter update laws for \(L\) and \(C_2\) of Chua’s oscillator in Figure 1. The derived result for tuning \(L\) is realized with analog circuit components and \(L\) is continuously updated by using a voltage-controlled inductor-gyrator made up of CFOAs.

II. SYSTEM MODEL

A. Chua’s Oscillator

In this paper, adaptive controllers are designed to tune parameters of a Chua’s oscillator shown in Figure 1. Various parameters of a Chua’s oscillator include \(L\) as a linear inductor, \(R\) and \(R_0\) as linear resistors, \(C_1\) and \(C_2\) as linear capacitors, and others that correspond to the Chua’s diode. The state equations of the Chua’s oscillator are given by

\[
\frac{dv_1}{dt} = \frac{1}{C_1} \left( G(v_2 - v_1) - g(v_1) \right),
\]

\[
\frac{dv_2}{dt} = \frac{1}{C_2} \left( G(v_1 - v_2) + i_L \right),
\]

\[
\frac{di_L}{dt} = \frac{1}{L} (-v_2 - R_0 i_L),
\]

where \(v_1, v_2,\) and \(i_L\) are voltage across \(C_1\), voltage across \(C_2\), and current through \(L\), respectively, and \(G\) is the con-
ductance of the resistor $R$ ($G \equiv \frac{1}{R}$). Furthermore $g(\cdot)$ is the nonlinear voltage-current $(v-i)$ characteristic of the Chua’s diode described by

$$g(v_R) = \begin{cases} G_0 v_R + (G_b - G_a) E_1, & \text{if } v_R \leq -E_1 \\ G_d v_R, & \text{if } |v_R| < E_1 \\ G_0 v_R + (G_a - G_b) E_1, & \text{if } v_R \geq E_1 \end{cases} \tag{2}$$

where $G_a$, $G_b$, and $E_1$ are known real constants that satisfy $G_b < G_a < 0$ and $E_1 > 0$.

### B. Master/Slave System

The adaptive control framework of this paper considers a unidirectional coupling between a master Chua’s oscillator and a slave Chua’s oscillator. Specifically, the master Chua’s oscillator operates autonomously whereas the slave Chua’s oscillator synchronizes its state to the state of the master Chua’s oscillator. A depiction of this configuration is shown in Figure 1. The state equations of the master Chua’s oscillator are given by

- $\dot{v}_1 = \frac{1}{C_1} \left(G(\bar{v}_2 - \bar{v}_1) - g(\bar{v}_1) + G_{u1}(v_{u1} - \bar{v}_1)\right)$, \hspace{1cm} \text{(3a)}
- $\dot{v}_2 = \frac{1}{C_2} \left(G(v_1 - \bar{v}_2) + \bar{i}_L\right)$, \hspace{1cm} \text{(3b)}
- $\dot{i}_L = \frac{1}{L} \left(-\bar{v}_2 - R_0 \bar{i}_L\right)$, \hspace{1cm} \text{(3c)}

where $v_{u1} = v_1$ since it is the output of a voltage follower op-amp and $G_{u1}$ is the conductance of the coupling resistor $R_{u1}$ in Figure 1 ($G_{u1} \equiv \frac{1}{R_{u1}}$). Note that $C_2$ and $L$ are tunable parameters for which we provide adaptive parameter update laws in Section III.

![Sync_Circuit](image)

**Fig. 1: Master/slave Chua’s oscillator coupling.**

### III. ADAPTIVE SYNCHRONIZATION

Given two Chua’s oscillators with unidirectional coupling and one mismatched parameter, we design an adaptive controller to synchronize the two oscillators and tune the slave oscillator’s parameter to the master oscillator’s parameter. Using the ideal Chua’s oscillator model (1), two cases are considered to tune either $L$ or $C_2$.

