Demand Response with Moving Horizon Estimation of Individual Thermostatic Load States from Aggregate Power Measurements

Evangelos Vrettos, Johanna L. Mathieu, and Göran Andersson

Abstract—We present an optimization-based state estimation method that allows us to estimate the states of individual thermostatically controlled loads (TCLs), such as air conditioners and space heaters, from measurements of the power consumption of small aggregations of TCLs. The state estimator can be used together with a controller to provide ancillary services to power systems such as frequency control. The main advantage of this method is that it is designed to work with existing communication infrastructure. We assume that aggregate power measurements are available from distribution substations every few seconds, while TCL state measurements are available from smart meters only every 20 minutes. We model TCLs as hybrid systems and propose a moving horizon state estimator (MHSE), which is formulated as a mixed-integer linear program. We demonstrate the performance of the MHSE in two case studies: (a) estimation of TCL states in the absence of external control actions, and (b) a power tracking problem with closed-loop control using broadcast control inputs. To demonstrate the robustness of the method, we conduct a parametric analysis with respect to aggregation size and diversity, process noise characteristics, and control trajectory characteristics. The results show that the method generally provides accurate estimates of TCL states, resulting in improved controller performance in most cases, and is implementable in real-time with reasonable computational power.

I. INTRODUCTION

Fluctuating renewable energy sources increase the need for power system ancillary services such as frequency control [1]. Traditionally, these services are provided by generators; however, there is a rising interest in using flexible electric loads to provide services [2] because they may be able to do so more effectively, at lower cost, and/or with less environmental impact. Active load participation in power systems is referred to as demand response (DR).

Thermostatically controlled loads (TCLs) such as air conditioners, space heaters, electric water heaters, and refrigerators operate with hysteresis controllers that include a temperature set-point and a dead-band. TCLs are good candidates for DR because (a) their power consumption can be shifted in time without compromising user comfort due to their thermal inertia; (b) they are easy to control; and (c) if aggregated, they can provide large amounts of DR. However, engaging large numbers of distributed TCLs in power system services may require significant investments in sensing, communication, and control infrastructure, in addition to new operational algorithms. Recent research has focused on methods to minimize the need for additional infrastructure by means of modeling, state estimation, and control strategies [3]–[6].

In this paper, we propose a novel moving horizon state estimation (MHSE) method that allows us to estimate the states of individual TCLs from measurements of the power consumption of small aggregations of TCLs, and periodic TCL state measurements. TCLs are modeled as stochastic hybrid systems (SHS), each with two states: a continuous state (temperature) and a discrete state (ON/OFF mode). Used together with a broadcast controller, the MHSE method allows an aggregator to control TCL populations to provide fast time scale services such as secondary frequency control (i.e. regulation) with high accuracy and minimal communication.

Several recent papers have investigated state estimation for TCL aggregations. Refs. [3], [4] proposed a method of modeling aggregations of TCLs with stochastic linear models based on Markov chains. One of the benefits of these models is that they can be used together with Kalman filters for state estimation. Ref. [6] proposed a state estimation approach based on a similar model and an MHSE method, while [5] proposed a four state aggregate system model, similar to that in [7], and a particle filter to estimate aggregate TCL states. However, with each of these approaches one is only able to estimate the fraction of TCLs in discrete bins in the state space, not the states of individual TCLs. Additionally, [6] did not consider measurement noise.

Estimating the states of individual TCLs instead of the states of aggregate models could improve control performance. Individual state estimates help us better estimate the effect of TCLs’ internal controllers and aggregator’s external control actions on TCL aggregate power consumption. Separate state estimation and control is not optimal for SHS since the separation principle does not apply to such systems [8]. However, it is common practice to heuristically separate the two tasks in the interest of simplicity. Control performance is generally improved with better state estimates; however, this must be verified through testing. SHS state estimation problems are often solved with ‘multiple model’ estimation schemes that involve a filter for each mode [9]. However, with large numbers of modes, as in our problem, these approaches are infeasible [10].

Our contributions are threefold. First, since the SHS TCL aggregation model is not amenable to common state estimation methods, we derive a stochastic mixed logical dynamical (MLD) model [11] from the SHS model. This allows us to represent a TCL aggregation as a linear system.

