Simultaneous Fault Detection, Isolation and Control Tracking Design using a Single Observer-based Module

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Abstract—In this paper, the problem of simultaneous fault detection, isolation and tracking (SFDIT) design for linear continuous-time systems is considered. An $H_\infty/H_\infty$ formulation of the SFDIT problem using a dynamic observer is developed. A single module based on a dynamic observer is designed which produces two signals, namely the residual and the control signals. The SFDIT module is designed such that the effects of disturbances and reference inputs on the residual signals are minimized (for accomplishing fault detection) subject to the constraint that the transfer matrix function from the faults to the residuals is equal to a pre-assigned diagonal transfer matrix (for accomplishing fault isolation), while the effects of disturbances, reference inputs and faults on the specified control output are minimized (for accomplishing fault-tolerant control and tracking problems). Sufficient conditions for solvability of the problem are obtained in terms of linear matrix inequality (LMI) feasibility conditions. Simulation results for an autonomous unmanned underwater vehicle (AUV) illustrate the effectiveness of our proposed design methodology.

Keywords: Simultaneous fault detection, isolation and tracking (SFDIT), Autonomous unmanned underwater vehicle (AUV), Linear matrix inequality (LMI).

I. INTRODUCTION

Model-based fault detection and isolation (FDI) schemes have attracted considerable interest over the past decades (see, e.g., [1]–[3] and the references therein). Among model-based approaches, the most common one is to use state observers or filters to construct residual signals and compare them with predefined thresholds. When the so-called residual evaluation function has a value larger than a threshold, an alarm is generated [1]. However, noise and disturbances may impact in significant ways the residual, leading to false alarms. Hence, fault detection filters have to be sensitive to faults and simultaneously robust to noise and disturbances. In [2], the fault detection filter design is formulated as an $H_\infty$-filtering problem, where the errors between residuals and faults are minimized. The authors of [3] considered the problems of $H_\infty$-index and multi-objective $H_\infty/H_\infty$ fault detection observer design via LMI conditions. Different performance indices are considered for the optimal selection of the post-filters as well as optimization of the fault detection filters in [4].

In most of the current literature, an open-loop model of the process is considered and/or is assumed that the controller maintains the stability of the closed-loop system upon the failure. This assumption may not be valid for many practical closed-loop feedback systems [5]. This motivates consideration of the problem of simultaneous fault detection and control (SFDIT) that has attracted interest in the past two decades [6]–[8]. The simultaneous design unifies the control and fault detection modules into a single unit which can result in lower complexity as compared to the case of separate designs. It is also a desirable approach since the design of each module should take into account the other unit’s considerations. The problem of SFDIT using dynamic observers for continuous-time linear time-invariant and linear switched systems is considered in [9] and [10], respectively. In [11], the problem of SFDIT for linear switched systems in the discrete-time and continuous-time under a mixed $H_\infty/H_\infty$ framework is considered. In [12], the problem of SFDIT using dynamic observer for an autonomous unmanned underwater vehicle (AUV) is considered. In [13], a brief survey of the integrated design of feedback controllers and fault detectors is presented.

To the best of our knowledge, the current literature in the field of SFDIT suffers from the following limitations and drawbacks. First, most of the literature that considers the problem of SFDIT can achieve the control objective of “regulation” but none of them consider the problem of “tracking” in the SFDIT design. Second, although most of the current references in the field of SFDIT can achieve acceptable fault detection [6]–[14], they cannot achieve fault isolation. On the other hand, it should be noted that the fault isolation objective in most of the current literature in the field of FDI is accomplished through the use of a bank of observers (see, e.g., [15]–[18]). For implementation of these methodologies one requires to have a bank of observers where each observer dimension is usually equal to the dimension of the entire system, and this puts a heavy computational burden on the FDI system. Therefore, reducing the dimension or the number of observers is of great significance and importance.