#### A. Tuning $L$

For the slave Chua’s oscillator, when capacitance $\tilde{C}_2$ matches the master oscillator’s parameter $C_2$ and inductance $\tilde{L}$ is the tunable mismatched parameter, (3) specializes to

$$\frac{d\tilde{v}_1}{dt} = \frac{1}{C_1} \left(G(\bar{v}_2 - \bar{\tilde{v}}_1) - g(\bar{v}_1) + u_1\right) \tag{4a}$$
$$\frac{d\tilde{v}_2}{dt} = \frac{1}{C_2} \left(G(\tilde{v}_1 - \bar{v}_2) + \bar{\tilde{i}}_L\right) \tag{4b}$$
$$\frac{d\tilde{i}_L}{dt} = \frac{1}{L} \left(-\bar{\tilde{v}}_2 - R_0 \bar{\tilde{i}}_L\right) \tag{4c}$$

where $u_1 \equiv G_{u1}(v_1 - \bar{v}_1)$. Subtracting (1) from (4) produces

$$\dot{e}_1 = \frac{1}{C_1} \left(G(e_{v1} - e_{\tilde{v}1}) - c(\bar{v}_1, v_1)e_{v1} + u_1\right), \tag{5a}$$
$$\dot{e}_2 = \frac{1}{C_2} \left(G(e_{v1} - e_{\tilde{v}1}) + e_{\tilde{i}L}\right), \tag{5b}$$
$$\dot{\tilde{e}}_L = \frac{1}{L} \left(-\bar{\tilde{v}}_2 + R_0 e_{\tilde{i}L}\right), \tag{5c}$$

where $e_{v1} \equiv \bar{v}_1 - v_1$, $e_{\tilde{v}1} \equiv \bar{\tilde{v}}_1 - \tilde{v}_1$, and $e_{\tilde{i}L} \equiv \bar{\tilde{i}}_L - i_L$ are the error states. Moreover, it is easy to show that $g(\bar{v}_1) - g(v_1) = c(\bar{v}_1, v_1)e_{v1}$ where $c(\bar{v}_1, v_1)$ is bounded by the constraints $G_a \leq c(\bar{v}_1, v_1) \leq G_b < 0$ [9].

Next, let the control law for $u_1$ be characterized as

$$u_1 = -G_{u1} e_{v1} \tag{6}$$

and let the parameter update law be given by

$$\frac{d}{dt} \left(\frac{1}{L}\right) = \gamma e_{\tilde{i}L} (\bar{v}_2 + R_0 e_{\tilde{i}L}) \tag{7},$$

where $\gamma$ is a positive constant.

**Theorem 1:** Two Chua’s oscillators (1) and (4) will synchronize and the parameter $L$ will converge to some constant under the control law (6) and the parameter update law (7) if the master system (1) remains on the trajectory of its chaotic attractor and $G_{u1}$ is chosen to satisfy the following inequality

$$G_{u1} > \frac{1}{2} G - G_a. \tag{8}$$

**Proof:** Consider the candidate Lyapunov function

$$V(e_{v1}, e_{\tilde{v}1}, e_{\tilde{i}L}, e_p) = C_1 e_{v1}^2 + C_2 e_{\tilde{v}1}^2 + L e_{\tilde{i}L}^2 + \frac{L}{2} e_p^2, \tag{9}$$

where $e_p \equiv \frac{1}{L} - \frac{1}{\tilde{L}}$. By computing the Lyapunov derivative and manipulating the result, it can be shown that

$$\dot{V} < - \left(-\frac{1}{4} G + \frac{1}{2} G_a + \frac{1}{2} G_{u1}\right) e_{v1}^2 - \frac{1}{4} G e_{\tilde{v}1}^2 - R_0 e_{\tilde{i}L}^2. \tag{10}$$

Using (8) and (10), it can be shown that $e_{v1}(t), e_{\tilde{v}1}(t), e_{\tilde{i}L}(t) \in L_2$. If (1) remains on its chaotic trajectory, the states of the master system are bounded. In this case, $v_2, i_L \in L_\infty$ and it follows that $e_{v1}(t), e_{\tilde{v}1}(t), e_{\tilde{i}L}(t) \in L_\infty$. Next, by Barbalat’s lemma, for any initial condition $e_{v1}(t), e_{\tilde{v}1}(t), e_{\tilde{i}L}(t) \to 0$ as $t \to \infty$. This implies that (1) and (4) will synchronize.