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with mixed-integer linear inequalities, which can be used within a mixed-integer linear program. To do this, we extend the MLD model for a single deterministic TCL proposed in [12] by a) considering external forcing from time-varying ambient temperature and process noise, b) expanding the model to handle TCL aggregations, and c) incorporating the effect of external broadcast control actions. Second, we propose an MHSE method based on mixed-integer linear programming (MILP) that is able to estimate the states of individual TCLs from aggregate power measurements, and periodic TCL state measurements. This is in contrast to [6], which uses an MHSE method to estimate the states of an aggregate system model, and does not consider TCL state measurements. Ref. [13] shows how MHSE can be used for MLD systems, while [14] derived sufficient conditions for asymptotic convergence of MHSE algorithms. However, the conditions are proved only for a deterministic system, and so we do not apply those results here. Instead, we investigate the effect of parameters that influence convergence – estimation horizon length, state penalties, etc. – empirically. Third, we illustrate how the MHSE method improves controller performance by conducting a parametric analysis with respect to aggregation size and diversity, process noise characteristics, and control trajectory characteristics.

The remainder of the paper is organized as follows. Section II describes the state estimation problem and Section III introduces the SHS and MLD models. In Section IV we describe the control scheme and in Section V we detail the MHSE method. Section VI presents two case studies and in Section VII, we give concluding remarks.

II. PROBLEM DESCRIPTION

We consider the problem of an aggregator managing a population of TCLs in a distribution network. The aggregator’s goal is to provide power system ancillary services. We assume that aggregate power measurements from medium-low voltage distribution substations are available to the aggregator at every time step. We also assume there exists a two-way communication system with latency and bandwidth constraints between the aggregator and each TCL. Specifically, we assume that TCLs interact with home energy management systems, which in turn communicate with smart meters that transmit data to the aggregator. TCL state measurements are obtained by home energy management systems every 10 seconds\textsuperscript{1} [15]; however, they can only be transmitted from smart meters to the aggregator every 20 minutes due to communication network constraints. Note that TCL state measurements are sent in packets that include all past state measurements since the last transmission. We also assume control signals are broadcast to all TCLs, i.e. TCLs are not individually addressed by the aggregator. The assumed measurements and their frequency are shown in Fig. 1.

Our goal is to estimate individual TCL states between consecutive state measurements with the overarching goal of improving control performance. This is a challenging task due to process and measurement noise. Process noise includes plant-model mismatch and errors in predictions of external forcing such as ambient temperature and consumer behavior. Measurement noise includes errors in aggregate power measurements. To obtain aggregate power measurements, we subtract the predicted uncontrolled load from the measured load at the substation, resulting in a noisy estimate of the aggregate power consumption of the TCLs.

The proposed MHSE method is efficient and computationally tractable for small TCL aggregations. In practice, larger TCL aggregations can be managed by solving parallel MHSE problems at several substations but as a single control problem. With this architecture, the proposed MHSE method is scalable and allows us to offer a significant amount of ancillary services. A major difference between this work and previous work is that the MHSE method efficiently utilizes measurements from medium-low voltage substations, instead of only aggregate measurements from high-medium voltage substations, as was assumed in [3]–[6].

III. MODELING

A. Individual TCL Modeling

Processes that evolve according to continuous dynamics, discrete dynamics, and logic rules can be modeled as hybrid systems [12]. Here, we use the two-state hybrid TCL model developed in [16]–[18]. Denote the TCL temperature at time step $t$ by $x_{c,t} \in \mathbb{R}$ and the ON/OFF state at time step $t$ by $x_{l,t} \in \{0,1\}$. A heating TCL’s stochastic discrete-time dynamics can be expressed as follows:

$$x_{c,t+1} = ax_{c,t} + bu_t x_{l,t} + fT_{\alpha,t} + w_t,$$

$$x_{l,t+1} = \begin{cases} 0 & \text{if } x_{c,t+1} \geq M \\ 1 & \text{if } x_{c,t+1} \leq m \\ x_{l,t} & \text{otherwise} \end{cases}$$

where $a = e^{-\Delta t/(CR)}$, $b = (1-a)RC_P P_{r}$, $f = (1-a)$, $\Delta t$ is the discretization time step, $C$ is the thermal capacitance, $R$ is the thermal resistance, $C_P$ is the Coefficient of Performance (COP), $P_r$ is the rated power, $w_t \in [0,1]$ is the fraction of the rated power consumed by the TCL at time step $t$ if it is ON, $T_{\alpha,t}$ is the ambient temperature, and $w_t$ is the process noise. Additionally, $M = T_{sp} + 0.5T_{db}$ and $m = T_{sp} - 0.5T_{db}$ are the

\textsuperscript{1}Apart from enabling estimation of individual TCL states, high resolution monitoring of TCLs is also necessary for auditing purposes.

Fig. 1. Source and frequency of power and load state measurements for a TCL aggregation in a distribution network.
upper and lower temperature dead-band limits, respectively, where $T_{sp}$ is the thermostat temperature set-point and $T_{db}$ is the dead-band width.

