Abstract—When unmanned aircraft systems operate in urban corridors, navigation accuracy is a priority due to proximity of buildings, obstructions, and other infrastructure. In most environments a Global Positioning System (GPS)/inertial measurement unit combination along with an air data system can provide accurate navigation capability. However, this is not possible in urban corridors where GPS has well-documented degradation. Other sensors such as vision-based systems and Long-Term Evolution transceivers have shown to be useful in urban settings, but modeling them individually is difficult without an in-depth understanding of each sensor and the factors dictating its accuracy. This paper proposes a framework to model location-dependent accuracy of navigation and how this changes within the urban environment. Results show that persistent machine vision can provide accurate navigation capability, but LTE with its current measurement delay does not have a noticeable positive effect on navigation accuracy.

I. INTRODUCTION

When operating in urban canyons, unmanned aircraft systems (UAS) must have persistent location awareness to avoid collisions with buildings, obstructions, and other infrastructure. In this environment, the Global Positioning System (GPS) quite frequently fails to provide sensor measurements needed for safe navigation. For example, Lu [1] reported that there were not enough visible signals to produce a GPS measurement 50% of the time in an urban section of Hong Kong. In the areas where a GPS measurement could be calculated, errors were greater than 20 meters for 40% of the test points. These poor levels of availability and accuracy make it necessary for other sensors to be included in a UAS urban navigation sensor suite.

The contribution of this paper is to synthesize a technique in which the UAS uses characteristics of its environment to make sensor accuracy decisions for state estimation. By tuning various accuracy parameters, a user can evaluate estimate quality for different sensor combinations in different types of urban environments. The key to this technique is the categorization of the navigable airspace based on its topology and expected signal availability based on knowledge of the building and obstruction locations. If the UAS has an accurate estimate of its position relative to the environment, it can match this position to characteristics about its environment to make conclusions about expected sensor properties. These sensor properties are used to generate measurements and error covariances, which are in turn passed to the estimator to be used in its correction step. The sensors to be considered include the traditional inertial measurement unit (IMU) and air data system (ADS), dual computer vision systems (VISION) for odometry and localization, GPS receiver, and a Long-Term Evolution (LTE) transceiver.

The rest of the paper is organized as follows. Section II describes and justifies use of a diverse sensor suite including the emerging LTE signal. Section III describes the UAS guidance, navigation, and control loop with focus on the multi-sensor fusion framework integrated into the sensor measurement block. The urban environment model is also presented. Section IV describes the sensor accuracy characterization process. Section V analyzes simulation results for three different mission altitudes, and Section VI concludes the paper.

II. BACKGROUND

In UAS navigation, traditional sensors include an IMU, ADS, and GPS receiver. The IMU and ADS can generally provide measurements at any location within the urban environment under steady or gently maneuvering flight conditions. The GPS receiver augments these two sensors with position and airspeed measurements based on signals received from at least four satellites. GPS works well in open spaces where line of sight to these satellites is unobstructed, but in urban canyons VISION sensors and possibly LTE transceivers must be considered as GPS signal strength degrades.

1) Inertial Measurement Unit: The IMU for a small UAS consists of 3-axis gyroscopes, 3-axis accelerometers, and a 3-axis magnetometer to provide measurements of the aircraft’s angular velocities as well as gravity and magnetic North vectors. These measurements are post-processed and filtered to convert data into roll, pitch, yaw and angular rate information accounting for any noise and bias in the data due to environmental conditions [2].

2) Air Data System: Most ADS include, at minimum, a static pressure port to generate altitude measurements and a dynamic pressure port, which along with the static port generates airspeed measurements. Static pressure ports also provide angle of attack and angle of sideslip measurements.
A. LTE Transceiver

LTE transceivers are complementary to GPS by providing two-dimensional horizontal position measurements using one of three standardized LTE network position methods, including the Observed Time Difference of Arrival (OTDOA) technique. Since OTDOA is independent of GPS and its accuracy increases as function of the number of hearable towers [3], it is an appropriate candidate sensor for urban navigation applications. The OTDOA technique uses time differences of positioning signals sent from hearable towers to an LTE transceiver within a smart phone using a position calculation process similar to GPS. The main drawback of LTE positioning is its measurement availability lag time, which is at least four seconds according to the Polaris Wireless website.

