Optimization Algorithm and Practical Implementation for 2-coil Wireless Power Transfer Systems

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Abstract— Wireless power transfer based on inductive coupling techniques has attracted a lot of research interest in recent years. It can be potentially applied in many industrial sections. However, the harsh outdoor industrial environment could induce circuit parameter drift. Only small drift may result in huge performance drop, as the circuits are very sensitive to the parameter changes. In order to address the problem, an optimization method to compensate for the effect of drifting parameters and track the maximum performance is introduced in this paper. A switch network which is able to dynamically change the inductance of the Tx coil is designed. When the parameter drift occurs, the tuning algorithm is able to adjust both the exciting frequency and the Tx coil inductance. By measuring the input power only, the best working point can be found out without requiring any measurement on the Rx side. Therefore, the algorithm can be implemented in the controller located on the Tx side without coordination and communication between both sides. In order to verify the effectiveness of the proposed method, a test rig is setup and practical experiments are conducted. The experimental results are also presented in the paper.

I. INTRODUCTION

Wireless power transfer (WPT) technology has been an increasing research interest in recent years. There are several WPT techniques being researched presently. Far-field techniques use propagating electromagnetic waves that transfer energy the same way radios transmit signals [1]. Inductive coupling (or near-field) techniques operate at distances less than a wavelength of the signal being transmitted. The early work of inductive coupling [2-4] mainly based on the pure physical theory, which could be difficult for the engineers from electrical background. Recent researches [5-7] start to use circuit and magnetic theory to analyze the principle of the magnetic resonance coupling, which makes the technology more tangible for a wider range of researchers.

A key requirement in all of the WPT applications is to deliver sufficient power to the load with high power transfer efficiency (PTE) and relative long distance. There are many configurations of WPT system developed recently including the 2-coil, 3-coil [8] and 4-coil [9] links. The multi-coil structures are able to increase the PTE in some cases but the price is the increasing complexity of the system structure. Each coil has its own parameters such as resistance, inductance and capacitance etc. Some parameters may be sensitive to the external environment like temperature, humidity. More coil loops means that more tuning work is required to get all the parameters aligned. It could be difficult to maintain the optimal performance in some harsh environment for the practical applications.

Potentially, inductive coupling WPT systems can be used in many industrial applications, such as power supplies for moving sensors and transducers [10], medical implants [11] and wireless electric car charging stations [12] etc. In power industry, due to the importance of High Voltage (HV) cables in the transmission network, on-site condition monitoring is a very important issue [13][14]. However, the power supply is always a major problem for the reliability of the monitoring devices which operate in the outdoor environment with no reliable power sources. Some applications applies the solar and wind power to solve the problem but these power sources are not guaranteed in case of continuous bad weather condition which could occur frequently in some places like the mountain areas in South West China.

The inductive coupling could be a potential solution for the problem. A tiny amount of the power can be recovered from the magnetic field around the HV power cables using induction coils and then transferred through the insulator (a few meters distance) to power the monitoring devices installed on the towers. For this kind of applications, robustness and reliabilities are more important issues than the performance. Therefore, the simple 2-coil structure may be the best option. Moreover, the WPT systems must be able to cope with the circuit parameters drift caused by changing weather conditions in the outdoor environment.

In this paper, a WPT test rig designed for practical HV power cable monitoring is setup. It is able to deliver 20W power for a distance of 2.2m. Due to relative low working frequency, there is no requirement for high performance power electronics. All the power electric components can be purchased with low costs. In order to track the best performance with the drifting parameters in potential practical applications, a dynamic optimization algorithm is designed and experiments are conducted to verify the proposed methods.

II. CIRCUIT AND MAGNETIC PRINCIPLE OF 2-COIL WPT SYSTEMS

A. Circuit Model

Fig. 1 shows the circuit model of the 2-coil WPT system using magnetically coupled resonator. Both the Tx Coil and Rx Coil have the same resonant frequency. When the Tx
Coil is energized by the resonant frequency, electric power can be transmitted through the magnetic field between the two coils. On the Tx side, an AC voltage source drives an RLC branch which create high frequency magnetic field around the Tx coil. The Rx coil recovers the energy from the field and drives a load $R_L$. In addition, both the Tx and Rx circuit have the parasitic resistance $R_{p1}$ and $R_{p2}$.