The SHS in (1)-(2) can be described using the MLD framework. Following the approach proposed in [12], we introduce the auxiliary binary variables $\delta_{1,t}$, $\delta_{2,t}$, $\delta_{3,t}$ and $\delta_{4,t}$ defined as follows:

$$\begin{align*}
\delta_{1,t} &= 1 \iff [x_{c,t} \geq M], \\
\delta_{2,t} &= 1 \iff [x_{c,t} \leq m], \\
\delta_{3,t} &= x_{t+1}, \\
\delta_{4,t} &= x_{t}. 
\end{align*}$$

Additionally, since (1) is bilinear between $x_{1,t}$ and $u_t$, we introduce the auxiliary continuous variable $z_{t} = x_{1,t}u_t = x_{4,t}u_t$ to achieve linearity of the model. Then, by defining $x_t := [x_{c,t}, x_{1,t}]^T$ and $\delta_t := [\delta_{1,t}, \delta_{2,t}, \delta_{3,t}, \delta_{4,t}]^T$, (1) can be rewritten as:

$$
\begin{bmatrix}
A & B_2 \\
B_3 & F_t
\end{bmatrix}
\begin{bmatrix}
x_t \\
z_t
\end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_1 + \\
\delta_2 + \\
\delta_3 + \\
\delta_4
\end{bmatrix}
+
\begin{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
+ \\
\begin{bmatrix}
f \\
0
\end{bmatrix}
+ \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
T_{\alpha,t} \\
0
\end{bmatrix}
+
\begin{bmatrix}
w_t \\
0
\end{bmatrix}
= x_{t+1}.
$$

Note that (7) is similar to (11a) in [12], except that we include two additional terms: ambient temperature and process noise.

The TCL internal hysteresis controller (2) can be described by the following logic relations:

$$\begin{align*}
[\delta_{1,t} = 1] &\quad \implies [\delta_{3,t} = 0], \\
[\delta_{2,t} = 1] &\quad \implies [\delta_{3,t} = 1], \\
[\delta_{1,t} = 0] \wedge [\delta_{2,t} = 0] &\quad \implies [\delta_{3,t} = \delta_{4,t}],
\end{align*}$$

which can be transformed into the following mixed-integer linear inequalities:

$$
E_2 \delta_t + E_3 z_t \leq E_1 u_t + E_4 x_t + E_5.
$$

The definitions of $E_1, E_2, E_3, E_4,$ and $E_5$ are omitted here due to space limitation, but can be found in (11b) of [12]. The entries of vectors $E_1, E_3 \in \mathbb{R}^{15}$ and matrix $E_4 \in \mathbb{R}^{15 \times 2}$ are 1, 0, or $-1$. Vector $E_5 \in \mathbb{R}^{15}$ and matrix $E_2 \in \mathbb{R}^{15 \times 4}$ additionally include $M$ or $m$ in some of their entries. Note that to keep the formulations generic, we have modeled the TCL power consumption $u_t$ as a continuous variable. However, in this paper we consider the most common case where TCLs operate at rated power when they are ON, i.e., $u_t = 1$. This simplification, the auxiliary variable $z_t$ is not needed.

B. TCL Aggregation Modeling

A heterogeneous aggregation of $n_{ag}$ TCLs can be modeled by drawing the TCL parameters, i.e. $R$, $C$, $C_p$, $P_n$, $M$ and $m$, from suitable distributions and stacking together models of individual TCLs, leading to the following state-space representation:

$$
\begin{align*}
x_{t+1} &= Ax_t + B_2 \delta_t + B_3 z_t + F_t + w_t, \\
y_t &= \begin{cases} C_1 x_t, & \text{if } t = jT_m, j \in \mathbb{N}, \\
C_2 x_t + v_t, & \text{otherwise}
\end{cases}
\end{align*}$$

where $x_t \in \mathbb{R}^{2n_w}$, $\delta_t \in \{0, 1\}^{4n_w}$, $z_t \in \mathbb{R}^{n_w}$, $w_t \in \mathbb{R}^{n_w}$, $E_5 \in \mathbb{R}^{15n_w}$, and $F_t \in \mathbb{R}^{2n_w}$ are stacked vectors, e.g., $x_t = [(x_{1,t})^T, \ldots, (x_{n_w,t})^T]^T$, $A$, $B_2$, $B_3$, $C_1$, $C_2$, $E_1$, $E_2$, and $E_4$ are block diagonal matrices with the matrices of the individual TCLs on the diagonals and appropriate dimensions; $C_1 = I$; $C_2 = [0, P_1^1, 0, P_2^1, \ldots, 0, P_n^1]$; $u_t = 1$; $v_t$ is the measurement noise; and $T_m$ is the period of TCL-level measurements. Note that the output of the system depends on the time step; every $T_m$ steps full state information is available, but at every other time step only the aggregate power is measured. Also, note that in (13) we consider noise in aggregate power measurements, whereas TCL-level state measurements are assumed noise-free.