B. Computer Vision-Based Sensors

Many urban canyon navigation algorithms use computer vision techniques to provide UAS odometry and/or localization information. These include the vanishing points method to improve attitude estimation [4] and homography-based feature detection and matching [5] to improve position, orientation, and velocity estimates from the IMU. Factors affecting measurement accuracy in urban environments are building surface material and texture, weather, shadows, time of day, and artificial light sources [6].

III. THE UAS FLIGHT CONTROL SYSTEM AND THE URBAN ENVIRONMENT

A. Urban Environment

The urban environment used in this research consists of a series of urban corridors (city blocks) aligned, without loss of generality, in a North-South orientation formed between columns of buildings. The buildings are modeled as rectangular three-dimensional polyhedra with spacing dependent on whether the building is on the interior of the urban corridor or at an end of the corridor, adjacent to the intersection. The UAS is assumed to have complete prior knowledge of all building, obstacle, and intersection locations, requiring localization only.

B. UAS Dynamics

The UAS guidance, navigation, and control algorithm generally consists of high level trajectory guidance, calculation of control inputs, state propagation using aircraft equations of motion, generation of sensor measurements, and state estimation. This research uses the traditional fixed-wing rigid body six degree of freedom equations of motion for aircraft. The twelve element state vector \( \mathbf{x} \) and four element control input vector \( \mathbf{u} \) are listed below:

\[
\mathbf{x} = [x_N \ x_E \ h \ V_T \ \alpha \ \phi \ \psi \ p \ q \ r]^T
\]

\[
\mathbf{u} = [\delta_a \ \delta_e \ \delta_r \ F_T]^T
\]

The inertial position states consist of the North position \( x_N \), East position \( x_E \), and mean sea level altitude \( h \). The body-fixed angular rates include roll rate \( p \), pitch rate \( q \), and yaw rate \( r \). The control inputs are aileron deflection \( \delta_a \), elevator deflection \( \delta_e \), rudder deflection \( \delta_r \), and total thrust \( F_T \).

Both the aircraft equations of motion and the UAS aerodynamic model used for this work can be found in Ducard Chapter 3, Appendix A, and Appendix F (UAS model) [7]. The general form of the state propagation equation and measurement equation are shown in (1) and (2) respectively with \( \mathbf{w}(t) \sim \mathcal{N}(0, \mathbf{Q}(t)) \) and \( \mathbf{v}_k \sim \mathcal{N}(0, R_k) \). \( \mathbf{Q}(t) \) and \( \mathbf{R}_k \) represent the process noise covariance matrix and the measurement noise covariance matrix respectively.

\[
\mathbf{x}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{w}(t) \tag{1}
\]

\[
\mathbf{y}(\mathbf{x}_k) = h(\mathbf{x}_k) + \mathbf{v}_k \tag{2}
\]

C. Guidance and Control

The high level trajectory guidance protocol instructs the controller to maintain straight and level trim conditions through the urban environment, (i.e. \( \mathbf{x}_{cmd} = \mathbf{x}_{trim} \)). For control, a linear quadratic regulator (LQR) [8] is utilized due to its straight-forward implementation and constant gains when the UAS dynamics are linearized about a trim point. The controller uses (3) as its control law, where \( K \) is the gain matrix. See [8] for more details on the gain matrix calculation.

\[
\mathbf{u}_{k-1} = K(\mathbf{x}_{cmd,k-1} - \mathbf{x}_{k-1}) + \mathbf{x}_{trim} \tag{3}
\]

D. Sensor Measurement Generation

Using a linear measurement model in (2) \( h(\mathbf{x}_k) \) simplifies to \( H_k \mathbf{x}_k \), where \( H_k \) is a row vector of eleven zeros and one in the location of the state whose measurement is to be taken. Each measurement is a sum of the true state and a zero-mean normally distributed measurement noise. For some sensors, such as those in the IMU, the measurement noise covariance can be experimentally determined through laboratory and flight testing and can be considered constant over most conditions. The ADS measurement noise covariance matrix is also considered constant, ignoring effects of buildings and environmental system discharge on air mass motion. While VISION sensor noise values are based on several environmental factors, modeling these factors is outside the scope of this paper and constant values will be used when VISION is available.

