Optimality and Stability of Event Triggered Consensus State Estimation for Wireless Sensor Networks*

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Abstract—This paper presents distributed state estimation methods through wireless sensor networks with event triggered communication protocols among the sensors. Optimal consensus filters are derived which apply to generic non-uniform and asynchronous information exchange scenarios among neighboring sensors. To obtain a scalable covariance propagation algorithm, the optimal filter is approximated by a suboptimal filter. Homogeneous detection criteria are designed on each sensor node to determine the broadcasting instants. Thus, a consensus on state estimates is reached with all estimator sensors for the suboptimal consensus filter. The purpose of event detection is to achieve energy efficient operation by reducing unnecessary interactions among the neighboring sensors. In addition, the performance of the proposed state estimation algorithm is validated using a simulation example.

I. INTRODUCTION

In recent years, wireless sensor networks have come into prominence with a range of applications [1], [2]. One of the most important applications is trajectory tracking. The Kalman filter and its various extensions are effective algorithms for tracking the state of known dynamic processes [3], [4]; while $H_{\infty}$ filter is specifically designed for robustness [5]. Typically, sensor networks are often deployed in environments with limited computational and communication resources. Communication over radio is the most energy-consuming function performed by these devices, so that the communication frequency needs to be minimized. These constraints dictate that sensor network problems are best approached in a holistic manner, by jointly maintaining estimation performance while reducing the number of transmissions.

One area that has received considerable attention during recent years is the utilization of an event triggered sampling to trade the computation for communication [6]. As pointed out in [7], event triggered state estimation is not a standard problem due to the non-standard information pattern. Information is obtained precisely only when an event occurs; if no event takes place, the information can only be inferred from the event condition. Currently, most research on event based state estimation focuses on centralized algorithms, either in stochastic [8], [9], [10] or deterministic settings [11], [12], [13]. However, some problems are difficult or impossible for a monolithic system to solve. This necessitates the use of wireless sensor networks [14]. Unfortunately, the current existing filter designs for wireless sensor networks, most of which are based on time triggered sampling, result in high power consumption and network congestion.

Based on the above observations, event triggered distributed state estimation approaches having a low transmission frequency are proposed which significantly reduce the overall bandwidth consumption, and increase the lifetime of the network. These approaches are an extension of [15] to the event triggered transmission case. Event triggered transmissions pose new challenges to existing design methodologies as novel requirements, like adaptivity, uncertainty, and nonlinearity, arise. Specifically, the sensor node will not receive any information from the neighbors if the events at neighboring sensor nodes are not triggered. In this case, the behavior of neighboring sensors has to be estimated with the aid of the system model and information obtained from neighbors at event instants. After an event has occurred, the sensor broadcasts its predictive state to its neighbors and the state of the internal system models will be reinitialized for both itself and its neighbors. Then, a modified consensus filter is proposed to accommodate the generic uniform and asynchronous information exchange scenario. The optimal filter gain has been given with the corresponding error covariance updating algorithm. But unfortunately the computational cost is not scalable with respect to the number of sensors. Then the optimal consensus filter is approximated by a scalable suboptimal filter. The formal stability analysis of the suboptimal filter is provided, and a specific event triggered transmission mechanism is constructed. Additionally, the event conditions can be checked without the knowledge of neighbors’ information.

II. PROBLEM FORMULATION

Consider a system whose state at time $k$ is $x(k) \in \mathbb{R}^n$. The time index $k$ of the state evolution will be discrete and identified with $\mathbb{N} = \{0, 1, 2, \ldots\}$.

Let $(\Omega, \mathcal{F}, P)$ be a probability space upon which $\{w(k), k \in \mathbb{N}\}$ is an independent sequence of Gaussian random variables, having zero mean and covariance matrices $\{Q(k)\}$. That is,

$$
\mathbb{E} \left[ w(k) w^T(l) \right] = Q(k) \delta(k, l),
$$

where $\delta(k, l) = 1$ if $k = l$, and $\delta(k, l) = 0$, otherwise. The state at the initial time $x(0)$ is a Gaussian random variable with mean $x_0$ and covariance matrix $P_0$.