![Circuit model of two-coil WPT system](image)

Normal the distance between the two coils can be several times of the coil radius, which makes the coupling coefficient $k$ is a very small value around 0.001 to 0.01. Only a small amount of the flux generated by Tx coil is able to penetrate the Rx coil. However, large amount of magnetic energy can still be transferred through the limited amount of flux with relatively high efficiency (around 30%). This phenomenon can be explained by the circuit of and magnetic theory.

### B. Circuit and Magnetic Principle

Kirchhoff’s voltage law (KVL) can be applied to analyze the both two circuit loops as

\[
I_1(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}) + j\omega L_2 M = V_s \quad (1)
\]

\[
I_2(R_2 + j\omega L_2 + \frac{1}{j\omega C_2}) + j\omega L_1 M = 0 \quad (2)
\]

where $R_1=R_{p1}$, $R_2=R_{L}+R_{p2}$ and the $M$ is the mutual inductance between the Tx and Rx coil. The relationship between the mutual inductance and coupling coefficient is defined as

\[
k = \frac{M}{L_1 L_2}
\]

In order to simply the two circuit equation (1) and (2), $Z_1$ and $Z_2$ are defined as the impedance of the two circuit loops as

\[
Z_1 = R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \quad , \quad Z_2 = R_2 + j\omega L_2 + \frac{1}{j\omega C_2}
\]

The two KVL equations (1) and (2) can be solved as

\[
I_1 = \frac{Z_2 V_s}{Z_1 Z_2 + \omega^2 M^2} \quad , \quad I_2 = -\frac{j\omega M_1 Z_2 V_s}{Z_1 Z_2 + \omega^2 M^2} \quad (3)
\]

Therefore, the input power $P_I$ which energizes the Tx coil and output power $P_O$ which are consumed on the load $R_L$ can be calculated as

\[
P_I = |V_s I_1| \cos(\angle V_s - \angle I_1), \quad P_O = |R_L I_2|^2 \quad (4)
\]

The overall wireless transmission efficiency is

\[
E = \frac{P_O}{P_I} = \frac{|R_L I_2|^2}{|V_s I_1| \cos(\angle V_s - \angle I_1)} \quad (5)
\]

An experimental test rig is setup in the laboratory whose measured circuit parameters are listed in Table I. In order to maximum the power transmission, both Tx and Rx coils are made with the same resonant frequency, which means they have the same inductance and capacitance.

### TABLE I. Circuit Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$, $L_2$</td>
<td>298uH</td>
</tr>
<tr>
<td>$C_1$, $C_2$</td>
<td>200pF</td>
</tr>
<tr>
<td>$R_{p1}$</td>
<td>7Ω</td>
</tr>
<tr>
<td>$R_{p2}$</td>
<td>5Ω</td>
</tr>
<tr>
<td>$R_L$</td>
<td>7Ω</td>
</tr>
<tr>
<td>$k$</td>
<td>0.0001 to 0.1</td>
</tr>
<tr>
<td>Frequency</td>
<td>620kHz to 680kHz</td>
</tr>
</tbody>
</table>

Using the data in Table I, the output power can be plotted in Fig 2, in which $P_o$ can be considered as a function of frequency and coupling coefficient $k$. Frequency splitting can be clearly observed as the value of $k$ is increased. When the coupling between the two coils decreases, the frequency separation decreases until the two peak values converge at $f_s$, which is very close to the resonant frequency of the two coils $f_0$ (detailed calculation reveals that $f_s\neq f_0$ but they are closed enough to be considered as a single value).
decreasing coupling \( k \) until it reaches the peak value \( P_{\text{max}} \). Then it drops dramatically.

III. OPTIMIZATION ALGORITHM FOR PRACTICAL APPLICATIONS

A. Effects of the Parameter Drift

In practical applications, it is very difficult to make two coils with the same resonant frequency. Even if the two coils were adjusted carefully initially, the capacitance and inductance could also change with the changing environment. Without perfect resonant frequency match, the performance of the WPT could degrade greatly.

For the sake of simplicity, it is assumed that the capacitance of the two coils is the same but the inductance is slightly different. \( L_2 \) has a constant value 298\( \mu \)H but \( L_1 \) changes from 288\( \mu \)H to 308\( \mu \)H. For a given distance (\( k=0.007 \)), output power is a function of frequency and Rx coil inductance \( L_1 \), which is plotted in Fig 4.

