Distributed Model Predictive Control for MLD systems: application to freeway ramp metering

Antonella Ferrara¹, Simona Sacone² and Silvia Siri²

Abstract—This paper deals with mixed logical dynamical systems controlled via model predictive schemes. For this class of systems a centralized MPC approach is firstly introduced, in which the finite horizon optimal control problem is a mixed-integer quadratic programming problem aiming at minimizing the deviations of the system variables from their equilibrium points. Since the application in real time of these MPC schemes is sometimes limited because of the high computational load necessary to solve the finite horizon optimal control problem, a distributed MPC scheme is proposed, characterized by two different and alternative algorithms. An important application is then introduced to assess the proposed approaches, i.e. ramp metering freeway traffic control. Referring to this applicative case, the two distributed control algorithms are compared, via simulation, with the centralized MPC scheme and with a completely decentralized one.

I. INTRODUCTION

There are many real applications in which the systems to be controlled are characterized by logic rules, discrete states, on/off inputs, piecewise linear functions. For such systems the formalism of mixed logical dynamical (MLD) systems has been proposed in [1]. An MLD system is normally characterized by linear equalities and inequalities involving both continuous and binary variables. Mixed logical dynamical systems have been proved to be equivalent to other classes of hybrid systems, i.e. linear complementarity systems, extended linear complementarity systems, piecewise affine systems, and max-min-plus-scaling systems, as shown in [2]. A computational framework for modeling this class of systems in discrete time has been proposed and discussed in [3].

When controlling MLD systems via Model Predictive Control (MPC) approaches, with the objective of minimizing the deviations of the system variables from given trajectories, the resulting Finite Horizon Optimal Control Problems (FHOCP) are solved through mixed integer quadratic programming. As it is well known, except specific cases, mixed-integer programming problems are NP-complete, and this means that in the worst case the solution times grow exponentially with the problem size [4]. As a consequence, the possibility of applying MPC schemes in real time can be in some cases limited by the computational load of the FHOCP to be solved at each time step. In the literature, several solutions have been proposed in order to overcome this drawback. Some of them are aimed to improve the efficiency of the optimization algorithm (see, for instance, [5] and the references therein), while others are focused on the reduction of the overall computational burden, as, for instance, [6].

Another possibility to devise MPC schemes for large scale applications is to break the control problem into small sub-problems to be solved locally, according to the principle of distributed control [7]. Distributed control schemes are based on a set of local controllers designed to operate on disjoint non overlapping pairs of inputs and outputs. The local controllers can exchange information, or be totally independent, giving rise to a decentralized control scheme. A wide number of distributed MPC algorithms for conventional dynamic systems have been proposed in the literature (see [8] for an overview).

In the present paper, two different distributed control algorithms for MLD systems and a completely decentralized control scheme for the same class of systems will be presented. The proposed distributed MPC approaches will be then assessed in the paper considering a relevant application of MLD systems, i.e. traffic control in freeway networks. In the present work, the adopted prediction model is the so called Cell Transmission Model (CTM) [9], [10], properly stated according to the MLD formalism. In this particular type of application, the dimensions of the FHOCP can become very large and, as mentioned before, the computational time necessary to solve the FHOCP (having a mixed-integer quadratic form) can be not adequate for real-time usage. Then, the design of distributed MPC schemes seems quite interesting and suitable in this area, as also proven by some works in the literature. For instance in [11] and [12] distributed approaches for freeway traffic control have been proposed in order to face the difficulty related to the real-time adoption of a centralized MPC approach for a relatively large network.

The paper is structured as follows. In Section II MLD models are described and the centralized MPC approach for controlling this class of systems is introduced. Section III presents the proposed distributed control algorithms and a completely decentralized control scheme. The application of the proposed approach to a freeway traffic system is discussed and presented in Section IV, while conclusive remarks are reported in Section V.
II. THE CENTRALIZED CONTROL SCHEME