IV. CONTROL PROBLEM

We use the closed-loop rule-based control algorithm proposed in [19] to make the TCL aggregation track a power trajectory. At each time step $t$, the controller receives a measurement of the aggregate power consumption of the aggregation $P_{agg,t}$ and, based on the desired set-point $P_{set,t}$, calculates the required change in power

$$
\Delta P_t = P_{set,t} - P_{agg,t}.
$$

However, at each time step, the actions of the internal thermostats will result in a change in aggregate power consumption $\Delta P_{int,t}$. Therefore, the controller must estimate $\Delta P_{int,t}$ so that it can calculate the effective change in power required by external control actions

$$
\Delta P_{eff,t} = \Delta P_t - \hat{\Delta} P_{int,t},
$$

where $\hat{\Delta} P_{int,t}$ can be estimated from state estimates $\hat{x}_t$ calculated with the MHSE algorithm described in Section V.

If $\Delta P_{eff,t} < 0$ additional OFF switching is required, whereas if $\Delta P_{eff,t} > 0$ additional ON switching is required. The TCLs that will be switched are determined according to a priority list based on their estimated state of charge (SOC)

$$
SOC_t = \frac{\hat{x}_{c,t} - m}{M - m},
$$

where $\hat{x}_{c,t}$ is the TCL’s estimated temperature. If $\Delta P_{eff,t} < 0$, the TCLs that are ON and within their dead-bands are candidates for OFF switching actions. An ordered list of these TCLs is obtained by ranking them in descending order based on $SOC_t$. The TCLs at the top of this list are hotter, and so have a higher switching OFF priority. Starting from the top of the list, we determine the number of TCLs that will be actively switched OFF such that their aggregate power consumption is as close as possible to $|\Delta P_{eff,t}|$. Analogously,
if $\Delta P_{\text{eff},t} > 0$, the TCLs that are OFF and within their dead-bands are ranked in ascending order based on $SOC_t$. We select the appropriate number of TCLs that will be actively switched ON from the top of the ordered list, since they are colder, and so have a higher switching ON priority. The $SOC_t$ of the last TCL of the priority list that is actively switched ON or OFF is called threshold SOC and is denoted by $SOC_{\text{th},t} \in [0,1]$.

At each time step, the controller broadcasts a pair $[SOC_{\text{th},t}, s_t]$, where $s_t \in \{0,1\}$ is a signal indicating whether an increase in consumption ($s_t = 1$ if $\Delta P_{\text{eff},t} > 0$) or a decrease in consumption ($s_t = 0$ if $\Delta P_{\text{eff},t} < 0$) is required. The TCLs that are outside of their dead-band are not controllable and ignore the control signal, whereas the rest respond based on their SOC. For each TCL, $SOC_{\text{th},t}$ can be mapped to a temperature threshold

$$x_{\text{th},t} = SOC_{\text{th},t}(M - m) + m.$$  

The desired control actions can be described by the state transitions in Tables Ia and Ib, which can be incorporated into the MLD framework of Section III by introducing

$$[\delta_{0,t} = 1] \leftrightarrow [x_{c,t} \leq x_{\text{th},t}].$$  

However, an equivalent formulation can be obtained without adding a new auxiliary variable in the following way. Set $M = x_{\text{th},t}$ and $\bar{m} = m$ if $s_t = 0$, and $M = M$ and $\bar{m} = x_{\text{th},t}$ if $s_t = 1$. With this notation and given that $SOC_{\text{th},t} \in [0,1]$, Tables Ia and Ib are equivalent to Table Ic by inspection.

Now, the state transitions of Table Ic can be described by the following logic relations:

$$[\delta_{1,t} = 1] \leftrightarrow [x_{c,t} \geq \bar{M}],$$  

$$[\delta_{2,t} = 1] \leftrightarrow [x_{c,t} \leq \bar{m}],$$

along with (5), (6), (8)–(10). Therefore, the external control actions can be directly incorporated into the MLD framework of Section III using (11). The only difference is that matrix $E_2$ and vector $E_5$, which now depend on $M$ and $\bar{m}$, are time-varying. The external control can be seen as a dynamic tightening of a TCL’s dead-band, which is visualized in Fig. 2. This control approach is similar to approaches based on temperature set-point control, e.g., as in [20]; however, in our approach user comfort is always guaranteed since $M \leq \bar{M}$ and $\bar{m} \geq m$. The control loop including the state estimation procedure is shown in Fig. 3.