**Remark 1:** Note that the results of Theorem 1 are also applicable if the Chua’s oscillator is on a periodic trajectory. As long as the attractor of the Chua’s oscillator is bounded, the results of Theorem 1 hold.
Remark 2: When the trajectories of (1) are driven on a chaotic attractor, its states will satisfy the qualities of PE as discussed in [16, 18, 22] and \( \bar{L}(t) \rightarrow L(t) \) as \( t \rightarrow \infty \).

Remark 3: In literature dealing with experimental implementation of Chua’s oscillator, \( R_0 \) is treated as a parasitic resistance and is often ignored [2, 13]. In this paper, we deliberately include \( R_0 \) to achieve our formal stability result.

B. Tuning \( \bar{C}_2 \)

For the slave Chua’s oscillator, when inductance \( \bar{L} \) matches the master oscillator’s parameter \( L \) and capacitance \( C_2 \) is the tunable mismatched parameter, (3) specializes to

\[
\begin{align*}
\frac{d\tilde{v}_1}{dt} &= \frac{1}{C_1} \left( G(\tilde{v}_2 - \tilde{v}_1) - g(\tilde{v}_1) + u_1 \right), \\
\frac{d\tilde{v}_2}{dt} &= \frac{1}{C_2} \left( G(\tilde{v}_1 - \tilde{v}_2) + i_L \right), \\
\frac{d\tilde{i}_L}{dt} &= \frac{1}{L} \left(-\tilde{v}_2 - R_0\tilde{i}_L\right). 
\end{align*}
\]

(11a)

(11b)

(11c)

Subtracting (1) from (11) produces the error dynamics

\[
\begin{align*}
\dot{e}_1 &= \frac{1}{C_1} \left( G(e_{v_2} - e_{v_1}) - c(\tilde{v}_1, v_1)e_{v_1} + u_1 \right), \\
\dot{e}_2 &= \frac{1}{C_2} \left( G(\tilde{v}_1 - \tilde{v}_2) + i_L \right) - \frac{1}{C_2} \left( G(v_1 - v_2) + i_L \right), \\
\dot{e}_L &= \frac{1}{L} \left(-e_{v_2} - R_0 e_{i_L} \right).
\end{align*}
\]

(12a)

(12b)

(12c)

Next, let the control law for \( u_1 \) be characterized as in (6) and let the parameter update law be given by

\[
\frac{d}{dt} \left( \frac{1}{C_2} \right) = -\eta e_{v_2} \left( G(\tilde{v}_1 - \tilde{v}_2) + i_L \right),
\]

(13)

where \( \eta \) is a positive constant.

**Theorem 2:** The two Chua’s oscillators (1) and (11) will synchronize and the parameter \( C_2 \) will converge to some constant under the control law (6) and the parameter update law (13) if the master system (1) remains on the trajectory of its chaotic attractor and \( G_{ni} \) is chosen to satisfy (8).

**Proof:** Theorem 2 can be proved in a similar manner as Theorem 1 by using the candidate Lyapunov function

\[
V(e_{v_1}, e_{v_2}, e_{i_L}, e_x) = \frac{C_1}{4} e_{v_1}^2 + \frac{C_2}{2} e_{v_2}^2 + \frac{L}{2} e_{i_L}^2 + \frac{C_2}{2\eta} e_x^2,
\]

(14)

where \( e_x = \frac{1}{C_2} - \frac{1}{C_2} \).

IV. TUNING \( \bar{L} \) IMPLEMENTATION

Over the years, several variations of the Chua’s oscillator have been developed [13]. Moreover, master/slave coupling between two Chua’s oscillators for state \( v_1 \) (and \( v_2 \)) is easily achievable with just one resistor and one op-amp (Figure 1). However, measuring and controlling the state \( i_L \) is not as trivial. Therefore variations of inductorless implementations of Chua’s oscillators have been developed [13]. This paper implements the adaptive controller presented in Section III-A which tunes the parameter \( \bar{L} \) to \( L \). This section describes the circuitry required for this task and it includes i) an inductorless implementation of Chua’s oscillator using an inductor-gyrator and ii) a physical realization of the adaptive parameter update law for \( \bar{L} \).

