Polynomial time sensitivity analysis of task schedules*

Moritz Niendorf1, Pierre T. Kabamba 1 and Anouck R. Girard

Abstract—By performing sensitivity analysis on an optimal task schedule, this paper derives a polynomial time method to determine whether the task schedule remains optimal after arbitrary changes to task costs occur. We consider fast reactive mission planning for unmanned aircraft in changing environments. Changing external conditions such as weather or threats may alter task costs, which can render an initially optimal task schedule suboptimal. Instead of optimizing the task schedule every time task costs change, stability criteria allow for fast evaluation of whether schedules remain optimal. This paper develops a method to compute stability regions for a set of schedules in a prototypical mission for unmanned aircraft, the traveling salesman problem, where the alternative schedules are part of a pre-approved mission plan. As the traveling salesman problem is NP-hard, heuristic methods are frequently used to solve it. The presented approach is also applicable to analyze stability regions for a tour obtained through application of the k-opt heuristic with respect to the k-neighborhood and is demonstrated with an example problem.

I. INTRODUCTION

This paper presents the polynomial time analysis of stability regions and tolerances for the best task schedule among a set of task schedules through application of sensitivity analysis to an integer linear programming (ILP) formulation of the task scheduling problem. In particular, this paper addresses task schedules that can be expressed as traveling salesman tours on a weighted graph and their stability regions and edge cost tolerances, where the edge cost tolerance is the supremum of cost increases of an edge (resp. infimum of cost decreases) under which the tour remains optimal, provided the other edge costs in the graph are unchanged. The stability region corresponding to a given tour is defined as the set of all possible edge cost combinations for which this tour is optimal.

Mission planning for unmanned aerial vehicles (UAV) involves tactical planning which refers to generating tactical goals and task scheduling, and path planning which is concerned with generating a flyable time-annotated path in the physical world. Both problems individually have been widely studied in the literature. This led to efficient algorithms for solving vehicle routing problems and the traveling salesman problem (TSP) on one hand and efficient path planning algorithms on the other hand. However, in dynamic or uncertain situations, such as during missions in adversarial environments, situations may occur in which to changes to path costs render a previously optimal task schedule suboptimal.

*This work was supported in part by AF grant FA 8650-07-2-3744.
1The authors are with the Department of Aerospace Engineering, University of Michigan, 48109, Ann Arbor, MI, USA.
\{mniendo, kabamba, anouck\}@umich.edu

A. Mission Overview

Consider the prototypical mission planning scenario in fig. 1. An unmanned aircraft starting from an airfield (1) is tasked to visit locations (2)-(6) and return to the airfield. We assume that a path planning algorithm computes flight paths between location pairs. The path length is assigned as edge weight to the edge connecting that location pair in a representation of the problem as a weighted graph. Then, tactical planning corresponds to solving a classical instance of the TSP on the weighted graph.

Assume that the UAV operator provides and approves three alternative schedules in the form of tours, where tour I (1,6,5,4,3,2,1) is cost optimal and tours II (1,6,2,3,5,4,1) and III (1,6,4,5,3,2,1) are backups. Shortly before take-off, new intelligence suggests the presence of anti-aircraft units that must be avoided between locations (3) and (4). The path planning algorithm computes a new flight path between this location pair and updates the edge weight. This raises the question of how stable the optimal schedule is with respect to changes in the problem data. Hence, this paper studies the edge cost tolerances and stability regions of task schedules in the context of dynamic mission planning for unmanned aircraft.

B. Literature Review

The traveling salesman problem as a prototypical example for many tactical planning problems has received a significant amount of attention in the control community. The TSP for a single vehicle with non-holonomic dynamics has been addressed in [1], [2].
The problem of assigning and scheduling tasks for heterogeneous teams of unmanned vehicles has been studied with a non-linear cost function [3], with a varying number of agents and dynamic tasks [4], as a patrolling problem with revisit deadlines [5] and as a patrolling and synchronization problem [6]. Multi-vehicle problems with heterogeneous teams are studied for static tasks [7], dynamic tasks [8] and for two depots [9]. The problem of visiting neighborhoods rather than specific target locations is addressed for the single vehicle case [10] and for multiple vehicles [11]. The 2-opt heuristic is combined with multi-query path planning algorithms for task scheduling under realistic path cost predictions to achieve task scheduling in obstacle rich environments [12].

However, the edge cost tolerance problem for the TSP has not yet been addressed extensively within the control community. Computing the exact values of the tolerances for all edges in a TSP is NP–hard and can be achieved by solving auxiliary instances of the problem [13]. Furthermore, the topology of stability regions and subsets thereof are described in ref. [13].