Let us consider nonlinear time-invariant discrete-time systems of the form
\[ x(k + 1) = f(x(k), u(k)) \]
\[ y(k) = G(x(k), u(k)) \]
where \( k \in \mathbb{N}, x(k) \in X \subset \mathbb{R}^n \) is the state system, \( u(k) \in U \subset \mathbb{R}^m \) is the control input and \( y(k) \in Y \subset \mathbb{R}^p \) is the system output. It is assumed that \( X \) and \( U \) are closed and bounded sets. Moreover, system (1) belongs to the class of analogously to [1], the concepts of equilibrium state and for system (2)-(4) in order to minimize the deviations of the inputs present binary parts. Moreover, \( x(k) \in X \subset \mathbb{R}^r, k \in \{0, 1\}^r \) and \( z(k) \in \mathbb{R}^r \), \( k \), are auxiliary binary and continuous variables, respectively, whereas \( A, B_1, B_2, B_3, C, D_1, D_2, D_3, E_1, E_2, E_3, E_4, E_5 \) are suitable matrices with constant coefficients. In MLD systems the auxiliary variables are used to represent physical laws, logical rules and nonlinear relationships by means of mixed linear relations (by exploiting the propositional logic).

A major feature of MLD systems is that the state and input vectors can be composed of both continuous and binary quantities, that is \( x \in [x_1, x_2]^T, x_1 \in \mathbb{R}^{n_1}, x_2 \in [0, 1]^{n_2}, n_c + n_d = n, u \in [u_1, u_2]^T, u_1 \in \mathbb{R}^{m_1}, u_2 \in [0, 1]^{m_2}, m_c + m_d = m \). In the present work, MLD systems characterized by \( n_d = m_d = 0 \) are taken into account, i.e. MLD systems in which neither the state systems nor the system inputs present binary parts. Moreover, completely well-posed MLD systems, according to the definition provided in [1], are considered in this paper. This means that, given \( x(k) \) and \( u(k) \), the values of \( \delta(k) \) and \( z(k) \) are uniquely defined through the inequalities (4), i.e. \( \delta(k) = \phi_\delta(x(k), u(k)), z(k) = \phi_z(x(k), u(k)) \).

In order to describe the control approach proposed in this work, some preliminary definitions are needed. First of all, analogously to [1], the concepts of equilibrium state and equilibrium pair are reported in the following.

Definition 1: A vector \( x^e \in X \) is an equilibrium state for the MLD system defined by (2)-(4) and for the input vector \( u^e \), if, starting at time step \( k_0 \in \mathbb{Z} \) with \( x(k_0) = x^e \) and with \( u(k) = u^e, \forall k \geq k_0 \), it is \( x(k) = x^e, \forall k \geq k_0, \forall k_0 \in \mathbb{Z} \). The pair \( (x^e, u^e) \) is an equilibrium pair. □

In the following, vectors \( \delta^e \) and \( z^e \) represent admissible values of the auxiliary variables with reference to the equilibrium pair \((\delta^e, u^e)\).

The objective of this work is to develop a control scheme for system (2)-(4) in order to minimize the deviations of the system variables from their equilibrium values. Specifically, a Model Predictive Control scheme is adopted in which at each time step an optimal control problem is solved over a finite time horizon, thus deriving an optimal control sequence. Only the first element of the control sequence is applied, and at the subsequent time step the optimization procedure is repeated. The finite horizon optimal control problem to be solved at a generic time step \( k \in \mathbb{N} \) can be stated as follows.

Problem 1: Given the system initial conditions \( x(k) \), find the optimal control vector \( u(h), h = k, \ldots, k + K_p - 1 \), minimizing the cost function
\[ J(k) = \sum_{h=k}^{k+K_p-1} L(x(h), u(h)) \]
\[ = \sum_{h=k}^{k+K_p-1} \|u(h) - u^e\|^2_{Q_1} + \|x(h) - z^e\|^2_{Q_2} + \|y(k) - x^e\|^2_{Q_3} \]
subject to (2), (4) and
\[ x(k + K_p) = x^e \]
where \( K_p \) is the time horizon. □

The control law derived by solving Problem 1 within a MPC scheme is named mixed integer predictive control (MIPC) law. In the standard MPC approach, at a generic time step \( k \) the FHOCP is solved and the optimal control strategy \( u^c(k + h|k) \), \( h = 0 \ldots, K_p - 1 \) is obtained. Let \( x^c(k + h|k), \delta^c(k + h|k) \) and \( z^c(k + h|k) \) be the optimal evolution of the nominal system state and the evolution of the corresponding auxiliary variables. The control action to be applied at time step \( k \) is \( u^c(k|k) \) and the procedure is repeated at time step \( k + 1 \).