V. STATE ESTIMATION PROBLEM

We propose an MHSE algorithm to estimate TCL states when TCL-level measurements are unavailable. At each time step $t \neq jT_m$, $j \in \mathbb{N}$, we solve a multi-period mixed-integer optimization problem with TCL temperatures, ON/OFF states, auxiliary binary variables, process noise, and measurement noise as optimization variables. Define the optimization vector at time step $t$, $x_t^{\text{opt}}$, as

$$x_t^{\text{opt}} := [\hat{x}_{t-N+1}|t, \hat{\psi}_{t-N+1}|t, \ldots, \hat{\psi}_{t}|t],$$

$$\psi_{k|t} := [\hat{\delta}_{k|t}, \hat{\zeta}_{k|t}, \hat{w}_{k|t}, \hat{v}_{k|t}], \ k \in [t - N + 1, t],$$

where $N$ is the estimation horizon, and $\hat{\psi}_{k|t}$ denotes the estimate of $\psi_{k|t}$ at time step $k$ using measurements up to time step $t$. Note that $\hat{x}_{t-N+1}|t, \ldots, \hat{x}_{t}|t$ can be determined from $\hat{\delta}_{k|t}, k \in [t - N + 1, t]$, and therefore these additional optimization variables are not needed. With this notation, the optimization problem can be expressed as follows:

$$\min x_t^{\text{opt}} \sum_{k=t-N+1}^{t} m_1 \left| \hat{y}_{k|t} - y_k \right| + m_2 \left| \hat{w}_{k|t} \right|$$

$$\ + m_3 \left| \hat{v}_{k|t} \right| + \sum_{k=t-N+1}^{t} m_4 \left| \hat{x}_{k|t} - \hat{x}_{k-1|t} \right|$$

$$\ + m_5 \sum_{i=1}^{m_6} \left| \hat{W}_i - W_i \right| + m_6 \left| \hat{V}_{k|t} - V \right|,$$

s.t. $\dot{x}_{k+1|t} = A\hat{x}_{k|t} + B_2\hat{\delta}_{k|t} + B_3\hat{\zeta}_{k|t} + F_t + \hat{w}_{k|t},$

$$\dot{y}_{k|t} = C_2\hat{x}_{k|t} + \hat{v}_{k|t},$$

$$E_x\hat{\delta}_{k|t} + E_3\hat{\zeta}_{k|t} \leq E_1u_t + E_4\hat{x}_{k|t} + E_5,$$

$$\hat{x}_{k|t} = x_{k|t}, \ \forall t \in [jT_m + 1, jT_m + N - 1],$$

$$\forall k \in [t - N + 1, jT_m],$$

where $\hat{\delta}_{k|t} \in \{0,1\}^{m_2}, \hat{\zeta}_{k|t} \in \mathbb{R}^{m_6}, \hat{w}_{k|t} \in \mathbb{R}^{m_2}, \hat{v}_{k|t} \in \mathbb{R}, m_1$ to $m_6$ are weighting factors, $W^i$ is a known statistic on the process noise of TCL $i$, $V$ is a known statistic on the measurement noise, and $W^i_{k|t}$ and $V_{k|t}$ are the current estimates of those statistics computed from $\hat{w}_{k|t}$ and $\hat{v}_{k|t}$.

The first term of (24) minimizes the difference between the output calculated from the estimated states (26) and
the measured aggregate power. The second and third terms penalize the process and measurement noise, where \( m_2 \) and \( m_3 \) may be a function of the noise variance/covariance matrices, \( Q \) and \( R \), if they are known. Note that there is no assumption on the probability distributions of the noises. The fourth term is needed to link the current estimation problem to the results of the previous estimation problems; the choice of \( m_4 \) is critical for the convergence of the estimator [14]. The fifth and sixth terms require that the current noise statistics are equivalent to their known values. However, if no noise statistics are known, the method can be applied without the last two terms.

For \( t \in [jT_m + 1, jT_m + N - 1] \), \( j \in \mathbb{N} \), noise-free TCL-level measurements are available, which are taken into account by introducing the equality constraint (28), and setting \( m_4 = 0 \) in (24). For \( t \geq jT_m + N \), (28) is not considered and \( m_4 \neq 0 \).

Let \( n_{a} = 4 \) denote the number of auxiliary binary variables per TCL, \( n_{w} = 1 \) denote the number of process noise variables per TCL, and \( n_{v} = 1 \) denote the number of measurement noise variables per time step. Then, the total number of variables is \( n_{var} = 2n_{ap} + N[n_{ap}(n_{a} + n_{w}) + n_{v}] \), of which \( n_{bi} = n_{ap} + Nn_{ap}n_{a} \) are binary variables. Therefore, the MHSE problem is a large scale mixed-integer optimization problem, even for small TCL aggregations and estimation horizons. For example, with \( n_{ap} = 20 \) and \( N = 10 \), the resulting problem has \( 820 \) binary variables. Due to the size of the problem, we use a 1-norm minimization in (24), which can be reformulated as a MILP, instead of least squares minimization as in [12]–[14], which leads to a quadratic integer program. Using the 1-norm enhances the tractability of our approach.