Edge cost tolerances are well defined with respect to the problem instance, i.e., the tolerances do not depend on the chosen optimal tour. Furthermore, it is shown in ref. [14] that the exact tolerance for every edge in case of additive cost functions can be computed by solving two instances of the TSP for each edge. The 1-tree relaxation of the TSP provides lower bounds on tolerances [15]. Sensitivity information based on a 1-tree relaxation is used to improve the efficiency of an implementation of the Lin-Kernighan (LK) heuristic [16] which is an improvement heuristic for the TSP [17]. It employs two or three pairwise edge exchanges and is one of the most successful methods for generating near optimal solutions for the symmetric TSP. Ref. [18] analyzes the sensitivity of the LK heuristic and derives the tolerances of a tour with respect to pairwise edge exchanges to improve the LK heuristic. Exact edge cost tolerances for some and lower bounds on the edge cost tolerance for all other edges can be obtained by analyzing a set of m-best tours. However, the quality of the approximation depends on m and the instance of the problem [19].

The more general case of tolerances for integer programming for combinatorial optimization is discussed in ref. [20].

As the above discussion indicates, both the traveling salesman problem and the vehicle routing problem have been studied extensively in the literature. Moreover, the sensitivity of the TSP has received substantial attention. However, the development of reactive algorithms based on sensitivity of the TSP is still an open issue. Hence, this paper aims at bridging this gap.

C. Original Contributions

The primary contributions of this work are as follows:

- An approach to sensitivity analysis for the TSP through a linear programming relaxation.
- A polynomial time method to compute the stability region of the best tour in a set of tours.
- A polynomial time stability test determining whether a tour remains optimal within a set of tours for arbitrary additive disturbances to the edge costs.
- A polynomial time tolerance computation method for all edges contained in at least one tour of the set of tours with respect to the tours in the set.
- The specialization of the above stability region and edge cost tolerance results to k-opt tours with respect to the k-neighborhood.

In contrast to previous work, we focus on providing measures to efficiently predict whether a previously found tour remains optimal after arbitrary cost increases or decreases occur to an arbitrary number of edges anywhere in the graph representing the problem. In particular, in comparison to ref. [18], we allow changing the cost of multiple edges and characterize the stability region for simultaneously increasing and decreasing edge costs. Furthermore, our approach is applicable to the more general k-opt heuristic or m tour problem. Our work is most closely related to the work in ref. [19]; however, we focus on stability regions with respect to an arbitrary set of tours. This provides us with methods to consider m-schedules, i.e., in the case of the TSP a set of m tours that for example was previously approved by a human operator.

D. Overview

The remainder of this paper is organized as follows: Section II introduces some of the concepts used in the paper; Section III illustrates the approach used to solve this problem; Section IV shows the application of the suggested approach; and Section V concludes the paper.

II. Background

The traveling salesman problem (TSP) can be formulated as follows: Given a set P of n cities \( \{p_1, \ldots, p_n\} \) and an edge cost matrix \( C \), where \( c_{ij} \) denotes the cost of traveling from city \( p_i \) to city \( p_j \) on edge \( e_{ij} \), find the Hamiltonian tour, \( (tour \ for \ short) \ T^* \), out of the set of all tours, where \( T^*(g) \) denotes city \( p \) visited at the g-th step, such that \( J(T^*) = \sum_{g=2}^{n+1} c_{T^*(g),T^*(g-1)} \) is minimal. Solving the TSP is known to be NP-hard [19]. Thus, multiple heuristics exist to compute approximate solutions for the TSP.

The k-opt heuristic is a widely and successfully used improvement heuristic for the TSP. It is the basis for the above mentioned LK–heuristic. Let the k-neighborhood \( T' \) of a tour \( T \) be the set of all tours obtained by permutation of any k cities in the tour. Then the k-opt heuristic is an iterative method that goes from one iterate tour to the next by doing the following: (a) construct the k-neighborhood of the tour, (b) select as the next iterate the best tour in the k-neighborhood obtained in (a). Hence, the k-opt heuristic terminates at a so-called k-opt tour that is guaranteed to be optimal in its k-neighborhood.

