Time-Domain Optimal Experimental Design in Human Postural Control Testing

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Abstract— We are developing a series of systems science-based clinical tools that will assist in modeling, diagnosing, and quantifying postural control deficits in human subjects. In line with this goal, we have designed and constructed an experimental device and associated experimental task for identification of the human postural control system. In this work, we present a Quadratic Programming (QP) technique for optimizing a time-domain experimental input signal for this device. The goal of this optimization is to maximize the information present in the experiment, and therefore its ability to produce accurate estimates of several desired postural control parameters. To achieve this, we formulate the problem as a non-convex QP and attempt to maximize a measure (T-optimality condition) of the experiment’s Fisher Information Matrix (FIM) under several constraints. These constraints include limits on the input amplitude, physiological output amplitude, subject control amplitude, and input signal autocorrelation. Because the autocorrelation constraint takes the form of a Quadratic Constraint (QC), we replace it with a conservative linear relaxation about a nominal point, which is iteratively updated during the course of optimization. We show that this iterative descent algorithm generates a convergent suboptimal solution that guarantees monotonic non-increasing of the cost function while satisfying all constraints during iterations. Finally, we present example experimental results using an optimized input sequence.

I. INTRODUCTION

In recent years, clinical researchers have expanded the study of the human postural control system through the application of control theoretic analysis techniques [1]–[3]. These studies often rely on accurate models of the underlying dynamics of the human in order to make the analysis tractable. However, humans possess a number of characteristics which may be impossible to measure accurately a priori, such as moments of inertia of body segments, center of mass (COM) locations, or feedback control gains. However, these parameters may be recoverable via examination of an experimental response. In the control sciences field, the set of techniques for recovering unknown or partially unknown model parameters from an experimental response are known as “system identification” techniques.

The design and optimization of system identification experiments is both a well-studied and ongoing problem in the literature [4]–[6]. Recent results in experimental optimization tend to favor the technique of optimizing the spectrum of the input signal [6], [7]. This technique poses a number of challenges for human experiments. Human subjects tend to fatigue quickly during motor control testing, which limits the feasible length of each trial. This issue makes frequency-domain techniques for optimal experimental design difficult to use, because the time sequence may be too short to produce accurate results at low frequency. Thus, it would be preferable to design input in the time-domain (for short input sequences). Additionally, it is difficult to adapt frequency-domain optimization techniques to the number and variety of constraints required for viable human testing, such as input constraints, output constraints, and constraints on signal predictability.

Our contributions in this work are as follows. We formulate a time-domain Quadratic Program (QP) designed to optimize the design of an experimental input for identification of a Linear Time-Invariant (LTI) human postural control model. In this approach, we maximize the trace of the experiment’s Fisher Information Matrix (FIM), an objective known as T-optimality [8], while ensuring that the system does not violate a number of input and state constraints. We formulate a novel quadratic constraint on the input sequence’s autocorrelation function to ensure that the input is both unpredictable to subjects and possesses the desired frequency characteristics. By computing an iterative linear relaxation of this autocorrelation constraint, we are able to formulate the problem as a tractable nonconvex Quadratically-Constrained QP (QCQP) which can be solved locally at each iteration. We show that this iterative algorithm generates a convergent suboptimal solution that guarantees monotonic non-increasing of the cost function while satisfying all constraints during iterations. Our approach is applied to optimize the design of a human seated balance identification experiment. We show simulation results for this design using model parameters derived from a preliminary set of subject parameters, and apply the optimized input to an experimental subject using a novel compliant robotic seat that we have developed.

The rest of this paper is organized as follows: in Section II, we present the dynamic model for the seated balance task. In Section III, we derive the QP formulation for the experimental optimization and present the constraints under which the optimization will operate. In Section IV, we show results from an input optimization for one subject, and apply the optimized input to the subject. Finally, in Section V, we
offer some concluding remarks.