The state of the system satisfies linear dynamics

$$
x(k + 1) = A(k) x(k) + B(k) w(k),
$$

where $A(k)$ and $B(k)$ are matrices depending on $k$. The objective is to estimate $x(k)$ based on the state measurements $y(k)$ received from neighboring sensor nodes. The measurement process is described by

$$
y(k) = C(k) x(k) + v(k),
$$

where $C(k)$ is a known matrix and $v(k)$ is a zero mean Gaussian white noise sequence with covariance matrix $R(k)$. The observation process is given by

$$
y(k) = C(k) x(k) + v(k),
$$

where $C(k)$ is a known matrix and $v(k)$ is a zero mean Gaussian white noise sequence with covariance matrix $R(k)$. The observation process is given by

$$
y(k) = C(k) x(k) + v(k),
$$

where $C(k)$ is a known matrix and $v(k)$ is a zero mean Gaussian white noise sequence with covariance matrix $R(k)$.
where $A(k)$ and $B(k)$ are matrices of appropriate dimensions. The state can be observed only indirectly through a sensor network whose communication topology can be modeled by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Here $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of sensor nodes. The edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ consists of the communication links between sensors. Each sensor $i \in \mathcal{V}$ has a linear sensing model

$$z_i(k) = H_i(k)x(k) + v_i(k), \quad z_i(k) \in \mathbb{R}^m,$$

(3)

where $H_i(k)$ and $v_i(k)$ are observation matrix and measurement noise of sensor $i$ at time $k$, respectively. Each pair $(A(k), H_i(k))$ is not assumed to be observable, but assume the pair $(A(k), H(k))$ with col$(H_1(k), \ldots, H_N(k))$ is observable. The measurement noise processes $\{v_i(k)\}$ and $\{v_j(k)\}$ for $i \neq j$ are independent Gaussian processes with zero mean and known covariances $\{R_i(k)\}$ and $\{R_j(k)\}$, respectively. Note also that $\{w(k)\}$ and $\{v_i(k)\}$ for all $i \in \mathcal{V}$ are independent processes. Sensor $i$ can only communicate with other sensors within its sensing radius $r$. All the other sensors that sensor $i$ can communicate with are referred to as the neighbors of sensor $i$, which can be mathematically defined as $\mathcal{N}_i = \{j \in \mathcal{V} | d_{ij} \leq r \}$ with $d_{ij}$ being the Euclidean distance between sensor $i$ and sensor $j$. Note that the underlying graph $\mathcal{G}$ of the wireless sensor network is not required to be connected, and could be a switching graph.

The local consensus filter is described for $k \geq 0$ by the equations

$$\dot{\hat{x}}_i(k) = \hat{x}_i(k) + K_i(k) [z_i(k) - H_i(k) \hat{x}_i(k)] + C_i(k) \sum_{j \in \mathcal{N}_i} [\bar{x}_j(k) - \hat{x}_i(k)], \quad i \in \mathcal{V},$$

(4)

where $K_i(k) \in \mathbb{R}^{n \times m}$ and $C_i(k) \in \mathbb{R}^{n \times n}$ are the filter gain and consensus gain, respectively. Here $\hat{x}_i(k)$ is sensor $i$’s estimation of the process state $x(k)$; $\bar{x}_i(k)$ is the estimate prior to assimilating the measurements at time $k$, and it is obtained by projecting the estimate $\hat{x}_i(k-1)$ via the transition matrix

$$\bar{x}_i(k) = A(k-1) \hat{x}_i(k-1).$$

(5)

The estimates $\hat{x}_j(k)$ for $j \in \mathcal{N}_i$ are constructed for sensor $i$ to estimate the neighboring sensors’ state using only the measurements received from the neighbors, while $\bar{x}_i(k)$ is constructed for sensor $i$ to know what neighboring sensors assume its state estimate by using only predictive state estimates broadcasted to the neighbors. Hence, these estimates are the same for all sensors, which can be simply constructed by the following equations

$$\hat{x}_j(k) = \gamma_j(k) \bar{x}_j(k) + [1 - \gamma_j(k)] A(k-1) \bar{x}_j(k-1),$$

(6)

where $\gamma_j(k) = 1$ or 0 indicates the sensor node $j$ does or not broadcast the data to its neighbors at the time instant $k$, respectively. The structure in (6) is chosen because the information exchange usually happens at the beginning of each discrete time instant. At that time, only the predicted state is available instead of the estimated state.

