Abstract—A discrete-time resilient state feedback control scheme is presented to control nonlinear systems with locally conic type of nonlinearities and driven by finite energy disturbances. The resilience property is achieved in the presence of bounded perturbations in the feedback gain. The controller design is also robust as the design process addresses system models containing a higher degree of uncertainty by allowing perturbations in both the system parameters as well as the center and the boundaries of the cone in which the nonlinearity resides. Results are presented for various performance criteria in a unified framework using linear matrix inequalities (LMIs). Illustrative examples are included to demonstrate the efficiency of the proposed approach.

I. INTRODUCTION

In this paper, the design of a resilient state feedback controller to meet generalized performance criteria is presented. The design is intended for discrete time control of nonlinear systems. The system itself may have bounded perturbations on the system parameters and the nonlinearities considered in this design are constrained within a conic region whose boundaries are also uncertain. The inclusion of these perturbations on the system and the nonlinearities also guarantees a robust controller design.

When small perturbations in the controller gains cause significant performance deterioration, a “fragile” or “non-resilient” controller results [1]. An example of such a perturbation would be numerical round off errors when computing the gains which might occur when implementing controllers and observers with microprocessors. In other situations, manual tuning of the controller gains may be required to obtain the best performance. Thus, it is desirable to design a resilient controller that will have some tolerance to a change or readjustment of the control gain. We employ LMI techniques to obtain the resilient and robust controller design for performance criteria including asymptotic stability, $H_2$, $H_\infty$, input strict passivity, output strict passivity, and very strict passivity [2], [3].

Some recent representative work on robust and resilient control of various classes of nonlinear systems can be found in [4]-[14]. In [4] and [5] extensions to time-delay systems are presented; switched systems are treated in [6] and [7]; mixed criteria control is addressed in [8], while [9] and [10] consider singular systems. Fault-tolerant control for systems with time-varying delays is discussed in [11]; resilient control of networked systems is the focus of [12] and discrete-time sliding mode control is considered in [13]. A mixed criteria finite time control design is presented in [14] and [15] treats $H_2/H_\infty$ control for systems in the state-dependent nonlinear form. The present paper is an extension of [16]-[18] to the control of discrete-time uncertain nonlinear systems in the presence of uncertainty regarding the applied control gain.

The following notation is used in this work: $x \in \mathbb{R}^n$ denotes an $n$-dimensional vector with real elements. $A \in \mathbb{R}^{m \times n}$ denotes an $m \times n$ matrix with real elements. $A > 0$ implies that $A$ is a positive definite matrix. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalue of the symmetric matrix $A$. $l_2$ is the space of all real-valued vector functions of time. In $[A \; B \; C]$, *( denotes $B^T$. The following results are used in this paper:

Lemma 1. Schur complement

$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} > 0$ is true if and only if $C > 0$ and $A - BC^{-1}B^T > 0$.

Lemma 2. $A^T B + B^T A \leq \alpha A^T A + \alpha^{-1} B^T B$ is true for any $\alpha > 0$.

Lemma 3. Rayleigh’s inequalities

$P = P^T, \lambda_{\min}(P)\|x\|^2 \leq x^T Px \leq \lambda_{\max}(P)\|x\|^2$

Lemma 4.

$\lambda_{\max}(\Delta^T \Delta) = \lambda_{\max}(\Delta \Delta^T) = \lambda_{\max}(\Delta \Delta^\top)$

where $\Delta^T = [A \; B], \Delta^\top = [A \; 0 \; B]$, and $A, 0, B$ are matrices of appropriate dimensions [16].

II. PROBLEM FORMULATION

A linear state feedback control, $u_k = \bar{K}x_k$, where the feedback gain $K$ may be unintentionally perturbed as, $\bar{K} = K + \Delta K$, is assumed to be used to control a discrete-time nonlinear system,

$x_{k+1} = f(x_k, u_k, w_k)$  \hspace{1cm} (1)

where $x_k \in R^r$ is the state, $u_k \in R^u$ is the input, and $w_k \in R^w$ is an $l_2$ disturbance input. As mentioned before, undesired perturbations may be due to numerical errors or manual tuning.