As pointed out above, although there are certain published works in the field of SFDIT, none of them is capable of detecting and isolating simultaneous faults in the system as well as tracking the specified reference input. Therefore, in this paper,
we propose an $H_\infty/H_\infty$ formulation of the simultaneous fault detection, isolation and tracking (SFDIT) problem by using dynamic observer detector and state feedback controller for linear continuous-time systems. It should be noted that most of the existing FD and SFDC observers have been simply confined to traditional so-called “static” observers (classical Kalman-Luenberger observer). In order to distinguish from static observers, the term “dynamic” observer is used, which is an extension of the static observer in its configuration and embeds dynamics in the observer gain [19]. It has been stated in [9] that applying dynamic observers in the structure of SFDC block in contrast with the static observer-based SFDC, has a major advantage in the sense that one can obtain strict LMI conditions for designing the dynamic observer parameters and controller gains. In this paper, it is shown that by applying a single dynamic observer-based module whose order is equal to twice the order of the system, the SFDIT problem can be solved based on strict LMI feasibility conditions. Consequently, the computational complexity from the view point of the number and dimension of the required observer is significantly reduced in comparison with all the existing methodologies. Moreover, using this approach the system can not only detect and isolate the occurred faults but also able to track the specified reference input. Moreover, the proposed method can also handle isolation of simultaneous faults. To summarize, the main contributions of this paper in comparison with other methodologies in the fields of FD, FDI and SFDC are stated in Table I. It should again be emphasized that by using our proposed methodology one can achieve the isolation and the tracking objectives in addition to the SFDC problem objectives. This is the main advantage of our proposed methodology in comparison with the earlier works in [9]–[11].

| Table I: Comparison of the SFDIT scheme with other methodologies; where SO denotes the “single observer”, BGO denotes the “bank of general observers”, BDO denotes the “bank of dedicated observers”, NO denotes the “number of observers”, SF denotes the “single fault”, MF denotes the “multiple fault”, and $n_f$ denotes the number of faults in the system. |

A. System description

Consider the following linear time-invariant system:

$$
G : \begin{cases}
\dot{x}(t) = Ax(t) + B_1 u(t) + B_2 d(t) + B_3 f(t) \\
y(t) = C x(t) + D_1 u(t) + D_2 d(t) + D_3 f(t)
\end{cases},
$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $y(t) \in \mathbb{R}^{n_y}$ is the measured output, $d(t) \in \mathbb{R}^{n_d}$ is the external disturbance, and the unknown input $f(t) \in \mathbb{R}^{n_f}$ denotes a possible fault. The matrices $A, B_i's, C$ and $D_i's$ $(i=1,2,3)$ are assumed to be known constant matrices of appropriate dimensions. Note that the fault matrices $B_3$ and $D_3$ are specified according to the faults that are to be detected in the components, actuators, or sensors. For simplicity, the model uncertainties are assumed to be recast as disturbances, although other types of uncertainties can be formally investigated in our future work.

The reference model is described as follows:

$$
G_r : \begin{cases}
\dot{x}_r(t) = A_r x_r(t) + B_r u_r(t) \\
y_r(t) = C_r x_r(t)
\end{cases},
$$

where $x_r(t) \in \mathbb{R}^n$ is the state vector, $u_r(t) \in \mathbb{R}^{n_{ur}}$ is the energy bounded input vector and $y_r(t) \in \mathbb{R}^{n_y}$ is the output vector, respectively. $A_r, B_r$ and $C_r$ are constant matrices with appropriate dimensions. It is assumed that $A_r$ is Hurwitz and $x_r(t)$ is measurable to be used for control signal.

The following module $F$ is now proposed for the SFDIT problem:

$$
F : \begin{cases}
\dot{x}(t) = Ax(t) + B_1 u(t) + n(t) \\
\dot{y}(t) = C x(t) + D_1 u(t) \\
u(t) = -K (\hat{x}(t) - x_r(t)) \\
r(t) = y(t) - \hat{y}(t)
\end{cases},
$$

where $n(t) \in \mathbb{R}^n$ denotes the correction signal, the dynamics of which is given by:

$$
\begin{cases}
\dot{x}_d(t) = A_d x_d(t) + B_d r(t) \\
n(t) = C_d x_d(t) + D_d r(t)
\end{cases},
$$

Effectiveness of our proposed SFDIT strategy is it applies to a linear model of an AUV.