Standard notation will be used throughout the paper. Let \( \mathbb{R}, \mathbb{R}^+, \) and \( \mathbb{B} \) denote, respectively, the sets of real, positive real, and binary (i.e. \( \{0,1\} \)) numbers. The operators of expectation and covariance matrix are denoted by \( \mathbb{E} \) and \( \text{Cov} \), respectively. A random vector \( x \), which has a multivariate normal distribution of mean vector \( \mu \) and covariance matrix \( \Sigma \), is denoted by \( x \sim \mathcal{N}(\mu, \Sigma) \). An identity matrix of size \( n \times n \) is denoted as \( I_n \). A vector of zeros of length \( n \) is denoted as \( 0_n \). The Kronecker product is denoted by \( \otimes \). The vectorization of a matrix \( A \) is denoted by \( \text{vec}(A) \). Other notation will be explained as it is used.

II. SEATED BALANCE EXPERIMENT

Using a novel compliant robot seat (Fig. 2), we have designed a seated balance experiment based on the one performed in [2]. In the current experiment, the subject sits atop the compliant robotic seat which is free to pivot about an axis perpendicular to the coronal plane (Fig. 1). The angle of the lower body from vertical is \( \alpha_1 \) and the angle of the upper body from vertical is \( \alpha_2 \). Similar to the convention in [2], the portion of the subject and seat below the fourth lumbar (L4) vertebrae is lumped into a single rigid element with mass \( M_1 \) and moment of inertia (about the COM) of \( J_1 \). The COM is at a distance \( l_1 \) from the pivot point of the seat.

Similarly, the portion of the subject above the L4 vertebrae is lumped into a rigid element with mass \( M_2 \) and moment of inertia \( J_2 \) about the COM. The COM of the upper body is a distance \( l_2 \) from the L4 vertebrae. The L4 vertebrae itself is at a distance \( l_{12} \) from the seat pivot. The human can apply a control torque \( u_h \) about the L4 vertebrae, and additionally possesses an intrinsic rotational stiffness \( k_h \) and intrinsic rotational damping \( c_h \) about L4. We apply (through feedback) a virtual stiffness \( k_r \) and a virtual damping \( c_r \) about the pivot point, in addition to a torque disturbance \( u \).

The sum of these torques produce the total robot torque \( u_r \) about the pivot point, i.e. \( u_r = u - k_1 \alpha_1 - c_1 \alpha_1 \). The resulting dynamics can be determined by application of Lagrange’s equation to the model in Fig. 1.

We model the closed-loop dynamical structure of the coupled human/compliant robot system as shown in Fig. 3. The plant model \( P \) represents the dynamics of the system (linearized about the upright equilibrium) and has outputs \( z_h = [\alpha_1 \ \dot{\alpha}_1 \ \alpha_2 \ \dot{\alpha}_2]^T \) and \( z_r = [\alpha_1 \ \dot{\alpha}_1]^T \). There is a feedback controller \( R \) utilizing \( z_r \) such that the robot can simulate a desired dynamical system (in this case, a spring-damper system). The robot can additionally apply a torque disturbance \( u \) to the seat. Both of these signals are combined and converted into a torque through the robot motor \( M \). The model of the human has a feedback loop presumed to consist of a sensory delay \( e^{-\tau_s} \) implemented as a 5th-order Padé approximation, and an output feedback controller \( K \) such that (if we ignore delays), the human control is \( u_h = K z_h \), where \( K = [-K_1 \ -K_2 \ -K_3 \ -K_4] \). We also include an approximation of muscle dynamics using a first-order filter with time constant \( T_\omega \). Since we can experimentally only measure the angles \( (\alpha_1, \alpha_2) \) directly, we reduce the plant output \( z_h \) to \( y = [\alpha_1 \ \alpha_2]^T \) via the operator \( D_y \). Additive sensor noise \( w \) is also presumed to exist. We have determined a set of nominal model parameter values \( \theta_0 \) for one subject through a combination of parameters fitted in another study [2], tabulated data based on subject weight and height [9], and nonlinear least-squares fitting [10] (in the time domain) to a preliminary experiment, with

\[
\theta := [K_1 \ K_2 \ K_3 \ K_4 \ J_1 \ J_2 \ l_1 \ l_{12} \ l_2 \ \tau \ T_\omega]^T.
\]
III. EXPERIMENTAL OPTIMIZATION