Remark 1: When $\hat{x}_j(k)$ is replaced by $\bar{x}_j(k)$ for all $j \in \mathcal{N}_i = \mathcal{N} \cup \{i\}$, the consensus filter in (4) reduces to the distributed Kalman consensus filter proposed in [15]. It reflects that time triggered transmission is a special case of event triggered transmission when events happen at each time instant. Also different from the distributed event triggered consensus filter in [16], the local consensus filter in (4) for sensor $i$ can have access to the common estimate $\hat{x}_i$. Keeping a copy of this global estimate ensures consistency in the sensor network. This is crucial for event detector designs to decide the broadcasting instants if the deviation between the true predictive state of sensor $i$ and what the neighboring sensors assume its predictive state goes beyond some tolerance limit.

The problem now is to find the particular blending factor $K_i(k)$ for fixed $C_i(k)$ for all $i \in \mathcal{V}$ that yields a consensus state estimate for the sensor network under event triggered transmission that is optimal in some sense.

III. OPTIMAL EVENT TRIGGERED CONSENSUS FILTER DESIGN

Now we proceed to develop the consensus recursive equations. The optimization criterion used here is minimization of the mean-square estimation error of the random variable $x$ for each sensor. The following notation is defined to distinguish the different errors for sensor $i$ at time $k$:

$$\eta_i(k) = \hat{x}_i(k) - x(k) \text{ estimation error},$$

$$\tilde{\eta}_i(k) = \bar{x}_i(k) - x(k) \text{ prediction error},$$

$$\tilde{\eta}_i(k) = \hat{x}_i(k) - x(k) \text{ common error},$$

and the associated error covariance matrices with the estimates $\hat{x}_i(k), \bar{x}_i(k)$ and $\tilde{x}_i(k)$ are given by

$$P_{\eta_i}(k) = \mathbb{E} \left[ \eta_i(k) \eta_i^T(k) \right], \quad \tilde{P}_{\eta_i}(k) = \mathbb{E} \left[ \tilde{\eta}_i(k) \tilde{\eta}_i^T(k) \right],$$

$$P_{\bar{x}_i}(k) = \mathbb{E} \left[ \bar{x}_i(k) \bar{x}_i^T(k) \right], \quad \tilde{P}_{\bar{x}_i}(k) = \mathbb{E} \left[ \tilde{x}_i(k) \tilde{x}_i^T(k) \right], \quad \tilde{P}_{\tilde{x}_i}(k) = \mathbb{E} \left[ \tilde{x}_i(k) \tilde{x}_i^T(k) \right].$$

Returning to the optimization problem, it is to find the particular $K_i(k)$ for fixed $C_i(k)$ that minimizes the individual mean square estimation error $\mathbb{E} \left[ \| \eta_i^T(k) \eta_i(k) \| \right]$ for each sensor node $i$. The optimal consensus filter based on event triggered transmission is designed as follows.

Theorem 1: Consider a sensor network with topology $\mathcal{G}$ observing a linear time-varying process in (2). Assume every node measures the data using the sensing model in (3). Then, under suitable event triggered transmission mechanisms, the distributed event-based consensus filter in the form of (4), (5) and (6) has the optimal gain

$$K_i^* = \left[ \tilde{P}_i + C_i \sum_{j \in \mathcal{N}_i} (\tilde{P}_{ji} - \tilde{P}_i) \right] H_i^T \left( R_i + H_i \tilde{P}_i H_i^T \right)^{-1}$$

where

$$P_{\eta_j} = F_{ij} \tilde{P}_{\eta_j} F_{ij}^T + F_{ij} \sum_{s \in \mathcal{N}_j} \left( \tilde{P}_{js} - \tilde{P}_{ij} \right) C_s^T + K_i R_{ij} K_j^T$$

\footnote{The time index $k$ is omitted temporarily to save writing, and $k + 1$ is denoted simply as the superscript $^+$.}
\[
\sum_{r \in \mathcal{N}_i \setminus \{s\}} [\hat{P}_{rs} - \hat{P}_{ij}] F_j^T + C_i D_{ij} C_j^T,
\]
where \( D_{ij} = \sum_{s \in \mathcal{N}_j} \sum_{r \in \mathcal{N}_i \setminus \{s\} \setminus \{i\}} [\hat{P}_{rs} - \hat{P}_{ij}] - \hat{P}_{ij} \).