A. Inductor-Gyrator

Note that the parameter update law (7) is the rate of change for the reciprocal of the inductance \( \bar{L} \) and it requires the measurements of \( i_L, v_L \) and \( \tilde{v}_2 \). The use of CFOAs has led to Chua’s oscillator implementations with access to \( i_L \) measurements [13]. Each CFOA is a four port device in which the voltages on and currents through the various input-output terminals satisfy the following characteristics [25]

\[
i_y = 0, \quad v_x = v_y, \quad i_z = i_x, \quad \text{and} \quad v_w = v_z,
\]

(15)

with the current direction convention as shown in Figure 2(a).

![Fig. 2: (a) Current direction convention for the CFOA and (b) modified version of the inductor-gyrator of [23] with a reciprocal \( r_{DS} \) relationship to impedance.](image)

A grounded voltage-controlled impedance-gyrator is suggested by [23] using CFOAs and a JFET transistor. In this paper, we modify the impedance-gyrator suggested by [23] to that in Figure 2(b). The schematic in Figure 2(b) is drawn analogous to [23] for easy comparison and the modification is highlighted in box A. As seen below, the modification will enable us to implement the adaptive parameter update law for the reciprocal of \( L \) in (7). It can be shown that the modified gyrator has an impedance represented by

\[
\frac{Z_{am}}{Z_{am}} = \frac{R_{1m} R_{3m} Z_{2m}}{Z_{am}} \times \frac{1}{r_{DSm}(V_c)}.
\]

(16)

Since the modified gyrator is intended to be used as a voltage-controlled inductor, we choose \( R_{1m}, R_{3m}, Z_{2m} \) to be linear resistors \((Z_{2m} = R_{2m})\) and \( Z_{am} \) to be a linear capacitor \( C \left(Z_{am} = \frac{1}{C} \right)\). Now the impedance of the circuit of Figure 2(b)
can be shown to be
\[ \text{Zam}(s) = \frac{R_{1m}R_{2m}R_{3m}Cs}{r_{DSm}(Vc)}, \quad (17) \]
where \( s \) is the Laplace variable. Using (17), the characteristic differential equation for the circuit of Figure 2(b) is
\[
\frac{di_{am}}{dt} = \frac{r_{DSm}(Vc)}{R_{1m}R_{2m}R_{3m}C}V_{am}, \quad (18)
\]
which can also be obtained using the voltage-current relationships of (15). The current \( i_{am} \) through the modified gyrator is obtained from the voltage measured at node Q since
\[
V_Q = -i_{am}R_{1m}. \quad (19)
\]
The modified gyrator of Figure 2(b) replaces the inductor \( L \) in Figure 1. Analogously a modified gyrator similar to Figure 2(b) replaces the inductor \( L \) in Figure 1 but in this case the JFET is replaced by a known fixed resistor, \( R_{DS} \) (see Figure 3 for details).

By applying (17) and (19) for the modified gyrators used in Figure 3, we obtain the following
\[
V_Q = i_LR_{1m}, \quad \bar{V}_Q = \bar{i}_L\bar{R}_{1m}, \quad (20)
\]
\[
L = \frac{R_{1m}R_{2m}R_{3m}C}{R_{DS}}, \quad \bar{L} = \frac{\bar{R}_{1m}\bar{R}_{2m}\bar{R}_{3m}C}{\bar{r}_{DSm}(\bar{V}_c)}. \quad (21)
\]

**Remark 4:** For the physical realization of the Chua’s circuit requiring adaptation of \( \bar{C}_2 \) as in Section III-B, a modified version of the voltage-controlled capacitor- gyrator from [24] can be used.