The remainder of this paper is organized as follows. In Section II, the system description and problem formulation are provided. The LMI-based solution to the SFDIT problem is described in Section III. To demonstrate the validity of our proposed approach, its application to an AUV is given in Section IV, which is followed by conclusions in Section V.

**Notation:** For a matrix $A$, $A^T$ denotes its transpose, and $I$ and 0 denote the identity and zero matrices with appropriate dimensions, respectively. The Hermitian part of a square matrix $A$ is denoted by $\text{Herm}(A) = A + A^T$, and $*$ denotes the symmetric entries of a matrix.

## II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this section, we first describe the system and the SFDIT module governing equations. The SFDIT problem is then formulated for the linear continuous-time systems.

A. System description

Autonomous underwater vehicles have become an increasingly important tool in a number of applications over the past recent years. As AUVs become more common, and in particular as missions become longer and take place in less well-known environments, improving the reliability of the vehicle becomes even more important [20]. Therefore, design of reliable FDI and SFDC systems becomes one of the core technologies in the field of AUV research. To illustrate the
and \( \hat{x}(t) \in \mathbb{R}^n \) denotes the estimate of \( x(t) \), \( \hat{y}(t) \in \mathbb{R}^ny \) denotes the observer output, \( x_d(t) \in \mathbb{R}^n \) denotes the residual signal and the constant matrices \( A_d, B_d, C_d, D_d \) denote the observer parameters to be designed subsequently.

By substituting the SFDIT module equations (3) and (4) into the system equation (1), and defining the control output \( z(t) = \hat{y}(t) - y_r(t) \) (to achieve the tracking objective), the following closed-loop system dynamics \( G_C \) is obtained:

\[
G_C : \begin{cases} \\
\dot{\xi}(t) = A\xi(t) + \bar{B}_d d(t) + \bar{B}_f f(t) + \bar{B}_ru_r(t) \\
r(t) = C_1 \xi(t) + D_d d(t) + D_f f(t) \\
z(t) = \bar{C}_2 \hat{x}(t) 
\end{cases}, \tag{5}
\]

where \( e(t) = x(t) - \hat{x}(t) \), and:

\[
\bar{A} = \begin{bmatrix} A_r & 0 & 0 & 0 \\
B_d K & A - B_1 K & D_d C & C_d \\
0 & A - D_d C & -C_d \\
0 & 0 & B_d C & A_d \end{bmatrix},
\]

\[
\bar{B}_d = \begin{bmatrix} D_d D_2 \\
B_2 - D_d D_2 \\
0 \\
B_d D_3 \end{bmatrix}, 
\bar{B}_f = \begin{bmatrix} D_d D_3 \\
B_3 - D_d D_3 \\
B_d D_3 \\
B_d D_3 \end{bmatrix}, 
\bar{B}_r = \begin{bmatrix} 0 \\
0 \\
0 \\
C \frac{D_3}{D_2} \end{bmatrix},
\bar{C}_2 = \begin{bmatrix} D_1 K - C_r \\
C - D_1 K \\
0 \\
0 \end{bmatrix},
\xi^T(t) = \begin{bmatrix} x_r(t)^T \\
\hat{x}(t)^T \\
e(t)^T \\
x_d(t)^T \end{bmatrix}. \tag{6}
\]

Note that, since it will be shown subsequently in Theorem 1 that \( e \rightarrow 0 \) when \( f(t) = d(t) = u_r(t) = 0 \), and \( \hat{y} \rightarrow y \), therefore considering \( z(t) = \hat{y}(t) - y_r(t) \) as a control output guarantee the tracking problem. However, without loss of generality, it is also possible to consider \( z(t) = y(t) - y_r(t) \) as the control output.

In the next subsection the SFDIT design problem is transformed into an \( H_\infty \) optimization problem.