In [1] it has been proved that, under suitable assumptions, the MIPC law determined by solving Problem 1 stabilizes the system defined by (2)-(4). In case the considered MLD system (2)-(4) is affected by exogenous inputs that are additive and bounded signals, the Input-to-State practical stability can be proved, as shown in [13].

III. THE DISTRIBUTED CONTROL SCHEME

In this section, we will propose a control scheme and two distributed MPC algorithms to be used in the scheme itself. The controller distribution is based on the idea of subdividing the whole MLD system to be controlled into a given number of subsystems with scalar control. Relying on the classification reported in [8], the two distributed control algorithms proposed in this paper can be classified as follows.

Algorithm 1 (partially connected noniterative independent algorithm): each local controller computes the control action minimizing a local cost function and exchanging information with a given subset of the other subsystems only once within each sampling time.

Algorithm 2 (partially connected noniterative cooperative algorithm): each local controller computes the control action minimizing a partial cost function which includes the local cost function of the considered subsystem and the local cost functions of the subsystems with which it exchanges information (the optimization problem is solved once within each sampling time).

In order to better formalize the different approaches, let us denote with \( N_s \) the number of considered subsystems and
with $u_s(k)$ the control variable associated with subsystem $s$, $s = 1, \ldots, N_s$, at time step $k$. Let us denote with $\underline{x}_s(k)$ the vector gathering all the state variables of subsystem $s$, $s = 1, \ldots, N_s$, at time step $k$. Moreover, let $\mathcal{N}_s$ be the set of subsystems to which subsystem $s$ is connected (i.e. subsystem $s$ exchanges information with such subsystems). Fig. 1 is a sketch of a distributed control scheme in case the MLD system to control is divided in $N_s = 4$ partially connected subsystems. In particular, in this example it is $\mathcal{N}_1 = \{2, 3\}$, $\mathcal{N}_2 = \{1, 4\}$, $\mathcal{N}_3 = \{1\}$, $\mathcal{N}_4 = \{2\}$.

Assume to partition the global cost function in Problem 1 as follows

$$J(k) = \sum_{s=1}^{N_s} J_s(\underline{x}_s(k), u_s(k))$$

In Algorithm 1, by virtue of its noniterative and independent nature, the local controller of each subsystem solves its own optimization problem separately, by minimizing $J_s(\underline{x}_s(k), u_s(k))$ with respect to the sequence $u^0_s(k), \ldots, u^0_s(k + K_p - 1)$. Then, it uses the sequence $u^0_j(k), \ldots, u^0_j(k + K_p - 1), j \in \mathcal{N}_s$, computed by the local controllers of the connected subsystems, to determine the actual control sequence at sampling time $k$ for subsystem $s$, according to the rule

$$u_s(h) = \nu_s u^0_s(h) + \sum_{j \in \mathcal{N}_s} v_j u^0_j(h)$$

with $h = k, \ldots, k + K_p - 1$, $\nu_s + \sum_{j \in \mathcal{N}_s} v_j = 1$, and $\nu_s, v_j > 0$, for $s = 1, \ldots, N_s$, and $j \in \mathcal{N}_s$.

As for Algorithm 2, which is noniterative and cooperative, the local controller of subsystem $s$ solves the optimization problem by minimizing the partial (nonlocal) cost function

$$J_{\underline{x}_s}(k) = \eta_s J_s(\underline{x}_s(k), u_s(k)) + \sum_{j \in \mathcal{N}_s} \eta_j J_j(\underline{x}_j(k), u_j(k))$$