VI. CASE STUDIES

We demonstrate the performance of the MHSE method via two case studies. In case study A, we estimate the states of a TCL aggregation in the absence of external control actions to gain insight into the estimation mechanism. In this case, the TCLs are controlled only by their internal thermostats. In case study B, we perform state estimation on a TCL aggregation that is externally controlled to track a power trajectory. The purpose of this investigation is to quantify controller performance improvement with better TCL state estimates. To understand how process noise affects the estimation quality, we set the measurement noise to zero in both case studies. We also disregard the last two terms of (24) by setting \( m_5 = m_6 = 0 \). We use space heating TCLs and parameterize them by drawing the parameters from the uniform probability distributions listed in Table II. In our simulation, we consider a time-varying ambient temperature with an average value of \( 15.5^\circ C \).

The optimal estimation period depends upon the aggregation size and, in practical applications, would likely be determined by real-time computational limitations. For these reasons, we do not choose \( N \) based on the observability tests proposed in [12], but select it empirically. However, we investigate how the choice of \( N \) affects the quality of the estimation by conducting a parametric analysis in case study A. In addition, case study B was repeated for different aggregations, noise characteristics, and control trajectories to assess the robustness of the method. The weighting factors \( m_1 \) to \( m_4 \) in (24) are all chosen equal to 1 since preliminary simulations showed that this choice leads to good performance. All simulations were done in MATLAB using a 4 core machine (2.83 GHz) with 8 GB RAM, and the MHSE problem was solved using CPLEX.

A. Results for Case Study A

To understand the details of the MHSE method, we first consider an aggregation of 20 TCLs and run hourly simulations with a time step of 10 seconds. Only integer values of \( P_{th} \) are used, which complicates the estimation process since different TCLs may appear similar to the estimator. We assume that the process noise for each TCL follows a normal distribution with mean value \( \mu = 0 \) and standard deviation \( \sigma = 10^{-3} \), and that the TCL states at the beginning of the simulation are known, based on the discussion of Section II. The estimation horizon is fixed to \( N = 10 \) steps, and TCL measurements are received every \( T_m = 20 \) minutes.

We compare the MHSE against a simple model-based predictor, which receives TCL measurements whenever they are available and evolves the TCL dynamics assuming zero process noise, but does not utilize the noisy aggregate power measurements. Fig. 4 shows the actual, estimated, and predicted temperature trajectories of a single TCL. During the first 36 minutes, the MHSE behaves only marginally better than the model-based predictor. At this point, the estimator notices a significant mismatch between the expected and actual aggregate power. Afterwards, the temperature estimate converges to the actual one and the MHSE clearly outperforms the predictor. Notice that even small process noise can lead to significant discrepancy between the actual and predicted trajectories due to the discrete ON/OFF operation of TCLs.

The quality of estimates provided by MHSE and predictor at each time step \( t \) is quantified using the temperature Mean

\[
\text{TABLE II.}
\]

<table>
<thead>
<tr>
<th>( P_{th} )</th>
<th>( C )</th>
<th>( T_{ap} )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5, 9] kW</td>
<td>[5, 9] kWh/( C )</td>
<td>[19, 23] ( ^\circ C )</td>
<td>[1, 2, 5] kWh/( 24 ) hr</td>
</tr>
</tbody>
</table>

\[
C_{p} = 2, 3 \quad T_{ap} = [19, 23] \, ^\circ C \quad T_{db} = [0.25, 1] \, ^\circ C
\]

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A. Results for Case Study A

To understand the details of the MHSE method, we first consider an aggregation of 20 TCLs and run hourly simulations with a time step of 10 seconds. Only integer values of \( P_{th} \) are used, which complicates the estimation process since different TCLs may appear similar to the estimator. We assume that the process noise for each TCL follows a normal distribution with mean value \( \mu = 0 \) and standard deviation \( \sigma = 10^{-3} \), and that the TCL states at the beginning of the simulation are known, based on the discussion of Section II. The estimation horizon is fixed to \( N = 10 \) steps, and TCL measurements are received every \( T_m = 20 \) minutes.

