Constrained Optimal Control of Multi-Dynamometer Internal Combustion Engine Test Benches

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Abstract—Dynamical internal combustion engine test benches are commonly used in automotive development to enforce reproducible operating conditions for testing and calibration. The associated control problem consists in tracking simultaneously speed references and torque references and can be described in terms of a nonlinear optimal control problem. In most cases, only one internal combustion engine and one dynamometer are used, either an electrical dynamometer or a hydrodynamic dynamometer, each offering different advantages. In some cases, a so-called tandem configuration is meaningful, using a small electrical load machine for baseline operation and a hydrodynamic dynamometer for peak demands, or vice versa. Due to the asymmetric properties of the hydrodynamic dynamometer, the problem cannot be stated directly in terms of a Hamilton-Bellman-equation, but needs an extension. This paper presents such a solution based on a nonlinear mapping of the asymmetric input constraints and a dynamic extension to the state of the system and examines the usefulness of such an approach. Measurements at a test bench are used to evaluate the performance of the proposed control approach for typical cases.

I. INTRODUCTION

In optimal control problems a control input is determined such that a desired cost functional is minimized along the trajectory of the resulting closed-loop system. The standard approach to solve this problem is based on the solution of the first-order nonlinear Hamilton-Jacobi-Bellman (HJB) partial differential equation, see e.g. [1]–[3]. Several methodologies (see e.g. [4]–[6]) have been proposed to approximate the solution of the HJB partial differential equation in a neighborhood of the origin with a desired degree of accuracy as the determination of the explicit solution is hard or even impossible in practical cases. An approach to approximate the solution of the HJB partial differential equation by means of a dynamic extension to the state is presented in [7] and successfully applied to a test bench for internal combustion engines in [8] and the air path of a turbocharged Diesel engine in [9].

Input constraints appear in many real-world applications and should be considered when designing a controller. The problem of controlling systems with bounded inputs has been extensively addressed in the past (see e.g. [10], [11]), whereas optimal control in presence of bounded inputs has been rarely addressed. In [12] the authors have used nonlinear functions to map the input constraints to the state equations and some method to calculate the solution of the optimal control problem can be used subsequently.

A further challenge arises from the redundancy of actuators, see e.g. [13], [14]. This challenge has been rarely treated in the control of dynamic internal combustion engine test benches as setups with multiple dynamometers have been seldom used in the past. Both a hydrodynamic dynamometer and an asynchronous motor are coupled to an internal combustion engine in [15], the control of the dynamometers with special interest on stationary testing is done by means of feedback controls with gain scheduling.

This paper considers the design of an optimal multi-input multi-output controller of an internal combustion engine test bench equipped with both a hydrodynamic dynamometer and an electric dynamometer. The design of the controller is based on models of the mechanics of the test bench, the internal combustion engine and both dynamometers. The input constraints are mapped to the state equations of the system by nonlinear functions. The distribution of the load between the two dynamometers depends on the specific setup and application, and is treated in the determination of the references of the system states. By the use of the proposed approach and the extension with an electric dynamometer, test benches are provided that offer a sufficient dynamics to perform dynamic test cycles and a wide operation range.

An adaptation of this approach to other applications like a hybrid electric vehicle seems straightforward. In a vehicle with serial hybrid drive the power is provided by an internal combustion engine and an electric motor. The power delivered by each actuator is determined by the energy management e.g. depending on the state of charge of the battery. This management is often designed using optimal control-based techniques, see e.g. [16].

The paper is organized as follows: The test bench equipped with a hydrodynamic dynamometer and an electric dynamometer is described in Section II, where also the problem is formulated and the design of the dynamic control law is briefly introduced. Results are given in Section III. The paper is concluded with comments on the proposed methodology and a future outlook in Section IV.

II. SYSTEM DESCRIPTION & PROBLEM FORMULATION

The speed of a vehicle is the result of the engine torque and the load being the direct consequence of the road and vehicle conditions. The operation of an internal combustion engine in a vehicle is simulated at a test bench without this vehicle. Therefore, the load has to be computed and enforced by at least one dynamometer at the test bench. Indeed, a test bench guarantees reproducible conditions e.g. in terms of
The parameters $\theta_E$ and $\theta_D$ are the inertias of the internal combustion engine and the summed inertias of the dynamometers, respectively. The inertias of the connecting shaft and the measurement flange have been already included in these values. The constant $c$ is the stiffness of the shaft connecting the internal combustion engine and the hydrodynamic dynamometer, $d$ denotes the damping of this shaft.