where $\underline{\mathcal{I}}_s = \{\{s\} \cup \mathcal{N}_s\}$ is the set of indexes of the neighbors of subsystem $s$ including itself, $\sum_{s \in \underline{\mathcal{I}}_s} \eta_s = 1$, and $\eta_s > 0$, for $s \in \underline{\mathcal{I}}_s$. The minimization of (9) is realized with respect to the sequence of control variables related to subsystem $s$ itself $u^0_s(k), \ldots, u^0_s(k + K_p - 1)$, and the control variables of the neighbors $u^0_s(k), \ldots, u^0_s(k + K_p - 1)$, $j \in \mathcal{N}_s$. Then, the local controller of subsystem $s$ uses the sequence $u^0_s(k), \ldots, u^0_s(k + K_p - 1)$, $j \in \mathcal{N}_s$, computed by the local controllers of the connected subsystems, to determine the actual control sequence at sampling time $k$ for subsystem $s$, according to the rule

$$u_s(h) = \nu_s u^0_s(h) + \sum_{j \in \mathcal{N}_s} v_j u^0_j(h)$$

with $h = k, \ldots, k + K_p - 1$, $\nu_s + \sum_{j \in \mathcal{N}_s} v_j = 1$, and $\nu_s, v_j > 0$, for $s = 1, \ldots, N_s$, and $j \in \mathcal{N}_s$.

The two proposed distributed algorithms will be compared in simulation with a decentralized MPC, whose structure is depicted in Fig. 2. In this decentralized approach the local controllers are designed by minimizing $J_s(\underline{x}_s(k), u_s(k))$ with respect to the sequence $u^0_s(k), \ldots, u^0_s(k + K_p - 1)$ for each subsystem $s$ separately, in a sort of locally centralized way.

### IV. APPLICATION TO FREEWAY TRAFFIC

Let us now consider the application of freeway traffic control in order to test the proposed distributed control approach for MLD systems. To represent the dynamic traffic behaviour, we consider the first-order macroscopic traffic model known as Cell Transmission Model, which is based on the subdivision of the freeway into cells and on the discretization of the time horizon. Let $N$ be the number of cells, $K$ the number of time steps, $T$ the sample time and $L_i$ the length of cell $i$. Let us define the following quantities referred to a generic time step $k$:

- $\rho_i(k)$ traffic density of cell $i$ [veh/km];
- $\Phi_+(k)$ total flow entering, $\Phi_-(k)$ total flow exiting cell $i$ [veh/h];
- $\phi_i(k)$ mainstream flow entering cell $i$ from cell $i - 1$ [veh/h];
- $l_i(k)$ queue length in the on-ramp of cell $i$ [veh];
- $r_i(k)$ flow entering cell $i$ from the on-ramp [veh/h];
- $d_i(k)$ on-ramp demand referred to cell $i$ [veh/h];
- $s_i(k)$ flow exiting cell $i$ through the off-ramp [veh/h];
- $D_i(k)$ demand, $S_i(k)$ supply of cell $i$ [veh/h];
- $\beta_i \in [0, 1]$ split ratio, $F_i$ capacity [veh/h], $\rho_j$ jam density [veh/km], $w_i$ congestion wave speed [km/h], $v_i$ free flow speed [km/h], $p_i \in [0, 1]$ priority of on-ramp flow with respect to the mainstream flow of cell $i$. 

![Fig. 1: The distributed control scheme for partially connected subsystems in case $N_s = 4$](image1)

![Fig. 2: The decentralized control scheme in case $N_s = 4$.](image2)
The dynamic model is given by the state equations for the traffic density $\rho_i(k)$ and the queue length $l_i(k)$, $i = 1, \ldots, N$, $k = 1, \ldots, K$

\[
\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i} \left[ \Phi_i^+(k) - \Phi_i^-(k) \right] \\
l_i(k+1) = l_i(k) + T [d_i(k) - r_i(k)]
\]  

(11)  

(12)

where

\[
\Phi_i^+(k) = \phi_i(k) + r_i(k) \\
\Phi_i^-(k) = \phi_{i+1}(k) + s_i(k) \\
s_i(k) = \frac{\beta_i}{1 - \rho_i} \phi_{i+1}(k)
\]  

(13)  

(14)  

(15)

Referring to cell $i$, it is useful to define the demand of cell $i - 1$ and the supply of cell $i$, as follows

\[
D_{i-1}(k) = \min \left\{ (1 - \beta_{i-1}) v_{i-1} \rho_{i-1}(k), F_{i-1} \right\} \\
S_i(k) = \min \left\{ w_i(\bar{\rho}_i - \rho_i(k)), F_i \right\}
\]