**B. \( \bar{L} \) Parameter Update**

The parameter update law (7) is implemented using an analog multiplier (an AD633), a signal integration circuit, a reset switch, and a standard op-amp based voltage conditioning circuitry, as depicted in Figure 4. The AD633 has the ideal input-output relationship
\[
W = \frac{(X_1 - X_2)(Y_1 - Y_2)}{10} + Z. \quad (22)
\]
For the purposes of repeatability of our adaptive synchronization experiment, the intrinsic DC offset of the AD633’s output \( W \) is canceled by manually tuning the potentiometer \( R_{22} \). This is done prior to initiating adaptation. The integrator’s initial conditions are reset to zero using the switch SW1. Knowing (22) and using Figure 4, it follows that the expression for \( V_c \) in terms of \( i_L, i_\bar{L} \), and \( \bar{v}_2 \) is
\[
V_c = -\frac{1}{R_C\bar{R}_f} \int \left[ \frac{(\bar{R}_{1m}\bar{i}_L - R_{1m}i_L)(\bar{v}_2 + \bar{i}_L\bar{R}_{1m}\bar{R}_f)}{10} + V_{633off} + Z \right] dt, \quad (23)
\]
where \( V_{633off} \) is the DC offset of the AD633 output which is canceled with \( Z \). Note that in our experiments \( V_{633off} \) is found to be positive. If it happens to be negative, the supply voltage for voltage divider producing \( Z \) can be made +9V.
In an ideal case, we can assume that
\[
\bar{R}_{1m} = R_{1m}, \quad \bar{Z} = -V_{633off}, \quad \bar{R}_{f}R_{f} = R_0, \quad (24)
\]
\[
\bar{r}_{DSm} = -aV_c + c, \quad \text{where } a > 0 \text{ and } c > 0. \quad (25)
\]
Equation (25) is an approximation from the empirical results of [19].

By inverting \( \bar{L} \) in (21), substituting for \( \bar{r}_{DSm} \) using (25), and using (23)–(24), we obtain the following
\[
\frac{1}{L} = \frac{a}{\bar{R}_{2m}\bar{R}_{3m}C}\int \frac{1}{10\bar{R}_f\bar{C}_1} \int e_t(\bar{v}_2 + \bar{i}_L\bar{R}_0)dt + \frac{c}{\bar{R}_{1m}\bar{R}_{2m}\bar{R}_{3m}C}. \quad (26)
\]
Next, taking the derivative of (26) with respect to time yields
\[
\frac{d}{dt} \left( \frac{1}{L} \right) = \frac{a}{10\bar{R}_{2m}\bar{R}_{3m}C\bar{R}_f\bar{C}_1} e_t(\bar{v}_2 + \bar{i}_L\bar{R}_0), \quad (27)
\]
confirming that the schematic in Figure 4 implements the parameter update law (7) and that
\[
\gamma = \frac{a}{10\bar{R}_{2m}\bar{R}_{3m}C\bar{R}_f\bar{C}_1}. \quad (28)
\]

**C. Experimental Setup**

Figures 3 and 4 contain the complete circuit schematics that are used in the physical implementation of master/slave Chua’s oscillator synchronization with adaptive parameter update law for \( L \). The parts of the circuit constructed using two identical, dual layer printed circuit boards (PCBs), which utilize surface-mount technology (SMT) components, are highlighted in Figure 3. The remaining circuitry is built using solderless breadboards with through-hole (THL) components. The custom PCBs of the master/slave system are connected to a data acquisition device, two analog oscilloscopes, and a regulated ±9V power supply. Specifically, we used Measurement Computing’s USB-1608G data acquisition device, which is sampled at 62.5 kHz to collect the experimental data. Since USB-1608G can simultaneously sample only four signals at 62.5 kHz, we run each experiment twice. In the first experimental run, we use USB-1608G in differential mode to acquire \( \bar{V}_Q - V_Q \), \( \bar{v}_2 - v_2 \), \( \bar{v}_1 - v_1 \), and \( V_c - \text{GND} \) to examine synchronization between the master/slave system. In the second experimental run, we use USB-1608G to acquire \( v_2, V_Q, \bar{v}_2, \bar{V}_Q \) signals for parameter estimation. Photographic images of the experimental setup are shown in Figure 5. To test this setup for different parameter values of the master/slave system, we can simply vary the resistances (e.g., \( R_{2m}, i = 1, 2, 3) \) on the solderless breadboards. When appropriate, the setup is placed inside a grounded metal housing to eliminate electromagnetic interference (EMI).