The double summation can be written as

\[
D_{ij} = A P_{ij} A^T + BQB^T,
\]

Now expand the general form for \( P_i \) in (9), and rewrite it in the form

\[
P_i = \sum_{s \in \mathcal{N}_i} \left[ \hat{P}_{is} - \hat{P}_i \right] C_i^T + C_i \sum_{r \in \mathcal{N}_i} \left[ \hat{P}_{ri} - \hat{P}_i \right] - K_i H_i \left[ \hat{P}_i + \sum_{s \in \mathcal{N}_i} \left( \hat{P}_{is} - \hat{P}_i \right) C_i^T \right] - \left[ \hat{P}_i + C_i \sum_{r \in \mathcal{N}_i} \left( \hat{P}_{ri} - \hat{P}_i \right) \right] H_i^T K_i^T + \hat{P}_i + C_i D_i C_i^T + K_i (H_i \hat{P}_i H_i^T + R_i) K_i^T.
\]

Notice that the third and fourth terms are linear in \( K_i \), and that the last term is quadratic in \( K_i \). The matrix differentiation formulas may now be applied to (10). Minimize the trace of \( P_i \) because it is the sum of the mean-square errors in the estimates of all the elements of the state vector. Now differentiate the trace of \( P_i \) with respect to \( K_i \). The result is

\[
\frac{\partial \text{trace}(P_i)}{\partial K_i} = 2 K_i (H_i \hat{P}_i H_i^T + R_i) - 2 \left[ \hat{P}_i + C_i \sum_{r \in \mathcal{N}_i} (\hat{P}_{ri} - \hat{P}_i) \right] H_i^T \left( H_i \hat{P}_i H_i^T + R_i \right)^{-1}.
\]

Note that \( \hat{P}_i \) and \( \hat{P}_{ij} \) are needed to accomplish this. The error covariance matrix associated with \( \hat{x}_i \) is obtained by first forming the expression for the prediction error (with the time index reinserted)

\[
\tilde{\eta}_i (k + 1) = A(k) \eta_i (k) - B(k) w(k).
\]

Now note that \( \eta_i (k) \), \( \eta_j (k) \) and \( w(k) \) have zero cross-correlation, because \( w(k) \) is the process noise for the step ahead of \( k \). Thus, the expression for \( \tilde{P}_{ij} (k + 1) \) can be written as

\[
\tilde{P}_{ij} (k + 1) = A(k) P_{ij} (k) A^T (k) + B(k) Q(k) B^T (k).
\]

It remains to determine the update rules of \( \tilde{P}_{ij} \). The common error can be written as

\[
\tilde{\eta}_i (k + 1) = \gamma_i (k + 1) \tilde{\eta}_i (k + 1) + [1 - \gamma_i (k + 1)] \left[ A \tilde{\eta}_i (k) - B(k) w(k) \right].
\]

The expression for \( \tilde{\eta}_i (k + 1) \) can be given as

\[
\tilde{\eta}_i (k + 1) = A(k) C_i \sum_{r \in \mathcal{N}_i} (\tilde{\eta}_r (k) - \tilde{\eta}_i (k)) - B(k) w(k) + A(k) F_i (k) u_i (k) + A(k) K_i (k) v_i (k).
\]

In view of the definition, the expression of \( \tilde{P}_{ij} \) can be obtained. It is anticipated that the forms of \( \tilde{P}_{ij} \) and \( \tilde{P}_{ij} \) can be obtained similarly.

Remark 2: Notice that (8) is a general expression for the error covariance matrix with event triggered transmission strategy. However, a poor choice of the event generator may lead to either lack of consensus on estimates or lack of stability of the error dynamics of the filter. Theorem 1 provides the optimal \( K_i \) under the proposed strategy.
However, it does not mean the filter structure is optimal for any event triggered scheme. The optimal event triggered filtering problem is in general a nonlinear filtering problem due to the condition that triggers the events, which results in intractable computation. For the matter of convenience, the optimal event triggered estimator is usually approximated in a simple linear form [17].

