Negative Observations for Multiple Hypothesis Tracking of Dynamic Extended Objects

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Abstract—A novel approach to utilizing negative information to improve multiple hypothesis tracking (MHT) of extended objects is presented. Specifically, a missed detection of a tracked object is treated as an observation of object occlusion, the likelihood of which is described by an occlusion-based sensor detection model. This negative observation is used to update hypothesis weights in a method that is supplementary to any existing MHT framework, as it continues to inform the filter in the absence of traditional measurements. Experimental results are presented demonstrating that integrating this additional interpretation of the available information improves tracking performance.

I. INTRODUCTION

Scenes which include multiple extended objects pose serious challenges for tracking filters by way of occlusion phenomena. These challenges stem from the fact that fully occluded objects cannot be directly observed, while partially occluded objects generate partial observations that may be difficult to interpret or associate; complications intensify with highly dynamic scenes where object states are changing rapidly.

Occlusion is often ignored in tracking applications because of its challenging models and reasoning, leading to incomplete beliefs of the local scene $[1]–[3]$. Unresolved objects are quickly removed from the filter based on non-detection heuristics, and new tracks formed if/when objects re-emerge. This incomplete belief of the local scene hinders scene anticipation for proactive planning tasks, a vital component of safe driving $[4]$. Further, no effort is made to form correspondences between newly formed tracks and recently deleted tracks, thwarting any effort to reason about a particular track of interest over time.

Other work acknowledges occlusion, but does not exploit it beyond delaying occluded track deletion $[5]–[8]$. Ref. $[5]$ extends $[1]$ by introducing deleted as a potential track label, with some heuristic probability of assignment. In this way, occlusion is implied for object tracks not labeled as detected or deleted. Ref. $[6]$ further extends this concept to explicitly include an occluded track label with a corresponding heuristic probability of assignment. Both methods are naive in that they do not consider the geometry of the scene in their occlusion labeling, which is one of the most informative indicators of occlusion following a lack of detection. Refs. $[7]$ and $[8]$ consider scene geometry in their occlusion reasoning for undetected tracks, but do not utilize this information beyond track preservation. For highly dynamic, maneuvering objects, state uncertainties can become large under pure dynamics prediction, quickly leading to nonsensical beliefs with large portions of probability mass in visible regions of space. Highly uncertainty scene estimates complicate perception-dependent tasks such as data association and path planning. Robust occlusion reasoning enables more accurate and precise state estimates, improving the performance of downstream processes.

Our prior work leverages negative information, defined as inferences drawn from expected, but missing, observations, to develop a fully probabilistic occlusion likelihood model. This occlusion model is utilized to inform track existence, data association, and update the probability density of occluded object state estimates $[9]$. This work serves as a supplement to any multiple hypothesis tracking algorithm (MHT) for multiple extended objects by interpreting a missed detection of a tracked object as an observation of its occlusion, the likelihood of which can be computed with the occlusion model developed in $[9]$. In this way, hypothesis fitness can be evaluated during occlusion events, when sensor measurements are unavailable.

The remainder of this paper is structured as follows: Section II summarizes the occlusion likelihood model developed in $[9]$. Section III introduces the hypothesis fitness evaluation based on the occlusion observation, the key contribution of this work. Section V provides the details of the experiment demonstrating the benefits of the proposed occlusion-based MHT method. Lastly, section VI discusses the results of the experiment outlined in Section V.

II. BACKGROUND

An elegant approach for multi-object tracking from a moving sensory vehicle was developed in $[10]$ and $[7]$ for Cornell University’s submission to the DARPA Urban Challenge (DUC); the approach, which formally uses a Rao-Blackwellized particle filter to solve the joint data association and tracking problem, was extended to consider negative information in $[9]$. Specifically, the filter maintains the parameterized probability density of the object state, typical of most tracking filters, along with the most recent raw lidar sensor measurement assigned to each object, as seen from the object’s local coordinate frame. The laser scan measurement at each time step captures the approximate shape of the visible surface of the object, and is used to compute an expected lidar cluster for subsequent scans. The objects are assumed to be rigid bodies, thus maintaining this cluster does not increase the dimension of the state.
vector; simply estimating the evolution of the object-centric coordinate origin is enough to reproduce the entire cluster in sensor coordinates at future times.