The behavior of an internal combustion engine is rather complicated, see e.g. [24]. A simplified torque model of a Diesel engine is given by

$$ T_E = -\tau(m(\omega_E, \alpha)) T_E + \tau(m(\omega_E, \alpha)) m(\omega_E, \alpha), $$

(3)

where the parameter $\tau(\cdot)$ depends on the operating point of the engine and $m(\cdot)$ is a nonlinear static map including the static parts of the engine torque.

A hydrodynamic dynamometer consists of a rotor driven by the device under test and is composed of a turbine wheel and the shaft, and the stator unit composed of the housing and the supply, see Fig. 2. It operates based on the Foettinger principle, see [25]. However, in contrast to torque converters or clutches, only the rotor is rotating in a hydrodynamic dynamometer. The slip – calculated from the relative difference between the rotor and the stator speed – is always 100%. The large thermal impact mostly from incidence, friction and secondary circulation losses makes a fluid exchange indispensable.
Hydrodynamic dynamometers are input-redundant systems, as the same dynamometer torque $T_{D,h}$ can be achieved with an infinite number of combinations of inlet valve position $\gamma'$ and outlet valve position $\gamma''$. An additional condition is introduced to obtain a unique control input $(\gamma', \gamma'')$ achieving a given criterion of optimality in [21]. An operation of both valves around the center of their operational area is motivated by practical experiences. Furthermore, the fluid temperature at the outlet is considered in this condition. Both the dynamometer torque and the temperature of the working fluid at the outlet are simultaneously controlled via both valves.

The results in [21] allow to describe the hydrodynamic dynamometer combined with the inverse torque controller as

$$
\dot{T}_{D,h} = -\frac{1}{\delta_{D,h}} T_{D,h} + \frac{1}{\delta_{D,h}} T_{D,h,\text{set}},
$$

(4)

where $\delta_{D,h}$ denotes the time constant.

The frequency converter of the electric dynamometer disposes of an internal torque controller, it is

$$
\dot{T}_{D,e} = -\frac{1}{\delta_{D,e}} T_{D,e} + \frac{1}{\delta_{D,e}} T_{D,e,\text{set}}.
$$

(5)

The electric dynamometer builds up torque faster than a hydrodynamic dynamometer and an internal combustion engine, hence in many setups: $\frac{1}{\delta_{D,e}} \approx 5\delta_{D,e}$ and $\delta_{D,e} \approx 10\delta_{D,e}$.

The shaft torque $T_{ST}$ is besides the engine speed $\omega_E$ the controlled output of the test bench, therefore in a first step the first line of (1) is rewritten as

$$
\dot{T}_{ST} = -d \frac{\theta_E + \theta_D}{\theta_E \theta_D} T_{ST} + c (\omega_E - \omega_D) +
\frac{d}{\theta_E} T_{e} + \frac{d}{\theta_D} (T_{D,h} + T_{D,e}).
$$

(6)

Furthermore, the inputs to the test bench are limited as follows: The accelerator pedal position $\alpha$ is from 0% to $\alpha = 100\%$. The hydrodynamic dynamometer can only load the internal combustion engine, the hydrodynamic dynamometer torque is $0 \leq T_{D,h} \leq T_{D,h}$ and the electric dynamometer can both load and accelerate the internal combustion engine, $-T_{D,e} \leq T_{D,e} \leq T_{D,e}$.

The inputs are transformed by means of a nonlinear function, see [12]. This transformation will allow the inputs to the system to only reach values within the above specified limits. The transformation is given by

$$
\alpha = t_1 (u_1) = \frac{\alpha - \epsilon_E}{2} +
\frac{\alpha + \epsilon_E}{\pi} \arctan \left\{ u_1 - \tan \left( \frac{\pi}{2} \frac{\alpha - \epsilon_E}{\alpha + \epsilon_E} \right) \right\},
$$

$$
T_{D,h} = t_2 (u_2) = \frac{T_{D,h} - \epsilon_{D,h}}{2} +
\frac{T_{D,h} + \epsilon_{D,h} \pi}{\pi} \arctan \left\{ u_2 - \tan \left( \frac{\pi}{2} \frac{T_{D,h} - \epsilon_{D,h}}{T_{D,h} + \epsilon_{D,h}} \right) \right\},
$$

$$
T_{D,e} = t_3 (u_3) = \frac{T_{D,e} - \epsilon_{D,e}}{2} \arctan \left\{ u_3 \right\},
$$

where $\epsilon_E$ and $\epsilon_{D,h}$ are small offsets allowing the input to the system to become 0.