(16)  

(17)

The mainstream and on-ramp flows are obtained as

If

\[
D_{i-1}(k) + d_i(k) + \frac{l_i(k)}{T} \leq S_i(k)
\]

then

\[
\phi_i(k) = D_{i-1}(k), \quad r_i(k) = d_i(k) + \frac{l_i(k)}{T}
\]

else

\[
\phi_i(k) = \max \left\{ D_{i-1}(k), S_i(k) - d_i(k) - \frac{l_i(k)}{T}, \right\} \\
\phi_i(k) = \min \left\{ d_i(k) + \frac{l_i(k)}{T}, S_i(k) - D_{i-1}(k), p_i S_i(k) \right\}
\]

(18)

where the function $\max$ returns the maximum value.

Note that the considered traffic control approach is ramp metering, so that the control action is realized by varying the flows entering the cells through the on-ramps. The finite-horizon optimal control problems to be solved within the proposed MPC schemes adopt the CTM model for the predictions. Hence, the CTM model is rewritten in MLD form, by introducing suitable sets of inequalities and auxiliary variables, in order to avoid the nonlinearities present in (16), (17) and (18). For space limitations, we do not report all the inequalities and the auxiliary variables present in the MLD formulation (for more details on the CTM in MLD form and on the problem statement refer to [14]).

In the FHOCP to be solved at a generic time step $k$ over a prediction horizon $K_p$, the state variables are $\rho_i(h)$ and $l_i(h)$, $i = 1, \ldots, N$, $h = k + 1, \ldots, k + K_p$, the control variables are $r_i(h)$, $i = 1, \ldots, N$, $h = k, \ldots, k + K_p - 1$, and there are three sets of binary auxiliary variables $\delta_i,j(h)$, $j = 1, \ldots, 3$, $i = 1, \ldots, N$, $h = k, \ldots, k + K_p - 1$, and five sets of real auxiliary variables $z_i,j(h)$, $j = 1, \ldots, 5$, $i = 1, \ldots, N$, $h = k, \ldots, k + K_p - 1$. Then, the FHOCP to be solved at time step $k$ can be stated as follows.

**Problem 2:** Given the initial conditions on the density and the queue length $\rho_i(k)$ and $l_i(k)$, $i = 1, \ldots, N$, the demand of the cell before the first one $D_0(h)$, $h = k, \ldots, k + K_p - 1$, the supply of the cell after the last one $S_{N+1}(h)$, $h = k, \ldots, k + K_p - 1$, and the on-ramp demands $d_i(h)$, $i = 1, \ldots, N$, $h = k, \ldots, k + K_p - 1$, find the optimal control variables $r_i(h)$, $i = 1, \ldots, N$, $h = k, \ldots, k + K_p - 1$, minimizing

\[
\sum_{h=k}^{K_p} \sum_{i=1}^{N} \left[ \gamma_i^\rho (\rho_i(h) - \bar{\rho}_i)^2 + \gamma_i^l (l_i(h) - l_i^*)^2 \right. \\
+ \gamma_i^r (r_i(h) - \bar{r}_i)^2 + \sum_{j=1}^{3} \sigma_i^\delta (\delta_{i,j}(h) - \bar{\delta}_{i,j})^2 \\
+ \sum_{j=1}^{5} \sigma_i^z (z_{i,j}(h) - \bar{z}_{i,j})^2 \right]
\]

subject to the CTM model in MLD form.

For the distributed MPC scheme the basic subsystem is a *cluster of cells*, that is a subset of contiguous freeway cells which contains a single actuator, i.e. a single traffic light placed at the on-ramp. Moreover, $u_s(k)$, i.e. the control variable associated with subsystem (cluster) $s$, $s = 1, \ldots, N_s$, at time step $k$, corresponds to the on-ramp traffic volume in the cluster. Analogously, $x_s(k)$, i.e. the vector gathering all the state variables of cluster $s$, $s = 1, \ldots, N_s$, at time step $k$, includes the traffic densities of the cells of the cluster and the queue length of the on-ramp. Finally, $N_s$, i.e. the set of clusters to which cluster $s$ is connected, is given by the adjacent clusters. In other words, as represented in Fig. 4, we assume that each cluster of cells only communicates with the adjacent clusters (referring to Fig. 4, $N_1 = \{2\}$, $N_2 = \{1, 3\}$, $N_3 = \{2\}$).