**B. Problem formulation**

The simultaneous fault detection, isolation and tracking (SFDIT) problem to be addressed in this paper can be stated as follows.

**The SFDIT Problem:** The problem is to design for the system (1) the SFDIT module (3) and (4) such that, (a) the augmented system (5) is stable, (b) the effects of disturbances and reference inputs on the residual \( r(t) \) are minimized and the effects of faults on the residual \( r(t) \) are maximized (in order to accomplish fault detection), (c) each element of the residual \( r(t) \) is only sensitive to a specified potential fault (in order to accomplish fault isolation), and (d) the effects of the disturbance, fault and reference inputs on the control output \( z(t) \) are minimized (in order to accomplish the control performance and tracking).

Stated alternatively, our goal is to design the SFDIT block (3) such that the augmented system (5) is stable and the following optimization problem is satisfied:

\[
\text{minimize } \beta_1 \gamma_1 + \beta_2 \gamma_2 + \beta_3 \gamma_3 + \beta_4 \gamma_4 + \beta_5 \gamma_5 + \beta_6 \gamma_6 \\
\text{subject to:} \tag{7}
\]

\[
(\text{I}) \quad ||Tzd(s)||_\infty < \gamma_1, \quad (\text{II}) \quad ||Tzf(s)||_\infty < \gamma_2, \quad (\text{III}) \quad ||Tzu_r(s)||_\infty < \gamma_3, \quad (\text{IV}) \quad ||Trd(s)||_\infty < \gamma_4, \quad (\text{V}) \quad ||Trf(s)||_\infty < \gamma_5, \quad (\text{VI}) \quad ||T_{rf}(s)-T_0(s)||_\infty < \gamma_6,
\]

where \( T_0 \) is now selected as having the following structure:

\[
T_0 = J = \text{diag}(j_1, \ldots, j_{n_f}), j_i > 0, \forall i \in \{1, \ldots, n_f\}, \tag{8}
\]

and \( Tzd(s) = \bar{C}_2(sI - \bar{A})^{-1}\bar{B}_d, Tzf(s) = \bar{C}_2(sI - \bar{A})^{-1}\bar{B}_f, Tzu_r(s) = \bar{C}_2(sI - \bar{A})^{-1}\bar{B}_r, Trd(s) = \bar{C}_3(sI - \bar{A})^{-1}\bar{B}_d + D_2, Trf(s) = \bar{C}_3(sI - \bar{A})^{-1}\bar{B}_f + D_3, T_{rf}(s) = \bar{C}_3(sI - \bar{A})^{-1}\bar{B}_r. \)

The positive constant weights \( \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \) and \( \beta_6 \) can be used by the designer as a trade-off among the objectives (I)-(VII). The performance indices (I)-(III) are used to attenuate the effects of the disturbance, fault and reference inputs on the control output. The performance indices (IV) and (V) are used to attenuate the effects of the disturbance and reference inputs on the residuals. The performance indices (VI) and (VII) are used to guarantee a minimum level of sensitivity of the residuals to the fault signals and to ensure fault isolation.

**Remark 1.** It should be noted that in this work we have selected the simplest choice of \( T_0 \) (namely, a fixed diagonal matrix). Refer to [21] for more details on other choices for \( T_0 \). It should also be noted that the requirement of \( T_0 \) being diagonal is made to ensure the fault isolability in case that multiple faults may occur simultaneously.

In the next section, our SFDIT problem will be solved for the closed-loop system (5) subject to the conditions in equation (7).

**III. MAIN RESULT**

There are seven performance indices (I)-(VII) that must be satisfied simultaneously for solving the SFDIT problem for the closed-loop system (5). In the following a feasible solution to the SFDIT problem is obtained by considering these indices simultaneously.