A set of stable metadata, \( Y^i \sim \mathcal{N} (\bar{Y}, \Sigma^2_{y^i}) \) is extracted from each cluster and used to form filter innovations. Specifically,

\[
Y^i \sim \mathcal{N} \left( \begin{bmatrix} \bar{\theta}^i \\ \bar{\phi}^i \\ \bar{\rho}^i \end{bmatrix}, \begin{bmatrix} \sigma^2_{\theta^i} & \sigma^2_{\theta^i,\phi^i} & \sigma^2_{\theta^i,\rho^i} \\ \sigma^2_{\theta^i,\phi^i} & \sigma^2_{\phi^i} & \sigma^2_{\phi^i,\rho^i} \\ \sigma^2_{\theta^i,\rho^i} & \sigma^2_{\phi^i,\rho^i} & \sigma^2_{\rho^i} \end{bmatrix} \right) 
\]

where:

\[
\begin{align*}
\theta^i & \sim \mathcal{N} (\bar{\theta}^i, \sigma^2_{\theta^i}) & \text{cw most bearing to object/cluster } i \\
\phi^i & \sim \mathcal{N} (\bar{\phi}^i, \sigma^2_{\phi^i}) & \text{ccw most bearing to object/cluster } i \\
\rho^i & \sim \mathcal{N} (\bar{\rho}^i, \sigma^2_{\rho^i}) & \text{minimum range to object/cluster } i 
\end{align*}
\]

where clockwise (cw) and counterclockwise (ccw) designations are with respect to the mean bearing to the object, to avoid angle-wrap issues at the \( \pm \pi \) discontinuity.

The utility of this metadata is extended in [9] to serve as arguments to an occlusion likelihood function. Specifically, the likelihood that tracked object \( n \) is occluded is modeled as:

\[
L_{\text{occ}}^n = \sum_{i=1}^{N-1} \left\{ (-1)^{(i+1)} \sum_{j=1}^{(N-1)C_i} \prod_{k=1}^{i} L_{\text{occ}}^{n,C_j(k)} \right\} 
\]

where \((N-1)C_i\) denotes the number of \(i\)-combinations of the set of \(N-1\) tracked objects (excluding object \(n\)):

\[
(N-1)C_i = \frac{(N-1)!}{i!(N-1-i)!} 
\]

\(C_j^i(k)\) denotes the \(k\)th object in the \(j\)th \(i\)-combination, and \(L_{\text{occ}}^{n,m}\) is the likelihood that tracked object \(n\) is occluded by a particular single tracked object, \(m\), defined as:

\[
L_{\text{occ}}^{n,m} = \left( L_{B_1}^{n,m} + L_{B_2}^{n,m} + L_{B_3}^{n,m} - L_{B_2 \cap B_3}^{n,m} \right) \cdot I_R^{n,m} 
\]

where each term in parentheses corresponds to a specific occlusion geometry, \(j\), described by the following unique bearing conditions, \(L_{B_j}^{n,m}\), and a common range condition, \(I_R^{n,m}\):

\[
L_{B_1}^{n,m} = \Phi \left( \frac{\theta^n - \theta^m}{\sqrt{\sigma_{\theta n}^2 + \sigma_{\theta m}^2}} \right) \Phi \left( \frac{\phi^n - \phi^m}{\sqrt{\sigma_{\phi n}^2 + \sigma_{\phi m}^2}} \right) 
\]

\[
L_{B_2}^{n,m} = \Phi \left( \frac{\phi^n - \phi^m}{\sqrt{\sigma_{\phi n}^2 + \sigma_{\phi m}^2}} \right) \Phi \left( \frac{\rho^n - \rho^m}{\sqrt{\sigma_{\rho n}^2 + \sigma_{\rho m}^2}} \right) 
\]