IV. STABILITY OF EVENT TRIGGERED CONSENSUS FILTERS

As pointed out in [15], the computation complexity of solving the error covariance update equations is not scalable in the number of sensor nodes. In order to get a scalable algorithm, an approximate suboptimal filter for sensor $i$ is given in the following form:

\[
\hat{x}_i (k) = \tilde{x}_i (k) + K_i (k) (z_i (k) - H_i (k) \tilde{x}_i (k)) + C_i (k) \sum_{j \in \mathcal{N}_i} [\tilde{x}_j (k) - \tilde{x}_i (k)],
\]

where

\[
K_i (k) = \bar{P}_i (k) H_i^T (k) (R_i (k) + H_i (k) \bar{P}_i (k) H_i^T (k))^{-1},
\]

\[
P_i (k) = F_i (k) \bar{P}_i (k) F_i^T (k) + K_i (k) R_i (k) K_i^T (k),
\]

\[
\bar{P}_i (k + 1) = A (k) \bar{P}_i (k) A^T (k) + B (k) Q (k) B^T (k),
\]

\[
\tilde{x}_j (k + 1) = \gamma_j (k + 1) \tilde{x}_j (k + 1)
\]

\[
+ (1 - \gamma_j (k + 1)) A (k) \tilde{x}_j (k),
\]

\[
\tilde{x}_i (k + 1) = A (k) \hat{x}_i (k).
\]

The main assumption for such an approximation is that the consensus gains $C_i$ can be set to zero in covariance update equations in (9). After the approximation, the implementation of the estimator is simplified, and computation resources are therefore saved.

Now the main result on stability of the consensus filter will be presented in the following theorem.

Theorem 2: Consider the approximated consensus filter with the choice of consensus gain

\[
C_i (k) = \frac{2 F_i (k) \Gamma_i^{-1} (k)}{\lambda_{\max} (\mathcal{L}) \lambda_{\max} (\Gamma_i^{-1} (k))},
\]

where $\mathcal{L}$ is the Laplacian matrix associated with the graph $\mathcal{G}$ and $\Gamma (k) = \text{diag} \{\Gamma_1 (k), \ldots, \Gamma_N (k)\}$ with $\Gamma_i (k) = F_i^T (k) A^T (k) \bar{P}_i^{-1} (k + 1) A (k) F_i (k)$. Suppose $A (k)$ and $R (k)$ are invertible for all $i \in \mathcal{V}$ and $k \geq 0$. Then, the noise free error dynamics of the consensus filter is globally asymptotically stable under the event condition

\[
[\tilde{x}_i (k) - \bar{x}_i (k)]^T \sum_{j \in \mathcal{N}_i} [\tilde{x}_i (k) - \tilde{x}_j (k)] \leq 0. \tag{11}
\]

Furthermore, all estimators asymptotically reach a consensus on state estimates.

Proof: Given the consensus estimator of sensor node $i$,

\[
\tilde{x}_i (k + 1) = A (k) [\tilde{x}_i (k) + K_i (k) (z_i (k) - H_i (k) \tilde{x}_i (k))]
\]

\[
+ A (k) C_i (k) \sum_{j \in \mathcal{N}_i} [\tilde{x}_j (k) - \tilde{x}_i (k)]
\]

the noise free error dynamics of the consensus filter can be written as

\[
\eta_i (k + 1) = A (k) F_i (k) \eta_i (k) + A (k) C_i (k) u_i (k). \tag{12}
\]

The stability result for (12) will be proved through the use of a Lyapunov function

\[
V (k) = \sum_{i=1}^{N} \tilde{\eta}_i^T (k) \tilde{P}_i^{-1} (k) \tilde{\eta}_i (k).
\]

Calculating the change $\Delta V (k) = V (k + 1) - V (k)$ in the Lyapunov function, it can be shown that

\[
\Delta V (k) = \sum_{i=1}^{N} \tilde{\eta}_i^T (k + 1) \tilde{P}_i^{-1} (k + 1) \tilde{\eta}_i (k + 1)
\]

\[
- \sum_{i=1}^{N} \tilde{\eta}_i^T (k) \tilde{P}_i^{-1} (k) \tilde{\eta}_i (k). \tag{13}
\]