To formalize the concepts for the study of edge cost tolerances and stability regions, we define these two related
concepts as follows, where potential additive disturbances to entries in \( C \) motivate sensitivity analysis:

**Definition 2.1 (Tolerance):** Let \( \tilde{C} \) be the edge cost matrix of a TSP, \( T^* \) be an optimal tour and \( e \) an edge between two cities. Then the additive upper tolerance \( \Delta \tilde{c}_{ij}^+ (e) \) (resp. lower tolerance \( \Delta \tilde{c}_{ij}^- (e) \)) of \( e \) with respect to \( T^* \) is the supremum of cost increases of \( e \) (resp. infimum of cost decreases of \( e \)) under which \( T^* \) remains optimal, provided the other elements of \( \tilde{C} \) are unchanged.

**Definition 2.2 (Stability region):** The stability region corresponding to a given tour for the traveling salesman problem is defined as the set of all cost matrices for which this tour is optimal.

### III. TECHNICAL APPROACH

#### A. Problem Formulation

The problem can then be formulated as follows: Given a set of tours \( \mathcal{T} \) for an instance of the TSP \( \mathcal{P} \) and let \( T^* \) denote the best tour within \( \mathcal{T} \), define the stability region of \( T^* \) with respect to \( \mathcal{T} \) and determine an expression to test whether an arbitrary disturbance to the edge cost matrix \( \tilde{C} \) causes another tour in \( \mathcal{T} \) to be better than \( T^* \).

#### B. 0-1 ILP formulation of the best tour problem

Solving the TSP can be understood as choosing the shortest tour \( T^* \) from the set of all possible tours for \( \mathcal{P} \), where \( \mathcal{P} \) is an instance of the TSP with \( n \)-cities. For the remainder of this paper we only consider symmetric instances of the TSP, i.e., \( \tilde{c}_{ij} = \tilde{c}_{ji} \). Let \( l \) denote the cardinality of the set of all edges. We introduce \( c \in \mathbb{R}^{l \times 1} \), a column vector containing all elements \( \tilde{c}_{ij} \) above the diagonal of the symmetric matrix \( \tilde{C} \), where the index of a vector element \( c_k \) is related to a matrix entry \( \tilde{c}_{ij} \) as given by the following relationship:

\[
k = n(i - 1) - \frac{(i - 1)i}{2} + j - i
\]

Furthermore, we enumerate the tours in the set of all possible tours and define a subset of tours \( \mathcal{T} \) of cardinality \( m \) with an induced enumeration, i.e., the \( r \)-th element in \( \mathcal{T} \) is the \( r \)-th element found when searching for elements of \( \mathcal{T} \) in the set of all possible tours.

**Definition 3.1 (x):** Let \( x \) be a column vector and let its dimension be the cardinality of \( \mathcal{T} \). Then for each element in \( \mathcal{T} \) there exists a vector \( x \) that is associated with it in the following way: All entries in \( x \) are binary and \( \sum_{r=1}^{m} |x_r| = 1 \). Thus, there exists only one non-zero entry \( x_r \) at position \( r \). The position \( r \) of the non-zero entry \( x_r \) then associates \( x \) with tour \( T_r \).

**Definition 3.2 (h):** Let \( h \) be a row vector of length \( l \). A vector \( h \) then identifies the edges \( e_{ij} \in T \) in the following way: For all edges \( e_{ij} \in T \) the corresponding entry \( h_k \), where \( k \) is computed using eq. (1), is one. All other entries are zero.

**Definition 3.3 (H):** Let \( H \) be a matrix of the following form: The \( r \)-th row of \( H \) is vector \( h_r \) which identifies the edges contained in tour \( T_r \in \mathcal{T} \).

Then, the problem of finding the best tour in \( \mathcal{T} \) can be formulated as follows:

\[
\begin{aligned}
\text{minimize} & \quad c^r H^T x \\
\text{subject to} & \quad \sum_{r=1}^{m} x_r = 1 \\
& \quad x_r \geq 0, \quad r = 1, \ldots, m, \\
& \quad x_r \in \mathbb{Z}, \quad r = 1, \ldots, m.
\end{aligned}
\]

This formulation is different from the classical ILP formulation of the TSP as for example given in ref. [21] by constraining and explicitly enumerating the set of candidate tours \( \mathcal{T} \). Let \( q = c^r H^T \) and substitute in eq. (2). Then, the cost coefficients \( q_r \) are linear combinations of the elements of the edge cost vector \( c \), where \( q_r \) represents the cost of the corresponding tour \( T_r \). Furthermore, introduce \( a^r \), a row vector of ones and appropriate length to rewrite the equality constraint, which yields the problem in canonical form with a single equality constraint:

\[
\begin{aligned}
\text{minimize} & \quad qx \\
\text{subject to} & \quad a^T x = 1 \\
& \quad x_r \geq 0, \quad r = 1, \ldots, m, \\
& \quad x_r \in \mathbb{Z}, \quad r = 1, \ldots, m.
\end{aligned}
\]

**Proposition 1:** Every solution \( x \) of the optimization problem (3) identifies a cheapest tour in \( \mathcal{T} \) through the scheme given in definition 3.1. Conversely, every cheapest tour \( T^* \) in \( \mathcal{T} \) is associated with a solution of optimization problem (3) through the same scheme.