\[
L_{B_3}^{n,m} = \Phi \left( \frac{\theta^n - \theta^m}{\sqrt{\sigma_{\theta n}^2 + \sigma_{\theta m}^2}} \right) \Phi \left( \frac{\rho^n - \rho^m}{\sqrt{\sigma_{\rho n}^2 + \sigma_{\rho m}^2}} \right) 
\]

\[
L_{B_2 \cap B_3}^{n,m} = \Phi \left( \frac{\phi^n - \phi^m}{\sqrt{\sigma_{\phi n}^2 + \sigma_{\phi m}^2}} \right) \Phi \left( \frac{\rho^n - \rho^m}{\sqrt{\sigma_{\rho n}^2 + \sigma_{\rho m}^2}} \right) 
\]

where \( \Phi(\cdot) \) is the standard normal cumulative distribution function.

Fig. 1 demonstrates the occlusion likelihood function from the point of view of the indicated sensor for an arbitrarily complex scene including rectangular objects; the top pane shows the contours that result from computing the likelihood over a 2-dimensional (2D) grid, while the bottom pane displays 1-dimensional (1D) slices through the likelihood function.

The occlusion model in (2) is leveraged by [9] in the following three ways:

- To fuse negative measurements: the probability density of unresolved object states is updated, maintaining a sensible belief of the local scene.
- To inform track existence: confidence in the existence of unresolved tracks decays with time constant a function of its occlusion likelihood.
- To inform data association: correspondences resulting in likely occlusion of resolved measurements are discounted.

The work in this paper extends the utility of the occlusion likelihood function in (2) to a parallel task of evaluating Multiple Hypothesis Tracking (MHT) hypotheses during occlusion events. Specifically, for unresolved tracks, hypotheses that predict a history of likely occlusion are favored over those predicting detectable object states.

III. MULTIPLE HYPOTHESIS

General object tracking problems require a filter to track different types of maneuvering objects without deterministic knowledge of their controls, intent, or type. If the set of
objects to be tracked have similar enough dynamic character, the classical approach is to choose a single steady-state motion model with enough process noise to account for maneuvers, and then rely on frequent and precise sensor measurements to maintain consistent and unbiased state estimates throughout the set of conceivable maneuvers.

In the case of occluded objects, traditional sensor measurements are not available. The negative measurements introduced in [9] help to maintain a sensible belief of the occluded object state; however, they are generally less precise than traditional positive measurements, and thus object maneuvers not explicitly considered in the dynamics model can induce biases in the state estimate. In these cases, a traditional multiple hypothesis framework is appropriate [11]–[15], where the hypotheses are evaluated based on the negative information in the absence of traditional positive measurements.

To derive the hypothesis evaluation, consider the discrete probability of an estimate of the state history of object \( n \) over the time window \([1, k]\), \( \hat{x}^n_{1:k} \), given a series of independent observations of object \( n \), \( o^n_{1:k} \):

\[
p(\hat{x}^n_{1:k}|o^n_{1:k}) = p(\hat{x}^n_k, \hat{x}^n_{1:k-1}|o^n_k, o^n_{1:k-1})
\]

\[
= p(\hat{x}^n_k, \hat{x}^n_{1:k-1}, o^n_k, o^n_{1:k-1}) \cdot p(o^n_k, o^n_{1:k-1})
\]

(5)

where the subscript \( k \) denotes a single time step, and subscripts of the form \( k_1 : k_2 \) denote a trajectory over the time window \([k_1, k_2]\). The joint probabilities in the numerator and denominator of (5) can be factored as:

\[
p(\hat{x}^n_{1:k}|o^n_{1:k}) = p(\hat{x}^n_k | \hat{x}^n_{1:k-1}, o^n_{1:k-1}) \cdot p(o^n_k | \hat{x}^n_{1:k-1}, o^n_{1:k-1})
\]

\[
\cdot p(\hat{x}^n_{1:k-1}|o^n_{1:k-1}) \cdot p(o^n_{1:k-1})
\]