The performance output is defined as

$z_k = C_1 x_k + D_1 u_k + E_1 w_k$  \hspace{1cm} (2)

The linear part of the system can be extracted from (1) as
\[ x_{k+1|\text{lin}} = \tilde{A}x_k + \tilde{B}u_k + \tilde{F}w_k \]  

The nonlinear part of the system \( \tilde{F} \) is thus

\[ \tilde{F} = f(x_k, u_k, w_k) - (\tilde{A}x_k + \tilde{B}u_k + \tilde{F}w_k) \]

We assume a known bound on the perturbations as,

\[ \Delta x \Delta x^T \leq \sigma I \]

It is assumed that there are uncertainties associated with the linear part, so that the system parameters are perturbed as \( \tilde{A} = A + \Delta A, \tilde{B} = B + \Delta B, \tilde{F} = F + \Delta F \) for some \( \sigma > 0 \) and for every \( x \in D^r, \ u \in D^r, \ w \in D^r \) where \( D^r \subset R^r, D^s \subset R^s, D^w \subset R^w \) are domains which include the origin. Note here that (7) describes the conic region within \( D^r \times D^r \times D^r \) in which the nonlinearity resides[20]. From (7), it is noted that the center of this region represented by the linear system parameters in (3) is uncertain.

The parameters that define the boundary of this region in (7) are also perturbed as follows,

\[ C_i = C_i + \Delta C_i, \tilde{D}_i = D_i + \Delta D_i, \tilde{E}_i = E_i + \Delta E_i \]

The perturbations \( \Delta C_i, \Delta D_i, \Delta F_i, \Delta E_i \) are assumed to be bounded as follows,

\[
\begin{bmatrix}
\Delta C_i^T & \Delta D_i^T & \Delta F_i^T \\
\Delta D_i^T & \Delta E_i^T \\
\Delta F_i^T & \Delta E_i^T
\end{bmatrix} \leq \gamma I
\]

By having perturbations on both system parameters of the linear part(center parameters) and the boundary (radius) of the nonlinearity as given in (7) and (8), robustness on both types of system uncertainties is achieved, so we will design a resilient controller which is also robust.

For a perturbed locally conic nonlinearity, the cone in which the nonlinearity resides is shown as the shaded region in Fig. 1 with solid center and boundary lines for the scalar case with no noise. The uncertainties for the center and boundaries of the nonlinear region are shown as the dashed lines in the figure. Our design methodology allows us to incorporate these center and radius uncertainties.

Many nonlinearities used in modeling engineering systems belong to this conic type of nonlinearity including locally sinusoidal nonlinearities such as pendulum systems, saturation nonlinearities such those in transistors and motors, dead zone nonlinearities in diodes and amplifiers, and piecewise linear functions such as Chua's circuit [3], [19], [20].

Consider the inequality

\[ V_k - V_{k+1} - \delta \|z_k\| + \beta \|w_k\| > 0 \]

where \( V_i \) is quadratic energy function and \( V_i = x_i^T P x_i, \ P > 0 \). The additional terms in the Lyapunov inequality in (11) are used to incorporate various performance criteria to be explained below.

Upon summation, inequality (10) becomes

\[ \sum_{k=0}^{\infty} \|z_{k+i}\|^2 + \epsilon \sum_{k=0}^{\infty} \|w_{k+i}\|^2 - \beta \sum_{k=0}^{\infty} z_{k+i}^T w_{k+i} > 0 \]

By using Lemma 3, we obtain

\[ \lambda_{\text{max}} (P) \|z_0\|^2 \leq \lambda_{\text{max}} (P) \|x_0\|^2 - \sum_{k=0}^{\infty} \delta \|z_{k+i}\|^2 + \epsilon \|w_{k+i}\|^2 - \beta z_{k+i}^T w_{k+i} \]

We can design various controllers for several performance criteria with this method. For example, setting \( \delta > 0, \beta = 0, \epsilon = 0 \) will yield a bound on the energy of the performance output in terms of the initial state \( x_0 \),

\[ \sum_{k=0}^{\infty} \|z_{k+i}\|^2 \leq \frac{1}{\beta} \lambda_{\text{max}} (P) \|x_0\|^2 \]

Minimizing \( \lambda_{\text{max}} (P) \) and maximizing \( \delta \) will give us a smaller bound on the energy of the performance output, i.e., \( H_2 \) control. Values for \( \delta, \beta, \epsilon \) can be specified to lead to other performance criteria such as \( H_\infty \) and various passivity types as listed in Table I [2], [3].