**V. EXPERIMENTAL RESULTS**

**A. Experimental Results**

To illustrate our ability to tune parameter \( \bar{L} \) to match \( L \), we provide our experimental results. Our experiment is designed to demonstrate the performance of the adaptive controller when the master Chua’s oscillator is in chaotic mode. The
components selected for the experiment are listed in Table I. The symbols $\|\|$ and $+$ are used to denote components connected in parallel and series, respectively.

To quantify how well the master/slave system synchronizes, we use the 2-norm of $[e_{t1}, e_{t2}, e_{t3}]^T$ as our measure, $(e_{\text{norm}} \triangleq ||[e_{t1}, e_{t2}, e_{t3}]^T||)$, and observe its evolution over time. Using the signals $i_{t1}$ and $i_{t2}$ we estimate $\hat{L}$ and $\hat{R}_0$ ($L_{\text{est}}, R_{\text{est}}$) with a sliding window least square algorithm. Similarly, using signals $i_{t2}$ and $i_{t3}$ we estimate $\hat{L}$ and $\hat{R}_0$ ($L_{\text{est}}, R_{\text{est}}$). Comparing these estimates allows us to examine how well $\hat{L}$ converges to $L$. The transient experimental data is displayed in Figure 6. The vertical bar in Figure 6(b) indicates when SW1 is opened so that $V_c$ switches from a zero value to a non-zero value, which can be used to indicate the initiation of adaptation. No vertical bar is seen in Figure 6(c) because in this case $V_c$ is not being measured explicitly and thus we are unable to pin-point the exact instance SW1 is opened for the second experimental run. Note that each point on Figure 6(c) represents a least square estimate of a window of 50 samples and the x-axis indicates the time when the leading sample is taken. Figures 6(d) and 6(e) re-plot the last 10 ms of Figures 6(b) and 6(c), respectively, to better visualize the steady-state results. The average of each signal (except $V_c$) in Figures 6(d) and 6(e) is listed in Table I under

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### TABLE I: Experiment component values and estimates.