A. Quadratic Program

We reformulate the closed-loop model in Fig. 3 as the discrete-time LTI state-space model

$$x_{k+1} = A(\theta_0)x_k + B(\theta_0)u_k$$

$$y_k = C(\theta_0)x_k$$

where all elements in $H$ are assumed to be bounded, i.e., $\ell_{h_1} \leq H_{ij} \leq \ell_{h_2}$.

with $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}$, $y_k \in \mathbb{R}^p$, $w_k \sim \mathcal{N}(0, \Sigma) \in \mathbb{R}^p$, $\theta \in \mathbb{R}^{n_\theta}$, and sampling time $T$. The unknown true parameter vector $\theta_0$ is presumed to belong to a compact set $\Theta$ such that $\theta_0 \in \Theta = \{ \rho \in \mathbb{R}^{n_\theta} | \rho_{i, \text{min}} \leq \theta_i \leq \rho_{i, \text{max}} \}, \forall i = 1, \ldots, n_\theta$.

The system is defined over the time indices $k \in \mathbb{K} := \{0, \ldots, N\}$ such that $t_k = kT$. We define the error $e_k$ between the nominal output $y_k$ and the noisy output $\hat{y}_k$ for a given time index $k$ and the true parameter vector $\theta_0$ as

$$e_k(\theta_0) := \hat{y}_k - y_k := y_k - C(\theta_0)A(\theta_0)x_{k-1} - C(\theta_0)B(\theta_0)u_{k-1}. \quad (2)$$

Let us consider an experiment with an input sequence defined as $u := [u_0 \cdots u_{N-1}]^T$. The log likelihood function for a data set $y := [y_1^T \cdots y_N^T]^T$ given the true parametrization $\theta_0$ is

$$\ln p(y|\theta_0) = \sum_{k=1}^{N} \ln p(y_k|\theta_0) = -\frac{Np}{2} \ln 2\pi - \frac{N}{2} \ln |\Sigma| - \frac{1}{2} \sum_{k=1}^{N} e_k^T(\theta_0)\Sigma^{-1}e_k(\theta_0).$$

The maximum likelihood estimator for $\theta_0$ is then given by

$$\hat{\theta}_N = \arg \min_{\theta \in \Theta} \left( \frac{1}{N} \sum_{k=1}^{N} e_k^T(\theta)\Sigma^{-1}e_k(\theta) \right),$$

$$= \arg \min_{\theta \in \Theta} J_N(\theta).$$

Under mild conditions [11], [12], it can be shown that

$$\lim_{N \to \infty} \hat{\theta}_N = \theta_0 = \arg \min_{\theta \in \Theta} \lim_{N \to \infty} \mathbb{E} \{ J_N(\theta) \} \text{ w.p.1},$$

and that the prediction error converges in distribution to a normally distributed random variable [11]–[13]

$$\sqrt{N} \left( \hat{\theta}_N - \theta_0 \right) \overset{d}{\to} \mathcal{N} \left( 0, \Sigma^{-1} \right),$$

where $\mathbb{I}(u; \theta_0)$ is the FIM.

For a MIMO system, the FIM is an extension of the SISO case given in [14] and [15]:

$$\mathbb{I}(u; \theta_0) = \mathbb{E}_{y|\theta_0} \left[ \begin{bmatrix} \frac{\partial \ln p(y|\theta)}{\partial \theta} |_{\theta=\theta_0} & \frac{\partial \ln p(y|\theta)}{\partial \theta} |_{\theta=\theta_0} \end{bmatrix}^T \right]$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \frac{\partial c_k}{\partial \theta} |_{\theta=\theta_0} \end{bmatrix}^T \Sigma^{-1} \begin{bmatrix} \frac{\partial c_k}{\partial \theta} |_{\theta=\theta_0} \end{bmatrix}.$$