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Design Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta = 0, \beta = 0, \epsilon = 0 )</td>
<td>Asymptotic stability</td>
</tr>
<tr>
<td>( \delta = 0, \beta = 1, \epsilon &gt; 0 )</td>
<td>Input strict Passivity</td>
</tr>
<tr>
<td>( \delta = 0, \beta = 0, \epsilon = 0 )</td>
<td>( H_2 ) controller</td>
</tr>
<tr>
<td>( \delta = 1, \beta = 0, \epsilon &lt; 0 )</td>
<td>( H_\infty ) controller</td>
</tr>
<tr>
<td>( \delta &gt; 0, \beta = 1, \epsilon = 0 )</td>
<td>Output strict Passivity</td>
</tr>
<tr>
<td>( \delta &gt; 0, \beta = 1, \epsilon &gt; 0 )</td>
<td>Very Strict Passivity</td>
</tr>
</tbody>
</table>

The LMI formulation presented in the following section enables one to design various controllers according to many different performance criteria in a common framework.
III. MAIN RESULTS

The following is the main result of this work:

**Theorem 1.** There exists a resilient state feedback controller \( u = \hat{K}x \) for the discrete-time system described by (1) and (7) with performance output (2), if LMI (14)-(17) for each case in Table I, is feasible for some \( Y, Q > 0 \) and \( \alpha, \rho > 0 \). Then the controlled system will satisfy (10). The controller will be robust and tolerate uncertainties on both the linear part and the boundary of the cone with the maximum perturbation bound \( \gamma \) given by (9).

**Case 1.** \( \text{H} \) controller, output and very strict passivity (noisy, \( \delta \neq 0 \))

\[
S_1 = \begin{bmatrix}
    s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} & s_{19} & s_{10} \\
    * & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} & s_{28} & s_{29} & s_{11} \\
    * & * & s_{33} & s_{34} & s_{35} & s_{36} & s_{37} & s_{38} & s_{39} & s_{21} \\
    * & * & * & s_{44} & s_{45} & s_{46} & s_{47} & s_{48} & s_{49} & s_{31} \\
    * & * & * & * & s_{55} & s_{56} & s_{57} & s_{58} & s_{59} & s_{41} \\
    * & * & * & * & * & s_{66} & s_{67} & s_{68} & s_{69} & s_{51} \\
    * & * & * & * & * & * & s_{77} & s_{78} & s_{79} & s_{61} \\
    * & * & * & * & * & * & * & s_{88} & s_{89} & s_{71} \\
    * & * & * & * & * & * & * & * & s_{99} & s_{81} \\
    * & * & * & * & * & * & * & * & * & s_{101}
\end{bmatrix} > 0 \quad (14)
\]

**Case 2.** \( \text{H} \) controller (non-noisy, \( \delta \neq 0 \))

\[
S_2 > 0 \quad (15)
\]

where \( S_2 \) is obtained from \( S_1 \) by cancelling the second row and the second column matrices.

**Case 3.** Input strict passivity (noisy, \( \delta = 0 \))

\[
S_3 > 0 \quad (16)
\]

where \( S_3 \) is obtained from \( S_1 \) by cancelling the third row and the third column matrices.

**Case 4.** Asymptotic stability (noisy, \( \delta = 0 \))

\[
S_4 > 0 \quad (17)
\]

where \( S_4 \) is obtained from \( S_1 \) by cancelling the second, third, and seventh row and the second, third and seventh column matrices.

The matrices elements, \( s_{ij} \), are given as:

\[
s_{11} = Q, s_{12} = \frac{\beta}{2}(Q + D_{-1}D_{-1}^T), s_{13} = Q + D_{-1}D_{-1}^T, s_{14} = Q + D_{-1}D_{-1}^T
\]

\[
s_{16} = Q + D_{-1}D_{-1}^T, s_{15} = Y + D_{-1}D_{-1}^T, s_{19} = \frac{\beta}{2}(E_x + E_y) - \varepsilon I - \frac{\alpha}{4}D D_{-1}^T
\]

\[
s_{21} = s_{11} + s_{22} = 4, s_{33} = 4, s_{35} = 4, s_{44} = 4, s_{55} = 4, s_{66} = 4, s_{77} = 4, s_{88} = 4, s_{99} = 4
\]

\[
0
\]

All other elements of matrix \( S \) not defined above are zero.