<table>
<thead>
<tr>
<th>Master Cha’s Oscillator</th>
<th>Adaptive Controller</th>
<th>Slave Cha’s Oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 = 22 , k\Omega , 0.1% , SMT$</td>
<td>$R_{\text{est}} = 500 , \Omega , 1% , \text{THL}$</td>
<td>$R_1 = 22 , k\Omega , 0.1% , SMT$</td>
</tr>
<tr>
<td>$R_2 = 22 , k\Omega , 0.1% , SMT$</td>
<td>$R_2 = 1 , \Omega , 0.1% , \text{THL}$</td>
<td>$\hat{R}_2 = 220 , \Omega , 0.1% , \text{SMT}$</td>
</tr>
<tr>
<td>$R_3 = 3.3 , k\Omega , 0.1% , SMT$</td>
<td>$C_t = 18 , nF , 3% , \text{THL}$</td>
<td>$\hat{R}_3 = 3.3 , k\Omega , 0.1% , \text{SMT}$</td>
</tr>
<tr>
<td>$R_4 = 220 , \Omega , 0.1% , SMT$</td>
<td>$R_{21} = 200 , k\Omega + 200 , k\Omega + 200 , k\Omega$</td>
<td>$\hat{R}_4 = 220 , \Omega , 0.1% , \text{SMT}$</td>
</tr>
<tr>
<td>$R_5 = 2.2 , k\Omega , 0.1% , SMT$</td>
<td>$R_{22} = 1 , k\Omega , 0.1% , \text{SMT}$</td>
<td>$\hat{R}_5 = 2.2 , k\Omega , 0.1% , \text{SMT}$</td>
</tr>
<tr>
<td>$R_6 = 2.2 , k\Omega , 0.1% , SMT$</td>
<td>$R = 3.4 , k\Omega , || , 3.4 , k\Omega , (0.1% , \text{each}) , \text{SMT}$</td>
<td>$\hat{R} = 3.4 , k\Omega , || , 3.4 , k\Omega , (0.1% , \text{each}) , \text{SMT}$</td>
</tr>
<tr>
<td>$R_7 = 2.2 , k\Omega , || , 2.2 , k\Omega , (0.1% , \text{each}) , \text{SMT}$</td>
<td>$C_t = 10 , nF , 0.1% , \text{SMT}$</td>
<td>$\hat{C}_1 = 10 , nF , 0.1% , \text{SMT}$</td>
</tr>
<tr>
<td>$R_8 = 220 , \Omega , 0.1% , \text{SMT}$</td>
<td>$C_2 = 100 , nF , 1% , \text{SMT}$</td>
<td>$C_2 = 100 , nF , 1% , \text{SMT}$</td>
</tr>
</tbody>
</table>
| $R_9 = 220 \, \Omega \, (0.1\% \, \text{each}) \, \text{SMT}$ | $L = 40.5 \, mH \, \text{according to (21)}$ | $L = 50.8 \, mH \, \text{(average)}$
| $\hat{L}_{\text{est}} = 51.8 \, mH \, \text{(average)}$ | $L_{\text{est}} = 51.8 \, mH \, \text{(average)}$ | $L_{\text{est}} = 51.8 \, mH \, \text{(average)}$
| $e_{\text{norm}} = 0.0157 \, \text{(average)}$ | $\hat{R}_{\text{est}} = 225.544 \, \Omega \, \text{(average)}$ | $\hat{R}_{\text{est}} = 225.6381 \, \Omega \, \text{(average)}$
| $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$
| $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$
| $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$
| $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$
| $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$ | $\hat{R}_{\text{est}} = 1 \, k\Omega \, 0.1\% \, \text{SMT}$

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“Estimates”. Experiments have been run for other values of $L$ and $R_0$ including parameters that produce periodic oscillation (as opposed to chaotic oscillations). Moreover, SPICE simulations and MATLAB simulations have also been performed for an ideal case and with component tolerances. The simulation results have been found to be in agreement with the experimental results but are omitted due to space constraints.

B. Discussion

By closely observing the plots in Figure 6 and estimates provided in Table I, we can infer the quality of synchronization and parameter matching. First, we assess the overall performance by examining Figures 6(b) and 6(c). Specifically, we observe that following the initiation of adaptation, $\epsilon_{\text{norm}}$ decreases significantly and approaches zero and $L_{\text{est}}$ approaches $L_{\text{est}}$. Given that this data is obtained from a physical experiment, with imperfect components, parasitic effects, and noise, some degree of experimental error is expected. Figures 6(d) and 6(e) give a zoomed-in view of the long term behavior of system. We observe that in fact $\epsilon_{\text{norm}}$ does not go to zero but continues to oscillate as indicated in Table I has an average value of 0.0157. Moreover, we observe that the least square estimates of $L_{\text{est}}$ and $L_{\text{est}}$ are slightly offset by 1mH on average. A similar effect is observed by [20] in a simulation study of adaptive synchronization of Chua’s oscillator with a mismatched parameter.

VI. Conclusion

In this paper we presented two adaptive controllers that are designed to match a parameter ($L$ or $C_2$) in two Chua’s oscillators with the presence of PE. We implemented one of the adaptive controllers using analog circuitry. To our knowledge, this is the first instance of adaptive synchronization with parameter matching wherein the Chua’s oscillators and adaptive controller are realized using analog circuits. Our results show that the adaptive controller achieves parameter matching while the system is chaotic.

REFERENCES

Fig. 5: Experimental setup: (a) close up of circuitry and data acquisition device, (b) full setup when switch SW1 is closed, and (c) full setup when switch SW1 is open.

Fig. 6: Experimental results.