![Fig. 4: The distributed control scheme for partially connected freeway clusters in case $N_s = 3$](image)

As regards Algorithm 1 and the decentralized control scheme, the problem data of each cluster $s$, in solving the FHOCP at time step $k$, are the current state of the cells of cluster $s$, the demand before the first cell of cluster $s$, the supply after the last cell of cluster $s$ and the on-ramp demands for the ramp of cluster $s$, from $k$ to $k + K_p - 1$. On the contrary, in case of Algorithm 2, the problem data...
of each cluster $s$, in solving the FHOCP at time $k$, are the current state of the cells of clusters belonging to $I_s$, the demand before the first cell of the first cluster in $I_s$, the supply after the last cell of the last cluster in $I_s$ and the on-ramp demands for the ramps of clusters belonging to $I_s$, from $k$ to $k + K_p - 1$.

In order to evaluate the effectiveness of the distributed MPC approach described in the previous sections, we have implemented the proposed control schemes with the C Sharp (C2) programming language and we have adopted the MILP solver Cplex 12.5 to solve each FHOCP. The freeway stretch considered for the simulation test is composed of 35 cells with 5 ramps. Then, according to the definition of cluster of cells defined above, 5 clusters are considered, as described in Table I.

<table>
<thead>
<tr>
<th>Clusters</th>
<th>Initial cell</th>
<th>Final cell</th>
<th>Cell with on-ramp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>13</td>
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</tr>
<tr>
<td>3</td>
<td>14</td>
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<tr>
<td>4</td>
<td>21</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>

The simulation covers an horizon of $K = 150$ time steps (the sample time has been set as $T = 10$ [s]). The cells are homogeneous and are characterized by $L_i = 0.5$ [km], $v_i = 105$ [km/h], $w_i = 40$ [km/h], $F_i = 8000$ [veh/h], $\bar{\rho}_i = 450$ [veh/km], $i = 1, \ldots, 35$. As regards the ramp parameters, the following values have been used: $\beta_{i-1} = 0.06$ and $p_i = 0.4$, $i = 13, 11, 18, 24, 30$. The initial conditions have been set equal to 80 [veh/km] for the traffic density in all the cells, except some central cells that present a higher density; in particular, $\rho_{19}(0) = 100$, $\rho_{20}(0) = 120$, $\rho_{21}(0) = 150$, $\rho_{22}(0) = 120$ [veh/km]. The initial conditions for the queue lengths at the on-ramps have been set equal to 0. As for the boundary conditions, the on-ramp demands have been set higher at the beginning (i.e. $d_i(k) = 1500$ [veh/h], $k = 0, \ldots, 29$, $i = 13, 11, 18, 24, 30$) and lower later (i.e. $d_i(k) = 750$ [veh/h], $k = 30, \ldots, K$, $i = 13, 11, 18, 24, 30$). The demand from cell 0 is, again, higher at the beginning (i.e. $D_0(k) = 5000$ [veh/h], $k = 0, \ldots, 24$) and then it decreases (i.e. $D_0(k) = 2500$ [veh/h], $k = 25, \ldots, K$), whereas the supply in cell $N+1$ has been fixed to $S_{36}(k) = 8000$ [veh/h].

With these data, the evolution of the traffic density in the no-control case corresponds to a highly congested situation, as depicted in Fig. 5. In order to reduce such a congestion, ramp metering is applied considering the control schemes described in the previous sections. In particular, our objective is to make a comparison among the centralized MPC scheme, the completely decentralized one and the cluster-based distributed approach with the two proposed algorithms, i.e. the partially connected noniterative independent algorithm (Algorithm 1) and the partially connected noniterative cooperative algorithm (Algorithm 2). The chosen set-points and the parameters of the proposed schemes have been tuned on the basis of simulation results and they are reported in the following: $\gamma_i^0 = \gamma_i^f = 1$, $\gamma_i^e = 0.5$, $\gamma_i^y = 0.1$, $j = 1, \ldots, 3$, $\gamma_{i,j} = 0.1$, $j = 1, \ldots, 5$, $i = 1, \ldots, 35$, $\rho_i^e = 105$, $l_i^p = 0$, $i = 1, \ldots, 35$, $K_p = 9$, $\nu_s = 0.6$, $s = 1, \ldots, 5$, $v_j = 0.4/|N_s|$, $j \in N_s$, $\eta_j = \eta_k = 1/|I_s|$, $j, k \in I_s$, $s = 1, \ldots, 5$.