**Theorem 1.** Consider the augmented closed-loop system (5) and let \( \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \) and \( \beta_6 \) be given positive constants. The closed-loop system (5) is stable and the performance indices (I)-(VII) are guaranteed to achieve simultaneous fault detection, isolation and tracking if there exist symmetric positive-definite matrices \( Q_{11}, P_{11}, X, A_k, B_k, C_k, D_k, M, J \) and positive scalars \( \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6 \) such that the following optimization...
The control gain $d$ are given by:

$$d = P_{12}^{-1} (A_k - P_{11} (A - D_d C) Q_{11} - P_{12} B_d C Q_{11} + P_{11} C_d Q_{12}^T) (Q_{12})^{-1},$$

where $P_{12}$ and $Q_{12}$ are invertible matrices which satisfy $P_{12} Q_{12} = I - P_{11} Q_{11}.$

**Proof.** First, note that by applying the bounded-real lemma (BRL) [22], condition (1) is satisfied if and only if there exists a positive definite matrix $P$ such that the following inequality holds:

$$\begin{bmatrix} \tilde{A}^T P + PA & PB_d^T C_d^T \\ -\gamma_2^T I & -I \end{bmatrix} < 0,$$

(11)

Let us assume that the matrix $P$ has the following structure:

$$P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}, \quad P_{12} = P_{11} P_{12}.$$

(12)

Using the above structure for $P$ the condition (11) can be rewritten as:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ \* & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} \\ \* & \* & \* & \* & \* & \* \\ \* & \* & \* & \* & \* & \* \\ \* & \* & \* & \* & \* & \* \\ \* & \* & \* & \* & \* & \* \end{bmatrix} < 0,$$

(13)

where:

$$\Omega_{11} = P_{11} A + A^T P_{11}, \quad \Omega_{12} = K^T B_{11} P_{11}, \quad \Omega_{13} = 0,$$

$$\Omega_{14} = 0, \quad \Omega_{22} = P_{12} (A - B_1 K) + (A - B_1 K)^T P_{12},$$

$$\Omega_{23} = P_{12} D_{d1} C, \quad \Omega_{25} = P_{11} D_{d2}, \quad \Omega_{24} = P_{12} C_d,$$

$$\Omega_{15} = K^T D_{11}^T C_r C_{26} = 0, \quad \Omega_{16} = 0, \quad \Omega_{46} = 0, \quad \Omega_{66} = -\gamma_2^T I,$$

(14)

$$\Omega_{33} = \text{Herm} \left( P_{22} \begin{bmatrix} A & -D_d C \\ B_d C & A_d \end{bmatrix} \right),$$

$$\Omega_{35} = P_{22} \begin{bmatrix} B_d - D_d D_d \end{bmatrix}.$$

Let us further assume that $X = P_{11}^{-1}$ and $Q = P_{22}^{-1}$, and $Q$ is partitioned into:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix},$$

(15)

Define the matrices $\Pi_1$ and $\Pi_2$ as follows:

$$\Pi_1 = \begin{bmatrix} Q_{11} & I \\ Q_{12}^T & 0 \end{bmatrix}, \quad \Pi_2 = P_{22} \Pi_1 = \begin{bmatrix} I & P_{11} \\ 0 & P_{12} \end{bmatrix}.$$ (16)

Now by pre- and post-multiplying the inequality (13) by diag($X, X, X^T, I, I$) and diag($X, X, X, X^T, I, I$), respectively, and defining $A_k = P_{11} (A - D_d C) Q_{11} + P_{12} B_d C Q_{11} - P_{11} C_d Q_{12}^T + P_{12} A_d Q_{12}^T$, $B_k = P_{11} D_d + P_{12} B_d$, $C_k = -D_d C Q_{11} - C_d Q_{12}^T$, $D_k = D_d$ and $M = KX$, the second inequality in the expression (9) is obtained.

The performance indices (11)-(16) can also be shown by employing the same procedure and following along the same lines as in the derivations of the performance index (1). These details are omitted due to space limitations.

Note that $X > 0$ and $P_2 > 0$ and by using the definition of $\Pi_1$ and $\Pi_2$ in equation (16), $P_2 > 0$ is equivalent to:

$$\Pi_1^T P_2 \Pi_1 = \Pi_2^T \Pi_1 = \begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0,$$

(17)
It is now straightforward to express the inequality (VII) as a matrix inequality, namely
\[ \|T_0\|^{-1} \geq 1 \iff \|J\|^{-1} \geq 1 \iff I - J \leq 0. \] This completes the proof of the theorem.