![Fig. 4. Output power as function of the frequency and \( L_2 \)](image)

In order to achieve that target, the circuit model on the Tx side is analyzed in details. The circuit formula (1) is separated into several terms as

\[
V_R = I_1R_1, \quad V_L = j\omega L_1 \]
\[
V_c = -j\frac{1}{\omega C_1}, \quad V_m = j\omega I_2 k\sqrt{L_1L_2}
\]

where \( V_c \) is the voltage across the capacitor, \( V_r \) is the voltage across the resistor, \( V_l \) is the voltage across the capacitance, and \( V_m \) is the voltage incited by the current \( I_2 \) on the Rx coil. Fig 5 is the corresponding circuit model in which the mutual
reactance from the Rx coil is considered as a voltage source controlled by the derivative of $I_2$.

$V_{S}$
$C_1$
$R_1$
$L_1$
$I_1$

and corresponding $f_{p2}$ and $L_{1,p2}$.  

6) Repeat the fifth step $n$ times until the $I_{1,pn}$, $L_{1,pn}$ and $f_{pn}$ are obtained.

Therefore, in this case, the peak or bottom in Fig 6 merge into a single peak which is align with the two peaks and a bottom. In this case, the frequency output power maximum at the resonant frequency but to limit the current at the resonant frequency. Therefore, the term $V_m$ on the Tx coil is weaker as well. Therefore, in this case, the peak or bottom value of $I_1$ is bigger than the case when the two resonant frequencies are perfectly matched.

C. Tuning of the Tx Coil Resonant Frequency

It is assumed $L_1$ is a tunable parameter. By the tuning of $L_1$, the Tx coil resonant frequency can be changed to align the Rx coil. In the case when $L_1$ is not equal to $L_2$, the two coils have different resonant frequency. A peak or bottom (depend on the coupling $k$) value of $I_{1p}$ can still be detected close to the Tx resonant frequency. However, because the Rx coil is not on its resonant frequency, the incited current $I_2$ is weaker, which results in the “counter force” term $V_m$ on the Tx coil is weaker as well. Therefore, in this case, the peak or bottom value of $I_1$ is bigger than the case when the two resonant frequencies are perfectly matched.

D. Tuning Algorithm

Based on the analysis of the circuit model in Fig 8, it can be seen the value of the $I_1$ can reflect the output power. By measuring the current $I_1$, the optimization algorithm can be developed. It is assumed that the possible range of $L_2$ is between $L_{2,\text{min}}$ to $L_{2,\text{min}}+nd_l$ with the step size $d_L$ and the frequency ranges from $f_{\text{min}}$ to $f_{\text{max}}$.

1) Set $L_1$ to a $L_{1,\text{min}}$.  

2) Keep changing the exciting frequency from $f_{\text{min}}$ to $f_{\text{max}}$.  

3) If the current $I_1$ has two peak values, record the value at the bottom between the peaks as $I_{1,p1}$ and the corresponding frequency $f_{p1}$ and inductance $L_{1,p1}$.  

4) If the current $I_1$ has a single peak value, the peak current is recorded as $I_{1,p1}$.  

5) Set $L_2$ to $L_{2,\text{min}}+d_L$ and the scan the frequency for the whole range. Record the current $I_{1,p2}$ and corresponding $f_{p2}$ and $L_{1,p2}$.  

6) Repeat the fifth step $n$ times until the $I_{1,pn}$, $L_{1,pn}$ and $f_{pn}$ are obtained.
7) Compare the value of \( I_{1,pi} \) \((i=0 \text{ to } n)\). If \( I_{1,pi} \) is the minimum value, then the corresponding \( L_{1,pi} \) and \( f_{pi} \) is the optimized parameter for the WPT system.

IV. IMPLEMENTATION OF WPT SYSTEM

Fig. 9 is the diagram of the practical application of the proposed WPT system. A DDS (Direct Digital Synthesizer) module AD9850 controlled by a MCU (Micro Controller Unit) controller is adopted to generate the accurate square wave exciting signal. The frequency signal is amplified by a gate driver module and then drives a MOSFET H-bridge to generate a high frequency AC voltage source. The Tx coil is energized by the AC and transmit the power to the Rx coil though the inductive coupling. On the Rx side, the electric energy receive by the coil is rectified to DC by a high speed bridge rectified made of Shockley Diodes. The DC is filtered by capacitors and then drives the load. Even though the H-bridge inverter can only generate square waves rather than sinusoidal ones, only the fundamental frequency which perfectly matches the resonant frequency can pass through the WPT system. Therefore, the theoretical analysis based on the AC source is still applicable for the case of H-bridge inverters.