Substituting (12) into (13), we have

\[
\Delta V (k) = \sum_{i=1}^{N} \tilde{\eta}_i^T (k + 1) (\Gamma_i (k) - \tilde{P}_i^{-1} (k)) \tilde{\eta}_i (k)
\]

\[
+ 2 \sum_{i=1}^{N} \tilde{\eta}_i^T (k) \Lambda_i (k) u_i (k) + \sum_{i=1}^{N} u_i^T (k) \Pi_i (k) u_i (k),
\]

where

\[
\Lambda_i (k) = F_i^T (k) A^T (k) \tilde{P}_i^{-1} (k + 1) A (k) C_i (k),
\]

\[
\Pi_i (k) = C_i^T (k) A^T (k) \tilde{P}_i^{-1} (k + 1) A (k) C_i (k).
\]

Using the error covariance matrix update rules, we get

\[
\bar{P}_i (k + 1) = A (k) F_i (k) \bar{P}_i (k) F_i^T (k) A^T (k) + W_i (k)
\]

where

\[
W_i (k) = A (k) K_i (k) R_i (k) K_i^T (k) A^T (k)
\]

\[
+ B (k) Q (k) B^T (k).
\]

Multiplying $\bar{P}_i (k)$ on both sides of $\tilde{P}_i^{-1} (k) - \Gamma_i (k)$ and using the matrix inversion lemma gives

\[
\bar{P}_i (k) [\tilde{P}_i^{-1} (k) - \Gamma_i (k)] \bar{P}_i (k) = [\tilde{P}_i^{-1} (k) + M_i (k)]^{-1},
\]

where

\[
M_i (k) = F_i^T (k) A^T (k) W_i^{-1} (k) A (k) F_i (k).
\]

Now multiplying the above equation from left and right by $\tilde{P}_i^{-1} (k)$ results in

\[
\tilde{P}_i^{-1} (k) - \Gamma_i (k) = \tilde{P}_i^{-1} (k) [\tilde{P}_i^{-1} (k) + M_i (k)]^{-1} \tilde{P}_i^{-1} (k),
\]

which is a symmetric and positive definite matrix. The first term in (14) is clearly negative semi-definite. Let $C_i (k)$ satisfy $\Lambda_i (k) = \sigma (k) I > 0$ for all $i \in \mathcal{V}$. This can be achieved by setting the consensus gain to

\[
C_i (k) = \sigma (k) [F_i^T (k) A^T (k) \tilde{P}_i^{-1} (k + 1) A (k)]^{-1}.
\]

Using this choice of $C_i (k)$, the second term in (14) becomes

\[
2 \sum_{i=1}^{N} \tilde{\eta}_i^T (k) \Lambda_i (k) u_i (k) = 2 \sigma (k) \tilde{\eta}^T (k) u (k).
\]

Defining

\[
\Psi = \text{diag} \{\tilde{P}_1^{-1} (k) - \Gamma_1 (k), \ldots, \tilde{P}_N^{-1} (k) - \Gamma_N (k)\}
\]