We compare the MHSE against a simple model-based predictor, which receives TCL measurements whenever they are available and evolves the TCL dynamics assuming zero process noise, but does not utilize the noisy aggregate power measurements. Fig. 4 shows the actual, estimated, and predicted temperature trajectories of a single TCL. During the first 36 minutes, the MHSE behaves only marginally better than the model-based predictor. At this point, the estimator notices a significant mismatch between the expected and actual aggregate power. Afterwards, the temperature estimate converges to the actual one and the MHSE clearly outperforms the predictor. Notice that even small process noise can lead to significant discrepancy between the actual and predicted trajectories due to the discrete ON/OFF operation of TCLs.

The quality of estimates provided by MHSE and predictor at each time step \( t \) is quantified using the temperature Mean

\[
\text{TABLE II.}
\]

<table>
<thead>
<tr>
<th>( P_{th} )</th>
<th>( C )</th>
<th>( T_{ap} )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5, 9] kW</td>
<td>[5, 9] kWh/( C )</td>
<td>[19, 23] ( ^\circ C )</td>
<td>[1, 2, 5] kWh/( 24 ) hr</td>
</tr>
</tbody>
</table>

\[
C_{p} = 2, 3 \quad T_{ap} = [19, 23] \, ^\circ C \quad T_{db} = [0.25, 1] \, ^\circ C
\]
Absolute Error (MAE) and the ON/OFF state 1-norm error ($L_1$) for the whole aggregation, which are defined as

$$\text{MAE}_t = \frac{1}{n_{ap}} \sum_{i=1}^{n_{ap}} |x_{c,t}^i - \hat{x}_{c,t}^i|, \quad (29)$$

$$L_{1,t} = \sum_{i=1}^{n_{ap}} |x_{1,t}^i - \hat{x}_{1,t}^i|. \quad (30)$$

The evolution of estimation errors over time is shown in Fig. 5. The ON/OFF state of each TCL is almost always correctly estimated by MHSE, whereas the predictor produces significant errors. Around 42 minutes, the ON/OFF estimate diverges from the actual state; however, later on, the estimation error goes to zero. Therefore, even in case of temporary wrong estimates, the ON/OFF state estimates eventually converge to the actual states. The temperature prediction error generally increases with time, whereas the MHSE temperature estimation error is kept at lower, though non-zero, values. A perfect temperature estimation is not possible because the estimator retrieves information only when differences between predicted and measured power occur. In between, the temperature estimation error might grow depending on the process noise.

We next investigate the applicability of the MHSE method for larger aggregations and its performance with different estimation horizons. Fig. 6 shows results from simulations with $n_{ap} = \{10, 20, 30\}$ and $N = \{5, 10, 15, 20\}$. To ensure a fair comparison, exactly the same process noise data were used to simulate a given aggregation for each different $N$. In all cases, an horizon of $N = 10$ performs very well, while further increasing $N$ does not reduce the estimation errors. Shorter horizons degrade estimation quality for aggregations of 20 or 30 TCLs; however, for very small aggregations of 10 TCLs even an horizon of $N = 5$ is enough. Note that since we have considered specific TCL aggregations and process noise time series, these results are only meant to provide intuition. More general conclusions regarding the best choice of $N$ could be obtained by Monte Carlo simulations.

The problem (24)-(28) can be solved quickly for the aggregation sizes considered here. Table III shows how the average computation time (first number) and the maximum computation time (second number) depend on $n_{ap}$ and $N$. Not surprisingly, increasing $n_{ap}$ or $N$ leads to longer solution times. Note that increasing $N$ has a larger impact on computation time since this introduces more binary variables in the problem. The reported computation times are likely acceptable for real-time estimation applications.

### B. Results for Case Study B

Our ultimate question is how much the controller performance improves with better TCL state estimates. For this purpose, we use the control scheme of Section IV to track $P_{set}$ considering two cases: (a) the model-based prediction approach and (b) the MHSE method. The power trajectory $P_{set}$ is calculated by superimposing a random signal upon the predicted TCL aggregation baseline, i.e. the power trajectory without external control actions, as in [19], [21]. We run simulations with horizon of 20 minutes and use exactly the same 20 TCL aggregation and process noise data as in the previous subsection. Since $T_m = 20$ minutes, there are no TCL measurements received apart from the initial measurements.

The model-based predictor leads to a MAE of 6.40 kW, whereas MHSE leads to a MAE of 4.04 kW, which represents an improvement of about 37%. The absolute tracking errors for both cases are shown in Fig. 7a. Note that we have smoothed the error time series using a moving window of 1 minute to enhance the visibility of the trend. Fig. 7b shows the broadcasted control signal $SOC_{th}$ for both cases. One can see that improved state estimates with MHSE also lead
to less aggressive control signals, as compared to the model-based prediction approach.