In order to formulate the FIM for the system in Eqn. (1), we note that we can determine the system output $y_k$ at an arbitrary time index $k \geq 1$ when the input sequence $[u_0, u_1, \cdots, u_{k-1}]^T$ and initial state condition $x_0$ are known. The complete solution to the discrete-time state-space system given in Eqn. (1) is

$$y_k = CA^kx_0 + C \sum_{i=0}^{k-1} A^{k-i-1}Bu_i.$$
Amongst a number of different optimality conditions \[8\], we choose the T-optimality condition, which will maximize the trace of the FIM \[16\]–\[18\], and in turn provides an objective that is quadratic in \( u \). We therefore use a cost \( J(u; \theta_0) \) defined by
\[
J(u; \theta_0) = -\text{trace} \left( \mathbb{E}(u; \theta_0) \right).
\]
(4)
Note that both the FIM and \( J(u; \theta_0) \) are functions of the input sequence \( u \), the initial condition \( x_0 \), and true parameters \( \theta_0 \) only. While the cost function \( J(u; \theta_0) \) is nonconvex in \( u \) \[18\], a general quadratic programming solver can be used to perform the unconstrained minimization
\[
u^* = \arg\min_u J(u; \theta_0).
\]
(5)

\[\text{B. Design Constraints}\]

In this paper, the quadratic optimization in Eqns. (4)-(5) is subject to the following constraints:

- **Input Limits.** Since the motor is only capable of producing a finite amount of torque, we apply a constraint such that
\[
-u_m \leq u \leq u_m, \quad u_m \in \mathbb{R}_+.
\]

- **Output Constraints.** There is a finite angular range over which both the robot seat platform and the human torso can move. We therefore apply the constraint
\[
-1^N \otimes y_m \leq GU \leq 1^N \otimes y_m,
\]
where \( 1^N \) is a vector of ones of length \( N \), and \( y_m \in \mathbb{R}_+^1 \) defines the maximum amplitude of each output individually. Additionally, the angular difference \( \hat{\alpha} = \alpha_2 - \alpha_1 \) is limited by both the structure and flexibility of the subject’s lower back. By reformulating the closed-loop system in Fig. 3, we can form a structure \( G_\delta \) similar to \( G \) where \( u \) is the input and \( \hat{\alpha} \) is the output. We then apply the constraint
\[
-\delta_\alpha \leq G_\delta U \leq \delta_\alpha, \quad \delta_\alpha \in \mathbb{R}_+.
\]

- **Human Control Constraint.** The human subject is only capable of generating a finite amount of torque \( u_h \). We can again reformulate the closed-loop system in Fig. 3 to form a structure \( G_\hat{u} \) similar to \( G \) where \( u \) is the input and \( u_h \) is the output. Then, we apply the constraint
\[
-u_{hm} \leq G_\hat{u} U \leq u_{hm}, \quad u_{hm} \in \mathbb{R}_+.
\]

- **Autocorrelation Constraint.** In addition to the preceding linear constraints, it was desired to constrain the autocorrelation of the input sequence so as to reduce predictability of the signal while maintaining desirable spectral characteristics. The autocorrelation of a discrete real time sequence \( u_k \) at lag \( j \) can be computed as
\[
R_{uu}(u; j) = \sum_k u_k u_{k-j}.
\]
We can reformulate this as the quadratic matrix multiplication
\[
R_{uu}(u; j) = u^T Q(j) u,
\]
where \( Q(j) \in \mathbb{B}^{N \times N} \) is a Toeplitz matrix containing ones on its \( j \)-th upper off-diagonal and zeros everywhere else, e.g.
\[
\begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0 \\
\end{bmatrix}
\]

We consider the term \( R_{uu}(u) \) (with \( j \) omitted) to be the autocorrelation vector for all lags \( j = \{0, \cdots, \frac{N}{2} - 1\} \).

Based on feedback from the initial subject, our preliminary experiment consisted of a Pseudo-Random Binary Sequence (PRBS) which only contained significant power below approximately 1 Hz. We desired the normalized autocorrelation of the first \( N/2 \) lags of the optimal input sequence autocorrelation to be within some region of the autocorrelation \( R^*_{uu} \) of this initial sequence, i.e.
\[
R^*_{uu} - \beta \leq \frac{R_{uu}(u)}{R_{uu}(u; 0)} \leq R^*_{uu} + \beta,
\]
where \( \beta > 0 \) is a scalar constant.