**Proof:** Sufficiency: Let \( x \) be a solution of optimization problem (3), then \( x \) identifies a tour through the scheme in definition 3.1. We claim that that tour is a cheapest tour. To prove that claim let us assume that another tour is strictly cheaper. Then, that tour is associated with another \( x \) and that other \( x \) gives a strictly better cost in optimization problem (3) because that other tour is strictly cheaper. Therefore, \( x \) cannot be a solution of optimization problem (3). This contradiction completes the proof of sufficiency.

Necessity: Let \( T^* \) be a cheapest tour in \( \mathcal{T} \). This tour is associated with an \( x \) through the scheme of definition 3.1. We claim that \( x \) is a solution to optimization problem (3). To prove that claim, let us assume that there is another \( x \) that gives a strictly lower cost. Then, that other \( x \) is associated with another tour that is strictly cheaper than \( T^* \). Therefore, \( T^* \) cannot be a cheapest tour. This contradiction completes the proof.

#### C. LP relaxation of the 0-1 ILP formulation

The computational complexity of computing tolerances for a large class of 0-1 programming and combinatorial optimization problems (including the TSP) is known to be as hard as the optimization problem itself [22]. However, under some conditions the optimal solution to the linear programming LP relaxation of an ILP is guaranteed to optimally solve the ILP [21]. These conditions are that the constraint matrix \( A \) of the ILP is totally unimodular and that the right hand side of the constraints is integer [21].
Definition 3.4 (Totally unimodular): An integer matrix $A \in \mathbb{Z}^{m \times n}$ is totally unimodular if each square submatrix $S$ of $A$ has $|S| \in \{0, \pm 1\}$.

Eq. (3) satisfies these conditions. Therefore, sensitivity analysis is applied to the linear programming relaxation of optimization problem (3):

\[
\begin{align*}
\text{minimize} & \quad qx \\
\text{subject to} & \quad a^T x = 1 \\
& \quad x_r \geq 0, \ r = 1, \ldots, m, \\
& \quad x_r \in \mathbb{R}, \ r = 1, \ldots, m.
\end{align*}
\]

(4)

Note, that the search space of optimization problem (3) is a subset of the search space of optimization problem (4). Therefore, optimization problem (4) is a relaxation of optimization problem (3).

Theorem 3.5 ([21]): Let $A$ be a totally unimodular matrix and let $b$ be an integral vector. Then the polyhedron $P := \{x | Ax \leq b\}$ is integral.

Theorem 3.6 ([23]): Given a linear program in standard form where $A$ is an $m \times n$ matrix of rank $m$, i) if there is a feasible solution, there is a basic feasible solution, ii) if there is an optimal feasible solution, there is an optimal basic feasible solution.

Theorems 3.5 and 3.6 guarantee that the optimal solution to optimization problem (4) solves optimization problem (3). The following lemma establishes the converse.

Lemma 3.7: Every optimal solution to problem (3) is an optimal solution to problem (4).

Proof: Let $x$ be an optimal solution to problem (3). Then $x$ is feasible for problem (4), because problem (4) is a relaxation of problem (3). Assume $x$ is not optimal for problem (4), then there exists an $\hat{x}$ that is feasible for problem (4) and strictly better than $x$. Theorem 3.6 guarantees that the optimal solution of problem (4) happens at a vertex. Therefore, there must exist a vertex $\mathbf{x}$ that is better than $\hat{x}$. That vertex $\mathbf{x}$ is feasible for problem (3), because it is a vertex, and is strictly better than $x$, because it is better than $\hat{x}$ which was itself strictly better than $x$. Therefore, $x$ cannot be an optimal solution of problem (3). This contradiction completes the proof.

D. Stability region based on $\mathcal{J}$

The LP relaxation (4) is in canonical form. Therefore, the stability region of the solution $x^*$ to optimization problem (4) can be computed using the method given in ref. [24] for linear programs.