The four control schemes mentioned above are able to reduce the congestion in the freeway; the traffic densities in the four controlled cases present very similar behaviours. Fig. 6 reports the behaviour of the traffic density when the distributed approach with Algorithm 2 is applied, showing a decrease of congestion in the whole freeway stretch. Reducing the densities in the mainstream with ramp metering strategies obviously implies the creation of queues at the on-ramps. The four control approaches considered in this comparative analysis present similar behaviours also in the formation of queues. In particular, Fig. 7 shows the behaviour of the queue lengths on the on-ramps when the centralized MPC is applied. Such a behaviour of the queues is almost the same found also in the decentralized approach and the distributed approach with Algorithm 2. A slightly different behaviour is obtained instead when the distributed approach with Algorithm 1 is applied, as shown in Fig. 8. More specifically, the root mean square error of the queue lengths obtained applying the distributed control approaches with respect to the centralized case are equal to 17.3 and 0.5.

The main difference among the proposed MPC approaches stands in the dimension of the finite-horizon optimal control problem. Table II reports the main characteristics of the
FHOCP to be solved in the different control schemes, specifying in particular the number of variables and the number of constraints of the problem, as well as the average CPU time (in seconds) to solve it. It is worth noting that all the experimental tests have been realized with a 2.2 GHz Intel(R) Core(TM) 2 Duo computer with 2 GB RAM.

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>Variables</th>
<th>Constraints</th>
<th>Average time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized</td>
<td>3636</td>
<td>8554</td>
<td>&gt;60</td>
</tr>
<tr>
<td>Decentralized-cluster 1</td>
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<td>1556</td>
<td>0.14</td>
</tr>
<tr>
<td>Decentralized-cluster 2</td>
<td>756</td>
<td>1790</td>
<td>0.17</td>
</tr>
<tr>
<td>Decentralized-cluster 3</td>
<td>756</td>
<td>1790</td>
<td>0.21</td>
</tr>
<tr>
<td>Decentralized-cluster 4</td>
<td>756</td>
<td>1790</td>
<td>0.20</td>
</tr>
<tr>
<td>Decentralized-cluster 5</td>
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</table>

TABLE II: Characteristics of the FHOCP.

Considering the average computational times reported in Table II, it is possible to conclude that the centralized case is not applicable in real cases for freeway stretches of the considered dimensions since the computational times are much higher than the sample time. In such cases, a distributed control scheme is surely more suitable. Even though in the proposed example the two proposed distributed control approaches have a similar effectiveness, it seems that in general the distributed MPC approach relying on Algorithm 2 is more effective than the one based on Algorithm 1. Indeed, in the distributed MPC approach relying on Algorithm 2 the local controller in each cluster computes the control action minimizing a cost function that involves not only the cluster itself but also neighbor clusters, then determining a less “local” control action. The main disadvantage of Algorithm 2, with respect to Algorithm 1, stands in the computational burden. In the considered example, Algorithm 2 could still be applicable in an on-line scheme but, for larger freeway stretches, it could be no more adequate, and consequently Algorithm 1 might be preferred.

V. CONCLUSIONS

A distributed MPC approach in order to control MLD systems is presented in the paper. The motivation underlying the proposed control scheme is the necessity, in practical applications, to alleviate the computational burden normally associated with conventional MPC. In many cases, the centralized on-line application of MPC is practically impossible since the optimization problem to be solved at each time step requires a high computational time. In the paper, two different distributed control approaches are presented and then applied considering a relevant application, that is traffic control in freeway networks. Their effectiveness is then tested in simulation considering a quite large freeway stretch for which the centralized conventional MPC is not adequate.

REFERENCES