Therefore, our constrained optimization problem can be formulated as follows:
\[
u^* = \arg\min_u J(u; \theta_0),
\]
(7)
subject to the constraints
\[
-u_m \leq u \leq u_m,
\]
\[
-1^N \otimes y_m \leq GU \leq 1^N \otimes y_m,
\]
\[
-\delta_\alpha \leq G_\delta U \leq \delta_\alpha,
\]
\[
-u_{hm} \leq G_\hat{u} U \leq u_{hm},
\]
\[
R^*_{uu} - \beta \leq \frac{R_{uu}(u)}{R_{uu}(u; 0)} \leq R^*_{uu} + \beta.
\]
(8)

Unfortunately, the optimization in Eqns. (7)-(8) is a non-convex QCQP, the solution of which is still an open research question. Therefore, we propose an iterative linearization technique to find a good solution to Eqn. (7) in the next section.

\[\text{C. Proposed Iterative Descent Algorithm}\]

Since we can not directly apply a quadratic constraint such as the one in Eqn. (6) to the quadratic program, we propose to compute a linear relaxation of the autocorrelation about a nominal vector \( \hat{u} \). This relaxation takes the form of a linearization based on a Taylor series expansion about \( \hat{u} \), i.e.
\[
\hat{R}_{uu} (\hat{u}; u; j) = \hat{u}^T Q(j) \hat{u} + \hat{u}^T (Q(j) + Q^T(j)) (u - \hat{u}).
\]

This constraint is made slightly more conservative than the true quadratic constraint in Eqn. (6) by shrinking the constraint boundary, i.e.
\[
R^*_{uu} - \beta + \gamma \leq \frac{\hat{R}_{uu} (\hat{u}; u; 0)}{R^*_{uu}} \leq R^*_{uu} + \beta - \gamma,
\]
(9)
where $\gamma$ s.t. $0 < \gamma < \beta$ is a small constant. Note that we normalize $\hat{R}_{uu}(\hat{u}; u)$ by $\hat{R}_{uu}^0$, which we define as $\hat{R}_{uu}^0 := R_{uu}(\hat{u}; 0)$. Now, by ensuring that $\hat{u} = u - \tilde{u}$ is constrained to be small, a locally optimal solution can be found that satisfies the linear constraint in Eqn. (9) but does not violate the quadratic autocorrelation constraint Eqn. (6).

To ensure that the linearization in Eqn. (9) is both always valid and more conservative than the true quadratic constraint Eqn. (6), we constrain the difference $\hat{u} = u - \tilde{u}$ such that

$$-\delta_u \leq \hat{u} \leq \delta_u, \quad \delta_u \in \mathbb{R}^+. \quad (10)$$

Therefore, when we allow only a small change in $u$, we may solve the following optimization:

$$u^* = \arg \min_u J(u; \theta_0), \quad (11)$$

subject to the constraints

$$-u_m \leq u \leq u_m,$$
$$-1^N \otimes y_m \leq GU \leq 1^N \otimes y_m,$$
$$-\delta_u \leq G_\beta U \leq \delta_u,$$
$$-u_{hm} \leq G_\alpha U \leq u_{hm},$$

$$R_{uu}^0 - \beta + \gamma \leq \frac{\hat{R}_{uu}(\hat{u}; u)}{\hat{R}_{uu}^0} \leq R_{uu}^0 + \beta - \gamma,$$
$$-\delta_u \leq \hat{u} \leq \delta_u. \quad (12)$$

An overall solution is found by computing a series of successive solutions $u_t^*$ to the problem of Eqn. (11) subject to the constraints in Eqn. (12). For each iteration $i$, we perform a local linearization Eqn. (9) of the quadratic autocorrelation constraint in Eqn. (6) about $\tilde{u} = u^{\star i-1}$ and solve for $u_t^*$. Each solution $u_t^*$ becomes $\tilde{u}$ in the next iteration of the solution. This is done so as to allow $u$ to traverse a wide range while not violating the input linearization constraint in Eqn. (10) at any point during the optimization. Each solution $u_t^*$ is found using MATLAB’s quadprog general quadratic programming solver in combination with the yalmip modeling toolbox. Note that the optimization relies on the true parameter vector $\theta_0$, which is only known approximately, i.e. $\theta_0$, during the optimization procedure. This is a common problem in system identification, and can be dealt with via a number of methods, such as iterative system identification techniques [19].