When there exist alternative optimal solutions to this problem, then there exists a set of optimal solutions in $\mathcal{J}$ with cardinality greater than one. This paper is not concerned with solving eq. (3), but with analyzing the sensitivity of one optimal tour $T^*$ that is associated with one $x$ through the scheme in definition 3.1. That tour could either be given by a human operator or, for example, obtained through the use of the k-opt heuristic. The stability region of that tour can be computed based on the LP relaxation.

Let there be a disturbance to cost vector $c$ characterizing the specific instance of the TSP $\mathcal{J}$, which results in an additive disturbance edge cost vector $\Delta c$. This affects the objective function coefficients $q_r$ as follows:

\[
q(\Delta c) = q + \Delta c^T H^T = c^T H^T + \Delta c^T H^T.
\]

(5)

Suppose there exists a solution with a basis $B$ that is optimal for $\Delta c = 0$. (The reader might refer to ref. [24] for more details on sensitivity analysis for linear programs).

Due to the equality constraint, $x^*$ only contains one non-zero entry $x_r$, which corresponds to the basic variable. Then the reduced cost vector is given by:

\[
\bar{q} = q_r a^T - q,
\]

(6)

where $q_r$ is the unperturbed objective function coefficient corresponding to $x_r$. Let $H_b$ denote the row of $H$ that corresponds to the basic variables of the optimal solution $x^*$. Then, let:

\[
\overline{P} = a H_b - H.
\]

(7)

In order to maintain basis $B$ as the optimal basis, it is necessary and sufficient for the vector parameter $\Delta c$ to satisfy the inequality [24]:

\[
-\Delta c^T \overline{P} \leq \bar{q}.
\]

(8)

In other words: By design of the equality constraint and from the fact that the solutions of interest are 0-1 integer feasible: For any chosen optimal solution, $\overline{P}$ has the following structure: The row corresponding to the optimal solution is all zeros. The other rows of $\overline{P}$ are structured as follows:

- Entries corresponding to edges that are contained in the chosen optimal tour and in the tour corresponding to the specific row are zero.
- Entries corresponding to edges that are contained in the chosen optimal tour but not in the tour corresponding to the specific row are 1.
- Entries corresponding to edges that are not contained in the chosen optimal tour but in the tour corresponding to the specific row are $-1$.

Thus, the number of $-1$ and $+1$ within one row is always identical, as for every removed edge from a tour, another edge must be added.

Proposition 2: Note that, as per theorems 3.5 and 3.6, every optimal solution to problem (4) is an optimal solution to problem (3). Then the stability region of every optimal integral solution to problem (4) is equal to the stability region of that solution as an optimal solution to problem (3).

Proof: Sufficiency: Let $x$ be an integral optimal solution to problem (4). Then by theorems 3.5 and 3.6 $x$ is also an optimal solution to problem (3). Let $\Delta c$ be an arbitrary perturbation to the data of problem (4) in the stability region of $x$, then $x$ is an optimal integral solution of problem (4) perturbed by $\Delta c$. By theorems 3.5 and 3.6 $x$ also solves the perturbed version of problem (3). Therefore, $\Delta c$ belongs to the stability region of $x$ as an optimal solution to problem (3).
Therefore, the stability region of $x$ as an optimal solution to problem (4) is a subset of the stability region of $x$ as an optimal solution to problem (3). This completes the sufficiency part of the proof.

Necessity: Let $x$ be an optimal solution to problem (3). Then by lemma 3.7, $x$ is also an optimal solution to problem (4). Let $\Delta c$ be an arbitrary perturbation to the data of problem (3) in the stability region of $x$, then $x$ is an optimal solution of problem (3) perturbed by $\Delta c$. By lemma 3.7, $x$ also solves the perturbed version of problem (4). Therefore, $\Delta c$ belongs to the stability region of $x$ as a solution to problem (4). Therefore, the stability region of $x$ as an optimal solution to problem (3) is a subset of the stability region of $x$ as an optimal solution to problem (4). This completes the proof.

Inequality (8) therefore defines the stability region of the optimal solution $x^*$ and its associated optimal tour $T^*$, such that for any edge cost perturbation vector $\Delta c$ that satisfies eq. (8), $T^*$ remains the optimal tour in $\mathcal{T}$.

E. Edge cost tolerances

Using eq. (8), the edge cost tolerances with respect to $\mathcal{T}$ can be computed as follows: To compute the tolerance for an arbitrary edge $e_{ij} \in T$, where $T \in \mathcal{T}$, with associated edge cost vector entry $c_{ik}$, set $\Delta c_w = 0$ for all $w \neq k$. This immediately yields the tolerance $\Delta c_k$ by identifying the most restrictive inequality resulting by row-wise evaluation of eq. (8).