**Sketch of the Proof.** Substituting \( V_i \) and \( V_{k+1} \) in (10), we have:

\[
x_k^T P x_k - \delta \|z_k\|^2 - \varepsilon \|w_k\|^2 + \beta z_k^T w_k
\]

\[
- (x_{k+1,lin} + \tilde{\gamma})^T P(x_{k+1,lin} + \tilde{\gamma}) > 0
\]

Applying Lemma 1 and moving all terms containing \( \tilde{\gamma} \) to the right hand side of the inequality, we get

\[
x_k^T P x_k - \delta \|z_k\|^2 - \varepsilon \|w_k\|^2 + \beta z_k^T w_k
\]

\[
- (x_{k+1,lin} + \tilde{\gamma})^T P(x_{k+1,lin} + \tilde{\gamma}) > 0
\]

From the non-negative definite matrix

\[
\alpha^{-\frac{1}{2}} P^{-\frac{1}{2}} \begin{bmatrix}
    \alpha^{-\frac{1}{2}} & \tilde{\gamma} P^{-\frac{1}{2}} \\
    \tilde{\gamma} P^{-\frac{1}{2}} & \alpha P^2
\end{bmatrix} \geq 0
\]

we obtain,

\[
\begin{bmatrix}
    \alpha^{-\frac{1}{2}} & \tilde{\gamma} P^{-\frac{1}{2}} \\
    \tilde{\gamma} P^{-\frac{1}{2}} & \alpha P^2
\end{bmatrix} \geq 0
\]

Then the following

\[
x_k^T P x_k - \delta \|z_k\|^2 - \varepsilon \|w_k\|^2 + \beta z_k^T w_k
\]

\[
- (x_{k+1,lin} + \tilde{\gamma})^T P(x_{k+1,lin} + \tilde{\gamma}) > 0
\]

is a sufficient condition for (19).

Moving all the terms in (22) to the left side and using Lemma 1, we have,

\[
x_k^T P x_k - \delta \|z_k\|^2 - \varepsilon \|w_k\|^2 + \beta z_k^T w_k
\]

\[
- \alpha^{-\frac{1}{2}} \beta P^{-\frac{1}{2}} x_{k+1,lin} + \tilde{\gamma} P^{-\frac{1}{2}} x_{k+1,lin} > 0
\]

Using (7), we obtain a sufficient condition for (23)

\[
x_k^T P x_k - \delta \|z_k\|^2 - \varepsilon \|w_k\|^2 + \beta z_k^T w_k
\]

\[
-(\tilde{C} x_k + \tilde{D}_1 u_k + \tilde{E}_1 w_k) > 0
\]

Substituting for \( z_k, w_k \) and \( x_{k+1,lin} \), (24) can be rewritten in quadratic form as

\[
\begin{bmatrix}
    x_k^T & w_k^T
\end{bmatrix}

\[
H
\]

\[
\begin{bmatrix}
    x_k & w_k
\end{bmatrix}
\]

\[
> 0
\]

where

\[
H = \begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22}
\end{bmatrix}
\]

with

\[
h_{11} = P - \delta \tilde{C} \tilde{D}_1 \tilde{C}^T + \delta \tilde{D}_1 \tilde{D}_1^T + \tilde{E}_1 \tilde{E}_1^T + \tilde{D}_1 \tilde{D}_1^T
\]

\[
-(\tilde{A} + \tilde{E}_1 \tilde{E}_1^T \tilde{P} - \alpha \tilde{P}^2) \tilde{D}_1 \tilde{D}_1^T
\]

\[
h_{12} = \frac{\delta}{2} \tilde{C} \tilde{D}_1 \tilde{C}^T + \frac{\delta}{2} \tilde{D}_1 \tilde{D}_1^T - \frac{\beta}{2} \tilde{E}_1 \tilde{E}_1^T \tilde{P} - \alpha \tilde{P}^2
\]

\[
-(\tilde{A} + \tilde{E}_1 \tilde{E}_1^T \tilde{P} - \alpha \tilde{P}^2) \tilde{E}_1 \tilde{E}_1^T \tilde{P} - \alpha \tilde{P}^2
\]

\[
h_{22} = -\delta \tilde{E}_1^T \tilde{E}_1 + \frac{\beta}{2} \tilde{E}_1 \tilde{E}_1^T - \tilde{E}_1 \tilde{E}_1^T \tilde{P} - \alpha \tilde{P}^2
\]

\[
(19)
\]

\[
(20)
\]

\[
(21)
\]