The state $x = [T_{ST} \omega_E \omega_D T_{D,h} T_{D,e}]^\top \in \mathbb{R}^6$ of the entire test bench system is extended by the additional state $z \in \mathbb{R}^3$ to fit to the structure required to apply the technique presented in [7] as follows

$$
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix} = f(x,z) + g(x,z)u
$$

(7)

with

$$
f_1 (x,z) = -d \frac{\theta_E + \theta_D}{\theta_E \theta_D} x_1 + c (x_2 - x_3) + \frac{d}{\theta_E} x_4 + \frac{d}{\theta_D} (x_5 + x_6),
$$

$$
f_2 (x,z) = \frac{1}{\theta_E} (-x_1 + x_4),
$$

$$
f_3 (x,z) = \frac{1}{\theta_D} (x_1 - x_5 - x_6),
$$

$$
f_4 (x,z) = -\tau (m (x_2, t_1 (z_1))) x_4 +
\tau (m (x_2, t_1 (z_1))) m (x_2, t_1 (z_1)),
$$

$$
f_5 (x,z) = \frac{1}{\delta_{D,h}} (-x_5 + t_2 (z_2)),
$$

$$
f_6 (x,z) = \frac{1}{\delta_{D,e}} (-x_6 + t_3 (z_3)),
$$

$$
f_7 (x,z) = -\frac{1}{\gamma_E} z_1,
$$

$$
f_8 (x,z) = -\frac{1}{\gamma_{D,h}} z_2,
$$

$$
f_9 (x,z) = -\frac{1}{\gamma_{D,e}} z_3
$$

(8)

and

$$
g (x,z) = \begin{bmatrix}
  0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta_{D,h}} & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta_{D,e}}
\end{bmatrix},
$$

(9)

where $\gamma_E, \gamma_{D,h}$ and $\gamma_{D,e}$ describe the time constants of asymptotically stable and sufficiently fast filters.

The following problem is tackled:

**Problem 1:** Given the dynamical system (7) and the cost functional

$$
J (e, z, v) = \frac{1}{2} \int_0^\infty \left( q (e, z) + v^\top v \right) dt,
$$

(10)

where $q : \mathbb{R}^9 \to \mathbb{R}_+$ is a positive semi-definite continuous function, $e_i, i = 1, \ldots, 6$ is the relative error between the state $x_i$ and the reference $x_{i, \text{ref}}$ and $v_j, j = 1, \ldots, 3$ needs to be added to the reference input $u_{i, \text{ref}}$, i.e. $u_j = u_{j, \text{ref}} + v_j$. Determine a control law $u$ such that the cost functional (10) is minimized along the trajectories of the resulting closed-loop system.

Note that in the following the approach presented in [9] is used to calculation an approximation of the solution of
the optimal control problem. The computation of a so called algebraic \( P \) solution is enough to obtain the dynamic control law

\[
\dot{\xi} = -kV_\xi^T(e,z,\xi)
\]

\[
u = -B^TV_\xi^T(e,z,\xi)
\]

with \( k > k \) being a sufficiently high constant, \( B = g(0,0) \) and \( V_\xi \) being the partial derivative with respect to time of

\[
V(e,z,\xi) = P(\xi) e^T + \frac{1}{2} e^T R e - \frac{1}{2} \xi^T R \xi
\]

with the weighting matrix \( R > 0 \). The construction of the algebraic \( P \) solution is outlined in [26].

III. RESULTS

In the following, three different set-ups are considered which differ in the maximum torque values and the objective of the control strategy. However, note that in all cases the extension of an existing test bench is aimed. Therefore, and due to the lack of a controller for a test bench equipped with both a hydrodynamic dynamometer and an electric dynamometer, the performance is compared with the original set-up.

A. Electric dynamometer with low maximum torque

A test bench already equipped with a hydrodynamic dynamometer is provided with a small, and therefore inexpensive, electric dynamometer for increasing the dynamics. In particular, the maximum torque values are as follows: \( T_E = 800 \text{Nm}, T_{D,h} = 1250 \text{Nm} \) and \( T_{D,e} = 300 \text{Nm} \).

The basic load is intended to be covered by the hydrodynamic dynamometer, the electric dynamometer is intended to improve the tracking during transients. Therefore, the references for the system states are calculated as follows: The references for the engine speed \( \omega_E \), the dynamometer speed \( \omega_D \), the shaft torque \( T_{ST} \) and the engine torque \( T_E \) directly arise from the reference trajectories. The reference for the hydrodynamic dynamometer torque is also given from the reference trajectory of the torque, the reference for the electric dynamometer torque is set to 0.