F. Application to tours obtained through the k-opt heuristic

Based on above analysis, the sensitivity of solutions obtained using the k-opt heuristic with respect to the k-neighborhood can be obtained as follows: Given a k-optimal tour $T^*$ construct its k-neighborhood $T'$ and let $\mathcal{T}$ be $T' \cup T^*$. Then, eq. (8) yields the exact stability region of $T^*$ with respect to $T' \cup T^*$.

G. Complexity

Computing the stability region for an optimal tour $T^*$ with respect to a set of tours $\mathcal{T}$ is of polynomial time complexity in the cardinality of $\mathcal{T}$, i.e., $m$ and the number of cities $n$. Rather than re-computing the cost of all tours in $\mathcal{T}$ and sorting them according to their length after edge cost changes occur, the method in this paper characterizes the set of all admissible edge cost changes independent of one specific instance of edge cost perturbations. Matrix $\mathcal{H}$ and $\mathcal{Q}$ can be computed before any edge cost changes occur. Then, when edge cost changes occur, evaluating eq. (8) correctly determines if the previously optimal tour remains optimal in the set of tours considered. This test requires the evaluation of a system of $m$ inequalities given by eq. (8). With $l$ being the size of the cost vector $c$ obtaining the set of inequalities requires the multiplication of the disturbance cost vector $\Delta c$ with matrix $\mathcal{H} \in \mathbb{Z}^{m \times l}$. Therefore, an optimality test can be performed in polynomial time as a function of $n$ and $m$.

Based on eq. (8), tolerances can also be computed in polynomial time. This requires $l$-times the identification of the most restrictive inequality of the system of $m$ inequalities.

For the special case of 2-optimal tours note that the cardinality of the set of tours grows quadratically with the number of cities: $|T'| = \sum_{m=1}^{n-2} m$.

IV. Example

The following section demonstrates the application of stability analysis to the solution to the Euclidean 6 city TSP using the 2-opt heuristic similar to the motivating example in fig. 1. The targets are located at the vertices of a regular hexagon with edge length one. The unperturbed edge costs correspond to the Euclidean distance between the targets. The 2-opt heuristic yields the optimal tour $T^* = (1, 2, 3, 6, 5, 4, 1)$. The cost of multiple edges is then perturbed by a disturbance vector $\Delta c = [1110\ldots 0]^T$. Fig. 2 depicts the stability region for changes in the corresponding edge costs $c_{12}, c_{13}, c_{14}$ obtained through evaluation of eq. (8), where the red cross indicates the disturbance vector. As the disturbance vector is within the stability region, $T^*$ remains optimal in its 2-neighborhood, which leads to the scenario depicted in fig. 3, where the incremental increase in edge cost is indicated at the respective edges.

Further increasing the magnitude of the disturbances in the disturbance vector to $\Delta c = [11120\ldots 0]^T$ yields a disturbance vector outside the stability region as shown in fig. 4. When re-starting the 2-opt heuristic from the nominal solution $T^*$, the heuristic finds a new locally optimal solution $T_* = (1, 2, 3, 6, 4, 5, 1)$ as shown in fig. 5.

V. Summary, Conclusions, and Future Work

In summary, this paper addressed the problem of sensitivity analysis for task schedules to changes in task cost. In particular, we analyze dynamically changing travel costs between cities in the TSP problem, which captures a large set of UAV task scheduling problems. Stability analysis is used to determine whether a previously computed tour remains optimal among a set of tours. This set could either be designed and approved by a human operator or obtained through the use of the k-opt heuristic. Sensitivity analysis of the linear programming relaxation yields the desired stability information. Through the special nature of the problem formulation, the sensitivity results from the LP relaxation translate back to the original problem. In this paper, we obtain a polynomial time method to determine whether a tour remains optimal after arbitrary task cost changes occur and more specifically in the traversal cost between arbitrary city pairs in the case of the traveling salesman problem. Furthermore, we present a polynomial time method to compute the stability region for the best tour in a set of tours and derive tolerances from it for all edges contained in at least one of the tours. Future work will focus on how to choose subsets of tours to obtain global stability information and methods to efficiently re-optimize tours after a tour has become suboptimal. Furthermore, we are also considering replacing the prototypical TSP with more realistic and detailed UAV scenarios.
REFERENCES