\[
(22)
\]
Separating the terms with $P(P - \alpha P^2)^{-1}$ and using Lemma 1 twice, we have,
\[
\begin{pmatrix}
    h_{11} & h_{12} & (A + \tilde{B}K)^T P \\
    h_{12}^T & h_{22} & F^T P \\
    P(A + \tilde{B}K) & PP & P \\
\end{pmatrix} > 0
\]
where
\[
h_{11} = P - \delta(C_x + D_x\tilde{K})^T (C_x + D_x\tilde{K}) - (\tilde{C}_x + \tilde{D}_x\tilde{K})^T (\tilde{C}_x + \tilde{D}_x\tilde{K})
\]
\[
h_{12} = \delta(C_x + D_x\tilde{K})^T E_x + \frac{\beta}{2}(C_x + D_x\tilde{K})^T (\tilde{C}_x + \tilde{D}_x\tilde{K})^T E_x
\]
\[
h_{22} = -\delta E_x^T E_x - \varepsilon I + \frac{\beta}{2}(E_x + E_x^T) - \tilde{E}_x^T \tilde{E}_x
\]

Pre- and post-multiplying (26) by $\text{Diag}(Q,I,Q,I)$ where $Q = P^{-1}$, and writing $\hat{y}$ as $KQ$ and using Lemma 1 twice more, the following inequality results
\[
\begin{pmatrix}
    \frac{\beta}{2}(QC_x^T + \tilde{Y}^T D_x^T) & O\hat{A} + \hat{Y}^T B^T & 0 & O\hat{C}_x^T + \hat{Y}^T D_x^T \\
    \frac{\beta}{2}(E_x + E_x^T) - \varepsilon I & F^T & 0 & \tilde{E}_x^T \\
    Q & 0 & I & 0 \\
    0 & 0 & \alpha^{-1} I & 0 \\
    0 & 0 & \delta^{-1} I & 0 \\
    0 & 0 & I & 0 \\
\end{pmatrix} > 0
\]

Moving all the terms with $\Delta (\hat{A}, \tilde{B}, \tilde{C}_x$, etc.) in (27), to the right hand, the resulting right hand side can be bounded using Lemma 2. Then by applying Lemma 4 twice and using (9), we have
\[
\begin{pmatrix}
    \frac{\beta}{2}(QC_x^T + \tilde{Y}^T D_x^T) & O\hat{A}_d + \hat{Y}^T b^T & 0 & O\hat{C}_x^T + \hat{Y}^T d^T \\
    \frac{\beta}{2}(E_x + E_x^T) - \varepsilon I & F^T & 0 & \tilde{E}_x^T \\
    Q & 0 & I & 0 \\
    0 & 0 & \alpha^{-1} I & 0 \\
    0 & 0 & \delta^{-1} I & 0 \\
    0 & 0 & I & 0 \\
\end{pmatrix} > 0
\]

A bound for the right side of (29) is found by applying Lemma 2 as
\[
\begin{pmatrix}
    0 & 0 & \frac{\beta}{2} D_x & \frac{\beta}{2} D_x \\
    -B & 0 & 0 & -B \\
    \Delta \Delta & 0 & 0 & \Delta \Delta \\
    -I & 0 & 0 & -I \\
\end{pmatrix} \leq \frac{1}{\sigma} \begin{pmatrix}
    0 & 0 & \frac{\beta}{2} D_x & \frac{\beta}{2} D_x \\
    -B & 0 & 0 & -B \\
    \Delta \Delta & 0 & 0 & \Delta \Delta \\
    -I & 0 & 0 & -I \\
\end{pmatrix} + \sigma
\]

Then, using the bound (5), together with Lemma 1, LMI (14) in theorem 1 is obtained.

The other cases in Theorem 1 are special cases of this general case. The other LMIs in Theorem 1 can be derived using a similar procedure. For the non-noisy case, from (25) we only have $x_i^T h_{11} x_i > 0$. For the $\delta = 0$ case, in (26) terms with $\Delta$ no longer exist, so (27) becomes a 5 by 5 block matrix. Then, going through the similar procedure from (25) to (30), LMIs (15), (16) and (17) are obtained. So this completes the sketch of the proof.