Note that $J(u_t^*; \theta_0) \geq J(u^{\star i+1}; \theta_0)$ by the construction. Since the value $J$ has a lower bound and is monotonically non-increasing during the iterations, it will converge to some value as iterations proceed. Therefore, this iterative descent algorithm generates a convergent suboptimal solution that guarantees monotonic non-increasing of the cost function while satisfying all constraints during iterations.

**IV. CASE STUDY**

We have performed a case study on a single subject to demonstrate our experimental optimization. Preliminary testing of this subject identified an initial parameter vector $\theta_0$. The limits applied to the optimization are listed, along with their sources, in Table I. We let $x_0 = [0.01 \quad 0_5^T]^T$, and since the sensor noise for both elements of $y_k$ were approximately equal and uncorrelated, we let $\Sigma = I$. The initial input $\dot{u}$ was a PRBS signal with the same frequency characteristics and amplitude (5 Nm) as that given to the subject in the preliminary test. Note that, in the preliminary test, the initial $\dot{u}$ was challenging enough that the subject required considerable practice to complete the trials successfully (defined as no contact occurring with the mechanical stops at $\alpha_1 = \pm 0.26$ rad on the device.)

For an input sequence with length $N = 300$ and a sampling time of $T = 0.05$ seconds, we were able to converge to a suboptimal input sequence ($E_{\text{step}} = 0.5 \times 10^{-3}$) in approximately 4 hours on a 2.2GHz Xeon server. However, most of this time is overhead in the yalmip modeling toolbox. It would be possible to compute the solution significantly faster with a more optimized numerical setup.

**A. Optimization Results**

The suboptimal input $u^*$ along with its bounds is shown in Fig. 4, along with the convergence of the cost function $J(u; \theta_0)$ with iterations. It can be seen that the solution $u^*$ produces an approximately 4-fold improvement relative to the initial $\dot{u}$ in the value of the objective function without violating any of the listed constraints. Only two constraints are active for the system given input $u^*$: the difference angle constraint $\vert \dot{\alpha} \vert \leq \delta_\alpha$ and the autocorrelation constraint in (11). The values of these signals when the system is given the input $u^*$ are shown in Fig. 5.
B. Experimental Application

We ran another experiment on the same subject using the suboptimal input sequence $u^*$ in Fig. 4. The subject was able to successfully complete the experiment (no mechanical stop contact), although the difficulty was very high. The resulting best-fit parameters $\hat{\theta}_N$ are shown in Table II. While some of the parameters differ from those in $\theta_0$, it is expected that these parameter estimates themselves will converge as the process of the optimal input design and the parameter estimation is repeated.

V. CONCLUSIONS

In this work, we have demonstrated a QP technique for generating an optimized experimental input for a human postural control identification experiment. To this end, we have formulated a quadratic objective function based on a measure of the FIM that will maximize the information present in the experiment for the proposed testing. We have formulated a set of output, input, and control constraints, in addition to a unique linearized autocorrelation constraint, such that the resulting input signal will be feasible for the proposed testing. The resulting solution $u^*$ converged to a suboptimal value without violating any of the prescribed constraints. We have additionally demonstrated an experimental application of this input signal in conjunction with our compliant robot.

This technique will necessarily produce a different suboptimal input sequence if $\theta$ is defined using different parameters. It is therefore important to ensure that only the minimum number of required parameters are subject to fitting. Additionally, because the sequence $u^*$ is only designed for a true parameter vector $\theta_0$, this technique should be employed as part of a broader iterative procedure [19]. After a $u^*$ is found, a subject can be tested using $u^*$ as the input and the resulting experimental response fitted to find $\theta_N$. The parameters $\theta_N$ should become $\theta_0$ in the next iteration of the input optimization and the process repeated until a desired level of convergence is achieved.

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