When the system is initially tuned, the resonant frequency of the two coils and the excited AC frequency are aligned. The system has the optimized performance. However, due to the changes of the environment, the coil resonant frequency could drift and result in the dramatic performance drop. Therefore, in order to track the optimal operating point with the changing circuit parameters, both the exciting frequency and the resonant of the coils must be adjusted using the algorithm introduced in Section III.

It is easy to achieve the tuning of the exciting frequency with the DDS technology. However, the tuning of the resonant frequency is very difficult as it require changing either the capacitance or inductance. In this paper, a switch network which is able to “digitally” tuning the coil inductance is introduced. The switch network which consists of four switches and four inductors with 1.5uH, 3uH, 6uH and 12uH inductance are connected in serial in the Tx coil loop. With four switches, the following overall inductance of the switch network can be achieved as shown in Table III.

<table>
<thead>
<tr>
<th>Inductance</th>
<th>Switching Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>0uH</td>
<td>Off Off Off Off</td>
</tr>
<tr>
<td>1.5uH</td>
<td>On Off Off Off</td>
</tr>
<tr>
<td>3uH</td>
<td>Off On Off Off</td>
</tr>
<tr>
<td>…</td>
<td>… … … …</td>
</tr>
<tr>
<td>21uH</td>
<td>Off On On On</td>
</tr>
<tr>
<td>22.5uH</td>
<td>On On On On</td>
</tr>
</tbody>
</table>

Using the switch network, the add-on inductance can be tuned digitally from 0uH to 22.5uH with a step size of 1.5uH. On the Rx coils, a compensation inductor with 11uH inductance is also added to the loop. Assuming both the two coils are tuned to have the same resonant frequency, the switch network would give a tunable frequency from -11uH to 11.5uH to cope with the parameter drift.

V. EXPERIMENTAL RESULTS

In order to verify the proposed optimization algorithm, a WPT test rigs has been setup in the laboratory as shown in Fig 10. The two coils are winded by 19 turns using Litz cable with 500 strands, which form 298uH inductance. Two parallel 100pF high voltage ceramic capacitors are connected in serial as the add-on capacitors. The detailed circuit parameters are listed in Table I.
TABLE III.  EXPERIMENTAL RESULTS

<table>
<thead>
<tr>
<th>Add-on Inductance</th>
<th>Experimental Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{p}$</td>
<td>Frequency</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>3uH</td>
<td>2.73</td>
<td>641KHz</td>
</tr>
<tr>
<td>4.5uH</td>
<td>1.77</td>
<td>636KHz</td>
</tr>
<tr>
<td>6uH</td>
<td>2.46</td>
<td>632KHz</td>
</tr>
<tr>
<td>7.5uH</td>
<td>2.63</td>
<td>631KHz</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

The radius of the both coils is 20cm and the distance between the two coils is 2.2m, which makes the coupling between the two coils is around 0.006. The photos of the prototype are shown in Fig 10.

By switching the SW1 to SW4, the resonant frequency of the Tx coil can be adjusted. The exciting frequency is changed by using the keyboard on the MCU based controller. Both the overall efficiency and the DC bus current on the Tx side are measured and the experimental results are shown in Table III.

![Picture of the WPT system](image)

From the experimental results, the maximum DC bus current does reflect the efficiency of the whole system. By selecting the smallest peak current on the resonant frequency, the most optimized operating point can be achieved.

VI. CONCLUSION AND FUTURE WORK

In this paper, the design and implementation of a practical 2-coil WPT system have been introduced. In practical applications, the environment change could cause the circuit parameter drift. It is proved that only slight drift could result in great performance drop. In order to address this problem, a dynamic parameter tuning algorithm has been introduced to optimize the performance of the whole system. The proposed algorithm only requires the measurement of the input power on the Tx and the involvement of the Rx side is not necessary. Therefore the communication between the both sides is not required in the simplified system. In order to verify the proposed method, practical experiments have been conducted. It shows that with the proposed algorithm, the maximum power and efficiency can be achieved.

In the future, the switch network in the current circuit would be replaced by a relay network which control by MCU controller. A dynamic tuning algorithm will be implemented using a computer program without manual interference. Therefore, the optimization program can be executed automatically to make sure the whole system is working in the optimized condition.

REFERENCES