The function \( q(e,z) \) in (10) is selected as \( q(e,z) = [e^T z^T] Q [e^T z] \), where the positive definite matrix \( Q \) is chosen as a diagonal matrix. Errors in the shaft torque \( T_{ST} \) and the engine speed \( \omega_E \) are weighted about 100 to 500 times as high as errors in other system states in (7). Further, note that the operating points have to be added to the inputs and the error in the states is calculated from the outputs which are observed using an observer, see e.g. [23].

The proposed method has been tested in a simulation environment. In addition to the dynamics described in Section II, the test bench simulator takes limitations of the actuators, operating point dependent dynamics, measurement noise, combustion oscillations and some other disturbances into account. The sampling time is 1 ms. Note that a real-time calculation of the control law is possible for this sampling time on a rapid prototyping system.

In Fig. 3 a comparison of the tracking of the shaft torque \( T_{ST} \) and the engine speed \( \omega_E \) for a test bench equipped with a hydrodynamic dynamometer and a multi-dynamometer test bench is shown. The combination with the electric dynamometer reduces the rise times in tracking both output variables and furthermore reduces the effect of the couplings in the system. Note that this improvement results to a certain amount from the increase of the manipulated variable and to another amount from the higher dynamics of the electric dynamometer.

The third to fifth graph of Fig. 3 shows the time histories of the inputs to the test bench – accelerator pedal position \( \alpha \), set value \( T_{D,h,set} \) of hydrodynamic dynamometer torque \( T_{D,h} \) and set value \( T_{D,e,set} \) of electric dynamometer torque \( T_{D,e} \).
Note that for both setups all inputs are within the constraints. Furthermore, the input range of the electric dynamometer is fully utilized during transients. In contrast, the internal combustion engine is only loaded by the hydrodynamic dynamometer during stationary phases.

B. Both dynamometers with medium maximum torque

The performance of the coupling of a hydrodynamic dynamometer with an electric dynamometer both with medium maximum torque is compared to a test bench with solely a hydrodynamic dynamometer. The maximum torque of both dynamometers is half the value of the hydrodynamic dynamometer in Subsection III-A. In particular, the following input constraints are given at the multi-dynamometer test bench: $T_E = 1150 \text{Nm}$, $T_{D,h} = T_{D,e} = 625 \text{Nm}$.

The references for the system states have been chosen such that the torque is divided between the two dynamometers, there is no preference on one of the two dynamometers. The weighting matrix $Q$ is selected as before.

Fig. 4 shows an improvement of the tracking of torque and speed reference profiles and a reduction of the effects of the couplings in the system when using the two dynamometers in comparison to a setup with just a hydrodynamic dynamometer. Further, the time histories of the inputs are shown. This configuration allows an increase of the maximum torque of an existing test bench equipped with a dynamometer with medium maximum torque. However, note that the individual components have to have a suitable mechanical strength.

C. Maximum recovery into electric grid

In this case the maximum torque of the internal combustion engine is higher than the maximum torque of an electric dynamometer, $T_{D,e} = 625 \text{Nm}$. Therefore, the test bench is extended by a hydrodynamic dynamometer with the maximum torque as in Subsection III-A.

The electric dynamometer shall be used as much as possible such that a large portion of the loading power can be fed back into the electric grid. The reference of the hydrodynamic dynamometer torque is 0 for desired torques below the maximum torque of the electric dynamometer. The weighting matrix $Q$ remains unchanged.

Fig. 5 represents the tracking of the torque and speed profiles as well as the time histories of the inputs. A comparison with a hydrodynamic dynamometer as in Subsection III-A is shown. As expected the dynamics is increased in this configuration in comparison to the hydrodynamic dynamometer. The hydrodynamic dynamometer is supplementary used for high torque values. In contrast, for small torque values this dynamometer is not loading the internal combustion engine.

IV. Conclusion and Outlook

This paper discusses the problem of simultaneously tackling speed and torque references at the crankshaft of an internal combustion engine equipped with a hydrodynamic dynamometer and an electric dynamometer. The input constraints are considered in the equations of the state and the redundancy of the actuators in the choice of the references of the system states, afterwards the state is extended to avoid the explicit solution of the HJB partial differential equation and the optimal control problem is solved approximatively.

The use of two dynamometers can enlarge the area of application of an existing test bench, the simultaneous use of both dynamometers improves the tracking of the references and reduces the influence of the couplings in the systems. In all cases under investigation the set values of the shaft torque and the engine speed are achieved quickly and without overshoots. Furthermore, the tuning of the proposed controller is simple and easily adopted to other test benches as well as the proposed approach seems to be easily adopted to other applications like hybrid electric vehicles.

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