IV. SIMULATION STUDY

Chua’s circuit with chaotic behavior is used in this example [19]. The state space model is given by
\[
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3
\end{bmatrix} = \begin{bmatrix}
    -\alpha_c & \alpha_c & 0 \\
    1 & -1 & 1 \\
    0 & -\beta_c & -\mu
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix} + \begin{bmatrix}
    1 & 1 & 1
\end{bmatrix} u + \begin{bmatrix}
    0 & 0
\end{bmatrix} w_k
\]
where
\[
f(x_i) = bx_i + 0.5(a-b)(|x_i+1|-|x_i-1|), w_k = e^{-k}
\]
with the parameters
$\alpha_c = 9.1$, $\beta_c = 16.5811$, $\mu = 0.138083$, $a = -1.3659$, $b = -0.7408$

The discretized Chua’s circuit system with the sampling period of $T=0.01$s is given as

$$
\begin{bmatrix}
    x_{1,k+1} \\
    x_{2,k+1} \\
    x_{3,k+1}
\end{bmatrix} =
\begin{bmatrix}
    0.909 & 0.091 & 0 \\
    0.01 & 0.99 & 0.01 \\
    0 & -0.1658 & 0.9986
\end{bmatrix}
\begin{bmatrix}
    x_{1,k} \\
    x_{2,k} \\
    x_{3,k}
\end{bmatrix}
\begin{bmatrix}
    0.091f(x_{1,k}) \\
    0.01 \\
    0
\end{bmatrix} +
\begin{bmatrix}
    0.01 \\
    0 \\
    0
\end{bmatrix} u_k +
\begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix} w_k
$$

The design parameters for the performance output are given in Table II.

**TABLE II. DESIGN PARAMETERS IN THE SIMULATION EXAMPLE**

<table>
<thead>
<tr>
<th>Asy. S. P.</th>
<th>$H_2$ Ctrl.</th>
<th>$H_\infty$ Ctrl.</th>
<th>Input S. P.</th>
<th>Output S. P.</th>
<th>Very S. P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

In all cases, $C_f$, $C_z$ are chosen to be $0.1*I_3$ and $D_f$, $D_z$ are chosen to be $[0.1; 0.1; 0.1]$. $E_f$, $E_z$ are chosen to be $0.1*I_3$ for the noisy cases and zero matrix for non-noisy cases.

The state variables of the open loop chaotic Chua’s circuit is shown in Fig. 2. The initial values of the state variables are chosen to be $[1;1;1]$.

![Figure 2. The state of Chua’s circuit without control](image)

To verify the robustness and resilience property of the controller, the system matrix and coefficients of the nonlinearity are perturbed as follows

$$
\alpha_c = 8.9$, $\beta_c = 17$, $\mu = 0.15$, $a = -1.4$, $b = -0.76$

And the feedback gains $K$ are perturbed as shown in Table III.

Feedback gains for each performance criterion are given in the first column of Table III, after the proposed control is applied to the unperturbed system. And if the feedback gains are perturbed within the bound of $\sigma$ as given in the second column of Table III for each cases, the systems can still be controlled and desired performance criterion is still achieved. As shown in Figs. 3 & 4, when using the perturbed feedback gain $K$, perturbed systems are still stabilized and the expected performance levels are still achieved, demonstrating the effectiveness of the proposed design method.

**TABLE III. FEEDBACK GAIN OF THE CONTROLLED THE SYSTEM, THE PERTURBATION BOUND $\sigma$ AND THE PERTURBED GAIN**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Feedback Gain $K$</th>
<th>$\sigma_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asy. S. P.</td>
<td>$[-0.86 -9.15 1.2]$</td>
<td>0.25</td>
</tr>
<tr>
<td>$H_2$ Ctrl.</td>
<td>$[-2.1 -13.28 0.9]$</td>
<td>0.168</td>
</tr>
<tr>
<td>$H_\infty$ Ctrl.</td>
<td>$[-2.32 -10.38 0.85]$</td>
<td>0.124</td>
</tr>
<tr>
<td>Input S. P.</td>
<td>$[-1.22 -7.03 0.36]$</td>
<td>0.093</td>
</tr>
<tr>
<td>Output S. P.</td>
<td>$[-0.8 -5.85 0.15]$</td>
<td>0.083</td>
</tr>
<tr>
<td>Very S. P.</td>
<td>$[-0.89 -6.24 -0.8]$</td>
<td>0.052</td>
</tr>
</tbody>
</table>

![Figure 3. The state variables of controlled perturbed Chua’s circuit for the two non-noisy cases. (a) Asymptotic stability, (b) $H_2$ control.](image)

Criteria in Table I are verified to be achieved in Table IV. According to Table I, the numerical values of the terms that can be calculated prior to the design process are given in the second column of Table IV, while values of the other terms are shown in the third column. From the calculation, all the performance criteria are verified as shown in the table. Notice that the case of asymptotic stability is verified from Fig. 4.

**TABLE IV. VERIFICATION OF THE PERFORMANCE CRITERIA**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Pre-calculated Values</th>
<th>Simulation Results</th>
</tr>
</thead>
</table>

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