The constant measurement noise assumption does not hold for GPS receivers and LTE transceivers. Since GPS accuracy relies on several factors including line-of-sight visibility to satellites [9], its measurement noise covariance matrix must correlate to the number of visible satellites. The same is true conceptually for LTE accuracy, based on the relative location of LTE network towers in the local area.
1) UAS Relative Location Determination: An intuitive method of categorizing UAS relative location in the urban environment is to report its vertical position with respect to building heights and laterally with respect to canyons, intersections, and gaps between buildings. The vertical categorization is known as altitude with respect to buildings or simply ALT and the lateral categorization is known as street-level position or simply SL. To determine the UAS ALT category, its altitude is compared to the tallest building along the current block or intersection it is traversing as shown in Figure 1.

The UAS is in the ALT − 1 category (Figure 1a) when its altitude is higher than the tallest building on the current block or the buildings bordering the intersection. It is in the ALT − 2 category (Figure 1b) when its altitude is higher than the shortest building on the block/intersection, but lower than the tallest building on that block/intersection. It is in the ALT − 3 category (Figure 1c) when it is lower than the roof of the shortest building on the block/intersection.

The UAS SL category is determined by comparing its lateral position to the surrounding buildings along the current block as shown in Figure 2.

The UAS is in the SL − 1 category when it is along a block with buildings on both sides of the street (Figure 2a). It is in the SL − 2 category when traversing an intersection between city blocks (Figure 2b). The UAS is in the SL − 3 category when there are only buildings on one side (Figure 2c).

2) Sensor Measurements and Noise Covariance Values: Measurements generated from (2) require corresponding noise covariance estimates. For sensors expected to have constant noise covariances, these values are sent to the filter each time a measurement becomes available. However, measurement and measurement noise covariance generation for GPS and LTE require prior knowledge of the the environment’s effect on accuracy, as proposed in Table I.

When populated by simulation or flight data for GPS or LTE, Table I gives measurement noise covariance values as a function of the current ALT and SL values for the UAS. In simulation, the UAS can use its true three-dimensional position to first determine its true ALT and SL categories then look up tabulated measurement noise covariance values. GPS/LTE sensors with real-time accuracy determination capability can be mimicked by using this measurement noise covariance value in the state estimation filter. To mimic GPS/LTE sensors without this capability, the estimated three dimensional position can be used directly. This initial model is a discrete representation of the continuously-changing properties found in the environment.

E. Extended Kalman Filter with Delayed Measurements

A continuous-discrete Extended Kalman Filter (EKF) [10] is used to generate the estimated state vector and covariance matrix by fusing all measurements from each available sensor in a Bayesian prediction-correction structure. The EKF is an extension of the Kalman Filter where the predicted estimated state vector is propagated using the non-linear system dynamics and the covariance matrix is propagated using the linearized state transition matrix evaluated using the previous corrected estimated state vector and current control input vector. The correction step is then identical to the Kalman Filter.

Since GPS and LTE position measurements are both delayed with LTE generally delayed by at least four seconds, state
augmentation (also known as stochastic cloning) is used to allow for proper accounting of these measurements in the EKF [11]. For a measurement with a known delay of \( m \) time-steps that becomes available at time-step \( k \), the state augmentation process keeps a copy of the state estimate at time-step \( k - m \) and appends it to the bottom of the estimated state vector, while expanding the covariance matrix accordingly. As the estimated state is propagated, the augmented states are not propagated, but corrected using intermediate measurements, while the covariance matrix is updated according to the EKF covariance propagation equations. At time-step \( k \), the true state vector at \( k - m \) is used to generate the time-delayed measurement. This measurement and augmented estimated state vector are used then used to calculate the corrected state estimate. Once the filter has completed the correction at time-step \( k \), the augmented states and their covariance matrices are marginalized out of the system and the process is repeated when the next delayed measurement is taken.