(6)

and:

\[
p(o^n_{1:k-1}) = p(\hat{x}^n_{1:k-1}|o^n_{1:k-1}) \cdot p(o^n_{1:k-1})
\]

(7)

Notice that the marginal probability of the history of observations up through \( k - 1 \), \( p(o^n_{1:k-1}) \), in the numerator cancels with that in the denominator. Further, due to the underlying filter assumptions that the system is Markov and observations are independent over time, the following independence assertions can be made:

\[
o^n_k \perp o^n_{1:k-1}
\]

\[
o^n_{1:k-1} \perp \hat{x}^n_{1:k-1} | \hat{x}^n_k
\]

\[
\hat{x}^n_k \perp \{o^n_{1:k-1}, \hat{x}^n_{1:k-1}\} | \hat{x}^n_{k-1}
\]

(8)

which give rise to the following equality expressions:

\[
p(o^n_k | o^n_{1:k-1}) = p(o^n_k)
\]

\[
p(\hat{x}^n_k | \hat{x}^n_{1:k-1}, o^n_{1:k-1}) = p(\hat{x}^n_k | \hat{x}^n_{1:k-1})
\]

\[
p(\hat{x}^n_{1:k-1} | o^n_{1:k-1}) = p(\hat{x}^n_{1:k-1})
\]

(9)

Subbing (9) into (5) via the factored terms in (6) and (7) reveals that the discrete probability of a particular estimate of the state history of object \( n \) can be updated recursively when provided a new observation, \( o^n_k \):

\[
p(\hat{x}^n_{1:k}|o^n_{1:k}) = c \cdot p(o^n_k | \hat{x}^n_{1:k}) \cdot p(\hat{x}^n_{1:k-1}) \cdot p(o^n_{1:k-1} | \hat{x}^n_{1:k-1})
\]

\[
\cdot p(\hat{x}^n_{1:k-1} | o^n_{1:k-1})
\]

(10)

where \( c = p(o^n_k)^{-1} \) is a normalizing constant independent of the state estimate. Note that (10) is the canonical form for Bayesian inference; the posterior is computed as the product of the prior and the likelihood of the observation, normalized by the marginal probability of the observation:

\[
\text{Posterior} : \quad p(\hat{x}^n_{1:k}|o^n_{1:k})
\]

Prior : \( p(\hat{x}^n_{1:k-1}|o^n_{1:k-1}) \)

Likelihood : \( p(o^n_k | \hat{x}^n_{1:k-1}) \)

(11)

In this case, the prior is a product of the posteriors at the previous time step, \( p(\hat{x}^n_{1:k-1}|o^n_{1:k-1}) \), and a transition model predicting it to the current time step, \( p(\hat{x}^n_{1:k} | \hat{x}^n_{1:k-1}) \).

When multiple hypotheses are used to explicitly consider a collection of object maneuvers, each hypothesis, \( i \), is characterized by a unique transition model at time \( k \), \( p(\hat{x}^n_k | \hat{x}^n_{1:k-1}, i) \). \( i \) is the occlusion likelihood of hypothesis \( \hat{x}^n_k \) assuming it originated from hypothesis \( \hat{x}^n_{1:k-1} \) \( \forall i \in [1, N^n_{\text{hyp}}] \), leading to \( N^n_{\text{hyp}} \) unique hypotheses of the object state trajectory, \( \hat{x}^n_{1:k} \). Each hypothesis is conditioned on an assumed series of maneuvers, the validity of which can be evaluated against the observations via (10):

\[
p(\hat{x}^n_{1:k}|o^n_{1:k}) = c \cdot p(o^n_k | \hat{x}^n_{1:k}) \cdot p(\hat{x}^n_{1:k-1}) \cdot p(o^n_{1:k-1} | \hat{x}^n_{1:k-1})
\]

\[
\cdot p(\hat{x}^n_{1:k-1} | o^n_{1:k-1})
\]

