Integrated Robust Optimal Design (IROD) of Header Height Control System for Combine Harvester

Punit Tulpule\textsuperscript{1} and Dr Atul Kelkar\textsuperscript{2}

Abstract—This paper presents an integrated control-structure design methodology that is robust to parametric variations and its application to the design of combine header height control. Traditional design process of controlled multibody systems is sequential wherein the structural design is accomplished first followed by the control design. Such sequential process can only provide suboptimal overall design and yield a suboptimal performance. In this paper, a sensitivity based integrated robust optimal design (IROD) methodology is used to determine optimal structural parameters concurrently with a controller with an objective of improving header height control performance for a combine harvester. The problem is to control the height of the heavy header assembly at a constant distance above the rough terrain to maximize the yield and efficiency during harvesting. The design methodology is demonstrated by choosing the pivot location of the hydraulic actuator on the header as a structural design parameter concurrently with controller parameters. The design obtained with this methodology was compared with the sequential design for tracking performance and robustness. It is shown that the integrated design obtained from IROD yields better performance, lower control power, and improved robustness than the sequential method. It also allows for higher vehicle speed and lower operational cost.

I. INTRODUCTION

In optimal design process of automatically controlled dynamical systems, the plant model is designed for cost and performance and then control engineers implement optimal controllers on this pre-designed model. In practice, this sequential approach of designing controlled systems has been very popular due to its simplicity, but sequential approach can only provide a suboptimal design. This is because, when dynamical system or plant model is designed, the available design space for control engineer is reduced. In order to overcome this shortcoming, many researchers have proposed integrated or simultaneous controller/plant design methods. In integrated design approach, some parameters of the system, like mass, lengths of bodies, stiffness, spring constant etc. are considered as design variables along with the controller parameters. It was shown that integrated approach provides better designs, but the design process is complex and computationally inefficient, and when robustness criteria are added to the design requirements, the problem becomes even more complex. Even for very simple case of linear system, linear controller and system parameter appearing linearly in the dynamic equations, the design process is non-convex and hence NP hard. Some noteworthy research on integrated design includes work done in the design of controlled flexible space structures under Control-Structure Interaction (CSI) program at NASA in late 80’s to early 90’s. \cite{1}, \cite{2}, \cite{3}, \cite{4}. Most of the early research on integrated design was based on gradient descent methods and the gradient was calculated using sensitivity analysis. In \cite{1} an approach based partial sensitivities of closed loop systems with respect to control parameters and system parameters was proposed. Gilbert and Schmidt \cite{5} proposed a more generalized approach using multistep optimization based on sensitivity. In \cite{6}, system/controller redesign strategy based on variations in control and system parameters was proposed. This is an approach where controllers and system parameters are iteratively redesigned. Integrated design methodology was applied to flexible high speed robotic arm based on time optimal control and finite element analysis in \cite{7}. Later, integrated design approach for mechanical systems based on rapid prototyping was proposed by \cite{8}. Several other algorithms were proposed in late 1990s, some of those are - parallel vector optimization for large structures \cite{9}, and genetic algorithm based evolutionary design methods for integrated design with sensor and actuator locations \cite{10}. Optimality conditions for combined plant/control optimization problem were proved in \cite{11}. Motivated from automatic differentiation of dynamical equations, integrated robust design methodology for minimum sensitivity, variability and maximizing tolerance was proposed and demonstrated in \cite{12}, \cite{13}.

In design process, robustness and tolerance is also considered for improved durability and reduced manufacturing cost. In modern robust control theory, the robustness is included in the design process by uncertainty analysis, but this approach introduces conservatism as designer finds an upper bound on uncertainty. Also, the uncertainty domain may not always be readily available. The final design depends on the designers skills of uncertainty analysis, choosing upper bound on the uncertainty domain and weighting functions. Due to these limitations sensitivity based robust optimal control design method was proposed, for multibody systems in \cite{12}, \cite{13}. Before the modern robust control theory was developed, for many years sensitivity analysis was considered as robust control theory. The sensitivity theory was introduced in the textbooks \cite{14}, \cite{15} in early 1960s. Many researchers \cite{16}, \cite{17} and \cite{18} and others in late 1960s considered trajectory sensitivity minimization problems by augmenting the sensitivity dynamics to the system dynamics. These

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researches were focused on the LQR control problems and the solution strategies were some variants of the maximum principle. Until the modern robust control theory based on uncertainty characterization was established, the sensitivity theory was the primary area of interest in robust control research. There is a large body of literature available in this area from the 1970s [19], [20], [21], [22], [23].