IV. Simulation

A series of simulations were conducted to evaluate UAS navigation accuracy in an urban environment at three different altitudes. This allowed for different ALT values to be used when determining measurement noise covariance values for GPS and LTE. Table II summarizes simulation parameters.

### Table II
**Simulation Parameters**

<table>
<thead>
<tr>
<th>Monte Carlo Runs</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>50 m/75 m/125 m</td>
</tr>
<tr>
<td>Time-Step, dt</td>
<td>0.01 sec</td>
</tr>
<tr>
<td>Process Noise Covariance Matrix, ( Q )</td>
<td>0.01 ( \times I_{12} )</td>
</tr>
</tbody>
</table>

The urban landscape shown in Figure 3 is used for all simulations. Each block had four buildings with the exception of the east side of the third block, where there were no buildings (\( SL - 3 \): adjacent open space).

A. Determining Available Sensors

The IMU, ADS, and GPS, and LTE sensors were assumed to be available for the duration of the simulation with measurements taken at the nominal sensor sampling rate unless the simulation specifically excluded one of these sensors. To take advantage of the VISION sensors along each wing of the UAS as shown in Figure 4, these sensors must be oriented towards vertical building faces along the urban canyon. VISION measurement for each sensor are assumed to be available any time the UAS is adjacent to a building that would be detected in the sensor’s field of view. VISION is not available in \( ALT - 1 \) (above all buildings) or \( SL - 2 \) (intersections).

B. Accuracy of Available Sensors

Table III shows IMU, ADS, and VISION sensor performance data used in the simulation including measurement noise values, sampling rates, and available states. The VISION data was taken from a VISION/IMU fused system [5].

The GPS receiver measurement accuracy covariance values, shown in Table IV, for both \( ALT - 1/SL1 - 3 \) and \( ALT - 3/SL - 3 \) were derived from [16] and [17]. The GPS inertial velocity measurement noise covariance value for all relative location categories was set to 0.22 \( m^2/s^2 \) [18].

### Table III
**Sensor Simulation Parameters** [5], [2], [12], [13], [14], [15]

<table>
<thead>
<tr>
<th>Measured States (Noise Covariance Values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VISION 50 Hz</td>
</tr>
<tr>
<td>( \frac{\varphi}{\theta}, \frac{x}{y}, h )</td>
</tr>
<tr>
<td>( 2.77, 3.39, 3.36 ) m^2</td>
</tr>
<tr>
<td>( 0.076, 0.81, 0.74 ) deg^2</td>
</tr>
<tr>
<td>IMU 100 Hz</td>
</tr>
<tr>
<td>( \varphi, \theta, \psi )</td>
</tr>
<tr>
<td>( 2.71, 1.65, 8.27 ) deg^2</td>
</tr>
<tr>
<td>( 0.6, 0.6, 0.6 ) deg/s, deg/s, deg/s</td>
</tr>
<tr>
<td>ADS 50 Hz</td>
</tr>
<tr>
<td>( h, V_T, \alpha, \beta )</td>
</tr>
<tr>
<td>( 1.5, 1, 1, 1 ) deg/s, deg/s, deg/s</td>
</tr>
</tbody>
</table>

### Table IV
**Location-Based GPS Measurement Noise Covariance Lookup** [16], [17]

<table>
<thead>
<tr>
<th>( \sigma^2_{N_E, h} (m^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SL - 1 )</td>
</tr>
<tr>
<td>( SL - 2 )</td>
</tr>
<tr>
<td>( SL - 3 )</td>
</tr>
<tr>
<td>( ALT - 1 )</td>
</tr>
<tr>
<td>3.64^2</td>
</tr>
<tr>
<td>ALT - 2</td>
</tr>
<tr>
<td>4.0^2</td>
</tr>
<tr>
<td>ALT - 3</td>
</tr>
<tr>
<td>6.5^2</td>
</tr>
</tbody>
</table>

Figure 3. Example urban landscape used in simulations

Figure 4. UAS VISION sensor geometry

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The LTE measurement noise covariance values, shown in Table V, were derived from [19] as this was the only available source identified with LTE positioning accuracy specified as a function of number of hearable towers. The simulation capped the accuracy at 20 hearable towers, consistent with a review of the Cell Reception website for Detroit, Michigan, which showed approximately 22 cellular towers (all will be assumed to be on the LTE network) in and around the downtown core. Further analysis would be needed to determine exact tower hearability maps for each urban environment.