(12)

Traditionally, the observation of object \( n \) comes in the form of positive information; a measurement, \( z^n \), the likelihood of which can be evaluated with the filters’ sensor model. Conversely, in the case of occluded objects, the observation is in the form of negative information; the non-detection of object \( n \), the likelihood of which can be evaluated via the occlusion model defined in (2). Therefore, the recursive weight update for the \( i \)-th hypothesis of the state of object \( n \), \( w^n_{k,i} \), can be written directly from (12) as:

\[
w^n_{k,i} = c \cdot p(o^n_k | \hat{x}^n_{1:k-1}) \cdot p(\hat{x}^n_{1:k-1} | w^n_{k-1})
\]

(13)

where:

\[
p(o^n_k | \hat{x}^n_{1:k-1}) = \left\{ \begin{array}{ll}
p(z^n_k | \hat{x}^n_{1:k-1}) & \text{if } R^n_k \\
L^n_{\text{occ}} & \text{if } \neg R^n_k
\end{array} \right.
\]

(14)

where \( R^n_k \) is the event that object \( n \) is resolved in the sensor data at time \( k \), \( p(z^n_k | \hat{x}^n_{1:k-1}) \) is the likelihood of measurement \( z^n_k \) assuming it originated from hypothesis \( i \)-’s belief of object \( n \), \( L^n_{\text{occ}} \) is the occlusion likelihood of hypothesis \( i \)-’s belief of object \( n \), defined in (2), and the normalizing constant, \( c \), can be found by marginalizing over the set of hypotheses:

\[
c = \left( \sum_{i=1}^{N^n_{\text{hyp}}} p(o^n_k | \hat{x}^n_{1:k-1}) \cdot p(\hat{x}^n_{1:k-1} | w^n_{k-1}) \right)^{-1}
\]

(15)

In this way, hypotheses that best predict the sensor information at each time step carry the heaviest weights, even when the object cannot be directly observed.
Note that in computing $L_{occ}^{n_k}$ with (2), other hypotheses of object $n$ must be excluded, as an object cannot be occluded by a different hypothesis of itself (only one of the hypotheses represents the true physical state of the object); in (4), $r_{occ}^{m_j} = 0 \forall j$ when $m = n$. Similarly, the contributions of hypotheses of other objects, $x_{k}^{m_j}$, must be scaled by their hypothesis weight, $w_k^{m_j}$, such that the likelihood that hypothesis $i$ of object $n$ is occluded by object $m$ is the weighted sum of the likelihoods that it is occluded by each of the mutually exclusive hypotheses of object $m$.

IV. PRACTICAL CONSIDERATIONS

A. Hypothesis Selection

The utility of this work does not depend on the specific choice of hypotheses, although filter performance, in general, relies on this set being sufficiently exhaustive. That is, if the true maneuver of a tracked object is not well-modeled, the filter is not likely to perform well. This artifact is not unique to multiple hypothesis filters; single hypothesis implementations are simply a special case where the solitary hypothesis is sufficiently exhaustive.

Unfortunately, an exhaustive set of hypotheses may be infeasible conceptually, computationally, or both. Therefore, the following two hypothesis types are proposed:

1) Compliant hypotheses - hypotheses of expected behaviors, usually characterized by constrained dynamics and relatively small uncertainty.

2) Anomalous hypotheses - general hypotheses to capture a host of unexpected, but feasible, behaviors, usually characterized by nondescript dynamics models with relatively high uncertainty.