Residual Evaluation Criterion:
Following the construction of the residuals \( r_i(t) \in \mathbb{R}, \forall i \in \{1, \ldots, n_o\} \), the final step in the SFDIT strategy is to determine thresholds \( J_{ih} \) and evaluation functions \( J_r(t) \). Various evaluation functions can be considered [1].

In this work, upper and lower threshold values are selected as \( J_{ih}^u = \sup_{t_i=0, u_i \in d} r_i(t) \) and \( J_{ih}^l = \inf_{t_i=0, u_i \in d} r_i(t) \), respectively, where \( d \) represents the disturbances and \( \mathcal{D} \) denotes the set of allowable disturbances. For instance, \( \mathcal{D} = \mathbb{L}_2 \) or \( \mathcal{D} \) can be selected as a set of Gaussian white noise. Based on the selected thresholds and the evaluation function taken as \( J_r(t) = r_i(t) \), the occurrence of a fault can then be detected and isolated by using the following decision logic: \( r_i(t) > J_{ih}^u \) or \( r_i(t) < J_{ih}^l \) \( \implies f_i \neq 0. \)

IV. Case study

Application of our methodology to a linearized longitudinal model of the Subzero II AUV is presented in this section to illustrate the effectiveness of our proposed method.

The linearized longitudinal model of the Subzero II AUV is described as follows [23]:
\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_1u(t) \\
y(t) &= Cx(t)
\end{align*}
\]
where \( A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}, x = \begin{bmatrix} u & w & q & \theta & z & n & \delta s^T \end{bmatrix}^T, u = \begin{bmatrix} M_d & \delta s_d \end{bmatrix}^T, y = \begin{bmatrix} u & z \end{bmatrix}^T, \end{align*} \]
\( u \) is the forward speed, \( w \) is the vertical speed, \( q \) is the pitch rate, \( \theta \) is the pitch angle, \( n \) is the propeller rotation speed, \( \delta s \) is the control surface deflection, \( M_d \) is the motor command, \( \delta s_d \) is the control surface command, and
\[
A_1 = \begin{bmatrix}
-0.5558 & 0.0474 & 0.0516 & 0.0038 & 0 \\
0.001 & -2.1258 & -0.3734 & -0.0175 & 0 \\
-0.0365 & -7.9966 & -8.7065 & -0.6474 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & -1.3009 & 0 \\
7.2126 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]
\[
A_2 = \begin{bmatrix}
0.0582 & 0.0797 \\
-0.0001 & -1.8211 \\
-0.0038 & -13.4438 \\
0 & 0 \\
0 & 0 \\
-4.0813 & 0.0492 \\
0 & -11.53
\end{bmatrix}, B_1 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The disturbance matrices \( B_3, D_3 \) are selected as \( B_2 = 0 \) and \( D_2 = I \) which represent the occurrence of sensor noise in the system (as this is common in industrial applications). The fault matrices \( B_3 \) and \( D_3 \) are selected as \( B_3 = 0 \) and \( D_3 = I \) which show the occurrence of sensor fault in the system.

The reference model matrices for the tracking objective are selected as \( A_r = \text{diag}(-3, -3, -4, -4, -5, -6, -7) \), \( C_r = C \), and:
\[
B_r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 27.9 & 0 \end{bmatrix}^T.
\]

It should be noted that selection of the model reference matrices are not unique and different choices of model reference matrices lead to different tracking errors. Therefore, according to the specifications of the Subzero II AUV model we have attempted to select the appropriate matrices for the model reference.