Figure 7c shows the evolution of estimation errors over time. The ON/OFF state estimation error of MHSE is generally larger than in case study A, which is due to the external control actions. Also, note that there exist intervals when the MHSE ON/OFF state and temperature errors are larger than the prediction errors. However, the controller performs better with MHSE than with prediction during the same intervals. This indicates that even when the MHSE provides poor TCL-level state estimates, it still improves controller performance.

To provide evidence that the MHSE generally improves control performance, we repeat the analysis for different combinations of TCL aggregations, process noise characteristics, and control trajectories. We consider normally distributed zero mean process noise with three different standard deviations $\sigma = \{5 \times 10^{-4}, 10^{-3}, 5 \times 10^{-3}\}$, which we refer to as “low”, “medium”, and “high” noise. We also consider two control trajectories: a) a “moderate trajectory” based on a Swiss secondary frequency control signal with a resolution of 10 seconds, and b) an “aggressive trajectory” in which we superimpose zero mean random noise drawn from a normal distribution with standard deviation $\sigma_t = 10$ kW on top of the secondary frequency control signal. For each of the six combinations we run 200 simulations with randomly sampled TCL aggregations of size $n_{ap} = 20$.

Figure 8 shows the resulting improvements in controller performance and the frequency of their occurrence. In addition, Table IV summarizes the results showing the probability with which the MHSE will result to more accurate control compared to model-based prediction approach. In the worst case, i.e., high noise and the moderate trajectory, the MHSE improves controller performance with a 0.67 probability. In case of low noise and the aggressive trajectory, improvement occurs with a probability as high as 0.99. As expected, increasing process noise deteriorates the performance improvement. Interestingly, the control performance is better with the aggressive trajectory. This observation is in accordance with [6], which also uses an MHSE approach. Last, note that there are cases in which the MHSE method leads to worse controller performance than the prediction approach. In some cases, such as those in Fig. 8c, this is mainly the result of poor TCL state estimates due to high noise and little information retrieval while tracking the moderate trajectory. Also, in small TCL aggregations recent control actions influence a lot the tracking capability in the next time steps, which may result in performance deterioration. This explains, for example, the outliers with negative controller improvement in Fig. 8d and Fig. 8e.

VII. CONCLUDING REMARKS

This paper presented a moving horizon state estimation (MHSE) method to extract temperatures and ON/OFF states of individual loads in small aggregations of thermostatically controlled loads (TCLs). The method assumes a two-way...
TABLE IV.
CONTROL IMPROVEMENT WITH MHSE

<table>
<thead>
<tr>
<th></th>
<th>Moderate trajectory</th>
<th>Aggressive trajectory</th>
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</thead>
<tbody>
<tr>
<td>Low noise</td>
<td>0.77</td>
<td>0.99</td>
</tr>
<tr>
<td>Medium noise</td>
<td>0.70</td>
<td>0.97</td>
</tr>
<tr>
<td>High noise</td>
<td>0.67</td>
<td>0.72</td>
</tr>
</tbody>
</table>

constrained communication infrastructure between an aggregator and each TCL, where control signals can only be broadcasted and TCL-level measurements can be received at a lower frequency than aggregate power measurements. The proposed method handles both process and measurement noise; however, only process noise was explicitly investigated in the case studies. We conducted simulations for a case without external control and a power trajectory tracking application.

Our results show that state estimates are generally accurate and result in controller performance improvement in most cases. Additionally, our investigations revealed a number of interesting observations. First, the MHSE method works better with diverse TCL aggregations and aggressive control trajectories. Second, the controller achieves good tracking even with temporarly poor state estimates for individual TCLs. This is important for populations of TCLs that are not easily identifiable, i.e. TCLs with very similar parameters and/or identical power ratings, since in this case accurate estimates of individual TCLs might be very hard to get. In such cases, clustering of TCLs based on their parameters could be used to simplify the estimation process. Third, for aggregation sizes up to 30 TCLs, an estimation horizon $N = 10$ seems to be a reasonable tradeoff between estimation accuracy and computation time.

There are many avenues for future work. Preliminary simulations show that measurement noise can be in principle handled by the proposed method, but this would probably require longer $N$ and lead to longer solution times. Also, penalizing process and measurement noise using $Q$ and $R$ in (24) only, might not be enough for good performance. A potential solution would be to filter the aggregate power measurements before inserting them into the MHSE problem.

From a theoretical point of view, it would be interesting to analyze the convergence properties of the proposed MHSE algorithm. As explained in Section II, ancillary services can be offered by considering multiple distribution substations, where MHSE is performed separately at each of them. A direct extension of this work will be to investigate the controller/estimator performance in this more realistic case. Furthermore, designing controller/estimator schemes that are robust in the presence of delayed measurements or control inputs is an exciting research direction. Last, for benchmarking purposes, we plan to compare the MHSE method against techniques that use aggregate system models.

REFERENCES