B. Computation Savings

As discussed at the beginning of section III, the need for multiple hypotheses may diminish for visible objects, as they frequently receive precise sensor measurements. Therefore, one could employ a variable number of hypotheses, such that when the filter must rely on negative information, a larger set of more detailed hypotheses (e.g. Compliant hypotheses) is utilized to accommodate the relatively imprecise observations. Conversely, when positive information is available, the improved precision of the observations enables a smaller set of more general hypotheses (e.g. Anomalous hypotheses). Initialization of the new set of hypotheses is based on a transformation of the most likely subset of old hypotheses.

The details of such an implementation are highly application dependent and are not unique to tracking objects unresolved in sensor data. Therefore, they are beyond the scope of this paper. An in depth discussion of adaptive computational resource management for multiple hypothesis tracking is provided in [15].

V. EXPERIMENT DETAILS

The negative observation MHT proposed in this paper is demonstrated for an autonomous driving scenario using experimental data logged from three Ibeo ALASCA XT range finders on Cornell University’s autonomous vehicle platform. The scenario involves the autonomous ego vehicle following a lead vehicle which becomes occluded by an occluding vehicle.

The following two tracking filters are run over the same set of logged data for comparison:

- $F_1$: Baseline Filter [9] + multiple hypothesis
- $F_2$: Baseline + multiple hypothesis with negative observation hypothesis evaluation

The MHT implementation chosen here is identical for filters $F_1$ and $F_2$, except for the use of negative observations for the occluded object in $F_2$. Because, the negative observation work presented here is agnostic to the specific MHT implementation or choice of hypotheses, a detailed presentation is beyond the scope of this paper. A high level description of the hypothesis implementation is provided below.

Two hypotheses are chosen according to the suggestion in IV-A. Specifically,

- Compliant hypotheses (C) - constant velocity constrained to the road network to reflect expected driving behavior. This is a single hypothesis on straight road segments far from intersections, and multiple hypotheses near intersections to account for all possible traversals.
- Anomalous hypothesis (A) - unconstrained hypothesis with constant velocity, constant heading dynamics, each with relatively large uncertainty. This is a single hypothesis accounting for deviations from expected driving.

Interaction, as described in [11], is administered between the anomalous hypothesis and each of the individual compliant hypotheses, but not directly among the compliant hypotheses. This is to reflect the expectation that a given driver may freely transition between compliant and anomalous driving behavior over time, but cannot follow more than one path through an intersection without exhibiting an intermediate period of anomalous behavior. Lastly, hypotheses are formed only for occluded tracks at the onset of their occlusion, as a single hypothesis is sufficient for visible objects frequently returning precise measurements, as discussed in section IV-B.

VI. EXPERIMENTAL RESULTS

Fig. 2 shows snapshots of the tracking results for both filters, $F_1$ (left column of axes) and $F_2$ (right column of axes). The first frame, $t = 2s$, shows the last view of the lead vehicle prior to occlusion as a view of the rear bumper, and is identical for both filters as positive observations (measurements) have been readily available heretofore.

The second frame, $t = 2.7s$, occurs shortly after the onset of occlusion and shows that filter $F_2$ has formed the two hypotheses of the occluded object character described in Section V. The anomalous driver hypothesis, A, has been updated by the negative measurements to have approximately half of its mass outside of the occupied road lane. This reflects the chosen design of the anomalous hypothesis being free to conform to the most likely regions of the

645
occlusion shadow, which projects off of the road at the onset of occlusion. Conversely, the compliant hypothesis (C) is constrained to the road network.

The third time frame, \( t = 30s \), shows that the anomalous driver hypothesis in \( F_2 \) follows the compliant hypothesis, C, more closely than in \( F_1 \). This is indicative of the negative observations giving rise to a dominant combined compliant hypothesis weight of 0.98, resulting in the anomalous hypothesis belief at each time step being dictated predominately by mass transitioned from the compliant hypothesis. This reflects \( F_2 \)'s negligible belief that there was prior anomalous driving behavior (2% probability), molded by the history of negative observations.