<table>
<thead>
<tr>
<th>ALT (−)</th>
<th>( \sigma_{N,E}^2 ) (m²) (# of Hearable eNBs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SL − 2</td>
<td>4.38 (10-15)</td>
</tr>
<tr>
<td>SL − 3</td>
<td>6.26 (7-10)</td>
</tr>
</tbody>
</table>

V. RESULTS AND ANALYSIS

The UAS ALT categorization is shown in Figure 5 for the three test altitudes. The twin dotted vertical lines represent the three intersections with urban canyons in between. As expected, at the highest altitude, \( h = 125 \) m, there is no ALT variation over the flight since the UAS is above the urban environment for the entire test. This environment allows for the best GPS accuracy and moderate LTE accuracy, but no VISION. For the \( h = 75 \) m test the UAS again starts in ALT − 3, but drops to ALT − 2 in the first intersection, and then drops again to ALT − 1 in the second canyon before transitioning back to ALT − 2 in the third canyon. Since this trajectory moves between all ALT categories, it utilizes all onboard sensors. When \( h = 50 \) m the UAS starts in ALT − 3 and stays in this category until it enters the second intersection where it drops to ALT − 2 before transitioning back to ALT − 3 in the fourth canyon. This trajectory utilizes the VISION sensors throughout since the UAS is always adjacent to buildings.

The longitudinal and lateral root-mean-squared (RMS) position error trajectories are shown in Figure 7 for the three test altitudes. Figure 7a) shows the longitudinal position RMS error trajectory and Figure 7b shows the lateral position error trajectory. Both will be analyzed together since the trajectories are similar. At \( h = 125 \) m, the RMS errors decrease steadily as GPS measurements become available. Upon receiving LTE measurements there are slightly larger drops in longitudinal error just before the first intersection and in the middle of the second intersection. At \( h = 75 \) m the error decreases until both VISION measurements become unavailable in the first intersection. At this point the error steadily climbs until the third canyon when one VISION measurement becomes available again. Even though the error begins to decreases in the third canyon with one VISION measurement, it increases slightly in the third intersection before decreasing again in the fourth canyon. In this canyon the rate of error decrease slows as the UAS is above some buildings in the canyon. The error reaches a larger value in the lateral position since the yaw angle also increases in the second canyon when only the IMU yaw measurement is available. At \( h = 50 \) m the error decreases sharply in the first canyon before leveling off near the first intersection. As with the \( h = 75 \) m tests, there is a slight increase in error in each of the intersections as both
VISION measurements become temporarily unavailable. The error does decrease again in the second canyon, but increases in the third canyon due to being in \( ALT - 2 \) and also because only one VISION measurement is available.

The error increased quickly in the absence of VISION, both in canyons and in intersections, but never reached the level of \( ALT \) error when VISION measurements were not available at all.

Figure 7. Root mean squared error trajectory for three altitudes

VI. CONCLUSIONS AND FUTURE WORK

This paper proposed a framework to characterize UAS state estimate accuracy in an urban environment. The simulation used an LQR controller, along with fixed-wing UAS dynamics, and an EKF with state augmentation to fuse measurements from the IMU, ADS, GPS, LTE, and VISION sensors. \( ALT \) (altitude with respect to buildings), \( 3\sigma \) error bound, and RMS results were shown for altitudes of 50 m, 75 m, and 125 m. VISION availability was the largest driver in estimation accuracy in both the longitudinal and lateral dimensions with similar error trajectories in both dimensions for all three altitudes. The error increased quickly in the absence of VISION, both in canyons and in intersections, but never reached the level of error when VISION measurements were not available at all. LTE had little effect on decreasing the \( 3\sigma \) error bounds or the RMS error at any of the altitudes due to its long delay and long sampling period.

Future work will focus more closely on studying the effect of LTE delay on accuracy as well as using other sensor measurements to improve accuracy. Also, simulations will be run using a regularized particle filter to determine the increase in performance using an ensemble-based technique.

ACKNOWLEDGMENTS

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