In the simulations conducted and without loss of generality the disturbance \( d(t) \in \mathbb{R}^2 \) that is injected to the system is band-limited white noise with a power of 0.0001 and the reference inputs \( u_{r1} \in \mathbb{R} \) and \( u_{r2} \in \mathbb{R} \) are selected as rectangular pulsed signals with amplitudes of 5 and 7 that are present from 10 s to 30 s and 50 s to 100 s, respectively. The fault signal \( f_1(t) \) is simulated as a rectangular pulsed signal with an amplitude of 0.25 that has occurred during the interval 50 s to 55 s, and the fault signal \( f_2(t) \) is simulated by a soft bias (slope = 0.1) that has occurred during the interval 75 s to 85 s.

It is desired that the system detects and isolates the occurrence of faults \( f(t) \in \mathbb{R}^2 \) in the presence of disturbances \( d(t) \in \mathbb{R}^2 \) and reference inputs \( u_r \in \mathbb{R}^2 \) and also tracks the desired output \( y_r(t) \in \mathbb{R}^2 \).

It is worth noting that by considering the worst case analysis of the residuals corresponding to the healthy operation of the subsystems being subjected to various disturbances, the threshold values \( J_{ih}^u \) and \( J_{ih}^l \) are selected.

Positive constant weights \( \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \) and \( \beta_6 \) are assumed to be equal to one, which implies the same level of importance in fault detection, isolation and tracking objectives (the objectives (1)-(VII)). The optimization problem of the Theorem 1 was solved and the SFDIT module matrices \( (A_d, B_d, C_d, D_d, K) \) were obtained. Moreover, the optimization parameters \( \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6 \) were obtained as \( \gamma_1 = 0.197, \gamma_2 = 0.4268, \gamma_3 = 0.4268, \gamma_4 = 0.1461, \gamma_5 = 1.0303 \) and \( \gamma_6 = 1.0303 \).

The residual signals are shown in Fig. 1, where the robustness against the disturbances, reference inputs and the enhanced fault sensitivity can be seen. Also the faults are satisfactorily discriminated from the disturbances and reference inputs. Note that by using our threshold test the faults \( f(t) \in \mathbb{R}^2 \) are effectively detected and isolated.

The system and reference outputs are shown in Fig. 2. From this figure it follows that the controlled system output can satisfactorily track the reference output in presence of disturbances and faults signals. Note that there are tracking errors in Fig. 2 during the occurrence of faults in the system. The source of these errors is due to the formulation of the SFDIT problem in condition (7). Indeed, the performance index (11) in condition (7) is used to attenuate the effects of fault signals on the control output \( z(t) \). This index guarantees that the energy of the fault signal (\( \|f(t)\|_2 \)) to the energy of
Fig. 1: The residual signals of (a) $r_1(t)$, (b) $r_2(t)$, where the solid lines denote the residual signals and the dash-dot lines denote the residual upper and lower thresholds.

Fig. 2: The system and reference outputs (a) $y_1(t)$ and $y_{r_1}(t)$, (b) $y_2(t)$ and $y_{r_2}(t)$, where the solid lines denote the system outputs and the dash-dot lines denote the reference outputs.

the control output ($\|z(t)\|_2$) is less than a satisfactory level represented by $\gamma_2$. Therefore, one cannot guarantee that the tracking error is zero during the occurrence of faults in the system, and there will always be some tracking errors during the presence of faults.

The above simulations and results demonstrate that indeed our proposed SFDIT scheme does achieve the simultaneous fault detection, isolation, and tracking requirements satisfactorily.

V. CONCLUSION

In this paper, an $H_{\infty}/H_\infty$ formulation of the simultaneous fault detection, isolation and tracking (SFDIT) problem for linear continuous-time systems using a dynamic observer has been developed and presented. An LMI approach for the SFDIT design has been introduced, which in addition to stabilizing the closed-loop system, simultaneously guarantees accomplishing fault detection, isolation and also control tracking objectives. Moreover, the SFDIT problem is solved such that each element of the residual vector is only sensitive to a specified fault, and therefore occurrence of simultaneous faults in the system can also be handled. Application of our methodology to the linearized longitudinal model of the Subzero II AUV is also presented to demonstrate and illustrate the effectiveness of the proposed approach.

REFERENCES