The fourth time frame, \( t = 42.7s \), shows the filters belief just prior to the lead vehicle re-emerging from the occlusion shadow. This frame shows an exacerbation of the effect described for the previous frame, where in \( F_2 \), mass transitioned from the dominant compliant hypothesis, C, dominates the anomalous driver hypothesis, A, causing the anomalous driver hypothesis to pool in the intersection. The anomalous driver hypothesis being dominated by recent transitions to anomalous behavior corresponds to a filter belief that past anomalous behavior is unlikely; this is a direct result of the negative observations. Conversely, the inability of the baseline filter to evaluate the likelihood of the hypotheses results in the \( F_1 \) anomalous driver hypothesis to propagate away from the road network at near constant velocity, representing a belief dominated by a prolonged history of anomalous behavior.

The fifth and final time frame, \( t = 42.8s \), shows that \( F_1 \)'s inability to identify the correct hypothesis leads to an imprecise belief of the object state, and ultimately a missed detection. Filter \( F_1 \) must then re-initialize a new object track for the lead vehicle after it emerges from the occlusion shadow. Filter \( F_2 \), on the other hand, has correctly identified the right-turn compliant hypothesis as the best representation of the true object behavior, leading to a precise object belief at the occlusion boundary in the right-turn lane, and enabling \( F_2 \) to reassociate the object as it emerges from the occlusion shadow.

Fig. 3 shows the time history of the hypothesis weights of the \( F_2 \) filter during the occlusion event. At the onset of partial occlusion, the anomalous driver hypothesis, A, immediately, albeit temporarily, emerges as the most likely hypothesis. As discussed above, this is due to the fact that the occluding vehicle has not yet completed its turn into the outgoing traffic lane to which the compliant hypothesis, C, is constrained; therefore, the peak occlusion likelihood projection is not aligned with the road lane. The anomalous driver hypothesis, by design, is completely free to conform to the negative information by way of the negative measurements introduced in [9], giving it an apparent advantage from the start. As the occluding vehicle completes its turn, the compliant hypothesis weight quickly recovers and dominates for the remainder of the run, as it consistently predicts states that support \( F_2 \)'s observations of the lead vehicle being unresolved.

Toward the end of the occlusion event, the compliant hypotheses begin to predict possible emergence from the occlusion shadow, at odds with the negative information. The degree of this conflict is greater for the left-turn hypothesis, enabling \( F_2 \) to correctly favor the right-turn hypothesis up to 5 seconds prior to the lead vehicle emerging from occlusion (\( t = 42.8s \)), confirming \( F_2 \)'s belief.

VII. CONCLUSION

This paper discusses a novel method to consider missing sensor measurements for tracked objects as observations of those objects in the evaluation of behavior hypotheses. Experimental results demonstrate that this approach enables the filter to evaluate hypothesis fitness in the absence of traditional positive measurements, leading to more precise and accurate beliefs about object states and behaviors that are unresolved in sensor data.

While this paper focused on occlusion as the sole factor affecting object detection, these ideas can be extended to include other sensor specific or environmental factors that influence the detectability of objects in the environment (e.g. sensor range, field of view, doppler blindness, etc.).

REFERENCES

Fig. 2. Snapshots of the tracking results for filter $F_1$ (left), and $F_2$ (right). The red rectangle denotes the ego vehicle, the blue point cloud denotes the lidar return from the occluding vehicle, and the green ellipses denote the $1$-$\sigma$ bound of the occluded object belief within each hypothesis, of which the solid lines denote the compliant hypotheses, and dashed lines denote the anomalous hypothesis. Each of the peripheral axes is plotted with respect to a coordinate frame fixed to the ego vehicle, while the central axes shows the global position of the ego-vehicle at each of the snapshots. In each frame the hypothesis weights are reported, with subscripts denoting hypothesis label.

Fig. 3. Top: Time history of the $F_1$ (black dashed), and $F_2$ (green, blue) hypothesis weights during the occlusion event. Bottom: Time history of the negative observation likelihood for each of the $F_2$ hypotheses. Vertical black dashed lines denote the snapshot times from Fig. 2.