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and noting that \( \Pi_i(k) = \sigma^2(k) \Gamma_i^{-1}(k) \), \( \Delta V(k) \) can be rewritten as
\[
\Delta V(k) = -\tilde{\eta}^T(k) \Psi \tilde{\eta}(k) + 2\sigma(k) \tilde{\eta}^T(k) u(k) + \sigma^2(k) \sum_{i=1}^{N} u_i^T(k) \Gamma_i^{-1}(k) u_i(k).
\]
Noting that \( u(k) = -(\mathcal{L} \otimes I) \tilde{\eta}(k) \), we have
\[
\Delta V(k) \leq -\tilde{\eta}^T(k) \Psi \tilde{\eta}(k) + 2\sigma(k) \tilde{\eta}^T(k) u(k) - \sigma^2(k) \lambda_{\text{max}}(\Gamma_i^{-1}(k)) \lambda_{\text{max}}(\mathcal{L}) \tilde{\eta}^T(k) u(k).
\]
Choosing
\[
\sigma(k) = \frac{2}{\lambda_{\text{max}}(\Gamma_i^{-1}(k)) \lambda_{\text{max}}(\mathcal{L})},
\]
\( \Delta V(k) \) becomes
\[
\Delta V(k) \leq -\tilde{\eta}^T(k) \Psi \tilde{\eta}(k) + 2\sigma(k) \tilde{\eta}^T(k) u(k).
\]
Choose the event condition for sensor \( i \) as,
\[
[\tilde{x}_i(k) - \hat{x}_i(k)]^T \sum_{j \in N_i} [\tilde{x}_i(k) - \hat{x}_j(k)] \leq 0
\]
so that the last term of \( \Delta V(k) \) is negative semi-definite. One can conclude that \( \Delta V(k) < 0 \) for all \( \tilde{\eta}_i(k) \neq 0, i \in \mathcal{V} \). Therefore, \( \tilde{\eta} = 0 \) is asymptotically stable for the error dynamics of the consensus filter without noise, which follows that \( \hat{x}_i(k+1) \) for all \( i \in \mathcal{V} \) is converging to \( x(k+1) \) for \( k \to \infty \). From the fact that
\[
\hat{x}_i(k+1) = A(k)\hat{x}_i(k), \quad x(k+1) = A(k)x(k),
\]
the implication that \( \hat{x}(k) \) is converging to \( x(k) \) as \( k \to \infty \) holds. Furthermore, since \( \tilde{\eta}_i = \hat{\eta}_i = 0 \) for all \( j \neq i \), all estimators asymptotically reach a consensus on state estimates, i.e., \( \hat{x}_1 = \cdots = \hat{x}_N \).

Remark 3: To calculate the consensus gain for each sensor, the largest eigenvalue of \( \Gamma^{-1}(k) \) has to be known to all agents at every time instant, which might be restrictive in a distributed environment. However, this could be amended by bounding it above by a constant. The particular choice of \( C_i \) in Theorem 2 is chosen to derive an event condition for each sensor. In order to reduce the need of a communication, each sensor node predicts the behavior of itself and its neighbors one step ahead. An event is generated by a sensor if the condition in (11) is violated; at the same time, the event condition is satisfied again by broadcasting its information to the neighbors.

Remark 4: According to the proof of Theorem 2, the result is also valid for linear time-varying deterministic systems. To the best of the authors’ knowledge, either control or estimation problems with event triggered sampling for linear time-varying systems have not been investigated despite of their importance. The algorithm above can be viewed as an event triggered estimation strategy for linear time-varying systems. The time-varying consensus filter is also a generalization of the steady-state filter for linear time-invariant systems with non-stationary noise covariance.

V. SIMULATION RESULTS
Suppose the discrete time model is
\[
x(k+1) = Ax(k) + Bu(k)
\]
with
\[
A = \begin{bmatrix} 0.9996 & -0.03 \\ 0.03 & 0.9996 \end{bmatrix}, \quad B = \begin{bmatrix} 0.375 & 0 \\ 0 & 0.375 \end{bmatrix},
\]
and \( Q = I \). Note that the state is moving on noisy divergent circular trajectories due to a pair of complex-conjugate eigenvalues with magnitude greater than 1. The initial state vector and the covariance matrix are set to be \( x_0 = [15 - 10]^T \), \( P_0 = 10I \), respectively.

![Fig. 1. The sensor network topology](image-url)
beginning at $k = 0$ and ending at $k = 200$. The result is shown in Fig. 3, where the estimate trajectory is obtained by taking the average of the estimates of all sensor nodes. From the figure, it can be seen the event triggered consensus filtering algorithm can effectively track the target even though the system model is unstable and the measurement noise covariance matrices $R_i$ for all $i \in \mathcal{V}$ are relatively large. The total number of broadcasting for the entire sensor network is 3034, which is 75.85% of the complete transmission scheme. The number of events for each sensor node under the event condition defined in (11) is shown in Fig. 4.

VI. CONCLUSIONS

Distributed state estimation algorithms were established for linear time-varying discrete-time systems with event triggered transmission using wireless sensor networks. Initially, the optimal event triggered consensus filter was constructed, and then it was approximated by a suboptimal filter. An event condition was presented for the suboptimal filter to reduce the number of transmissions for the wireless sensor network. The effectiveness of the proposed method was illustrated through a simulation example. Future work will address triggering rules for the optimal consensus filter to guarantee stability. The focus here is the interaction reduction among estimator sensors. Employing event detection rules between measurement sensor and estimator sensors would be an interesting extension.

REFERENCES