In current research we have proposed a new method based on relatively modern Linear Matrix Inequality (LMI) theory and the old sensitivity theory. The sensitivity theory allows easy automation of robustness evaluation of the system. The LMI is a powerful tool since, convex optimization algorithms like, interior point methods, can be used to solve many optimal control design problems. Earlier research was well documented in a research monograph [24]. Most of the optimal feedback control design problems can be easily written in form of Bilinear Matrix Inequality (BMI) constraint optimization problem. Some BMI constrained problems can be converted to equivalent LMI problems by linearization change of variables[25], [26] or methods based on projection lemma [27], but most of the BMI problems can not be converted to LMI problem. Hence, numerical solution strategies are proposed in the literature. There are mainly two types of theories, global and local optimization methods. Local optimization methods are based on fixing either controller or Lyapunov matrix variable and solve the resulting LMI problem, then fix other variable and solve the LMI problem. This process is repeated until local minimum is found. The global optimization methods suffer from computational problems, and local solution strategy obviously does not guarantee global convergence. Some of these approaches have been employed for variety of examples, like large flexible space structures, robotics, mechatronic systems, chemical processes, hybrid vehicles, power electronics and many other.

It can be proved that for general quadratic performance with linear systems, the integrated design problem can not be equivalently written in form of LMI constrained problem, hence mainly iterative methods were proposed [28], [29], [30], [31], [32]. In current research we have used the method proposed in [33], because it guarantees local convergence and requires solution of one LMI constrained problem at each step. This algorithm is simplified for the specific class of sensitivity augmented system by virtue of the lower block triangular structure.

A sensitivity and performance norm minimal designing algorithm for full state feedback controller synthesis problem using LMIs was presented in [34]. In a parallel work [35] and [32], the authors have developed and demonstrated a BMI based algorithm for robust optimal controller synthesis and integrated system-controller design via sensitivity augmented system. It was shown that the algorithm is a viable alternative to uncertainty based $\mathcal{H}_\infty$ or $\mathcal{H}_2$ or mixed controller design technique. This technique is applied to Combine harvester header height control problem. The combine harvester has heavy header and the height needs to be automatically controlled for higher efficiency of harvest and speed of operation as the harvester moves over rough terrain. The structure of this harvester imposes fundamental constraints on feedback control design [36]. A two DOF controller design using feed forward and feedback controller was presented in [37] and integrated plant/controller design based on only performance analysis was presented in [38] to overcome the bandwidth limitations due to the mechanical structure. In this research a generalized methodology for Integrated and Robust Optimal Design (IROD) is applied to this example and it is shown that the new method provides better tracking, robustness and control power than the robust $\mathcal{H}_\infty$ design method.

II. SENSITIVITY BASED ROBUST CONTROL SYNTHESIS

A. Introduction to Parametric Sensitivity

This section introduces the parametric sensitivity based robust optimal control design using Bilinear Matrix Inequalities (BMI). As discussed earlier, sensitivity analysis dynamics can provide a vital information about robustness of a system. For a continuous plant model, in time as well as parameter space, the parametric sensitivities can be obtained by differentiating the dynamical equations with respect to the variables representing uncertain parameters or parameters that are subject to change over operational time. Here, the continuity is implied in both, the time and parameter arguments. Performance sensitivity of a dynamical system can be defined as a small change in the performance for an arbitrarily small change in the design parameter.

Consider a system represented by differential equations

$$\dot{x} = f(x, u, b, t)$$  \hspace{1cm} (1)

From the definition, the dynamics of sensitivity can be obtained from -

$$\frac{d\dot{x}}{db} = \frac{df}{db} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial b} + \frac{\partial f}{\partial x} x_b$$  \hspace{1cm} (2)

If the function $f \in C^2$ i.e. $f$ is second-order continuous, then the order of derivatives with respect to time and parameter $b$ can be interchanged due to Clairout’s theorem [39] (which is also known as Schwarz’s theorem) and (2) becomes,

$$\dot{x}_b(x, x_b, u, b, t) = \frac{\partial f}{\partial x} x_b$$  \hspace{1cm} (3)

where, $x_b = \frac{dx}{db}$ is the sensitivity of the states $x$ with respect to small change in the parameter $b$. The sensitivity dynamics in (3) can now be augmented to the plant dynamics in (1) and solved simultaneously to obtain the system states, $x$, and sensitivity states, $x_b$. Now, the state vector for the augmented system is - $[x^T, x_b^T]^T$. As a special case of plant models, consider a linear system -

$$G(b) = \begin{bmatrix} A(b) & B_1(b) & B_2(b) \\ C_1(b) & D_{11}(b) & D_{12}(b) \\ C_2(b) & D_{21}(b) & 0 \end{bmatrix}$$  \hspace{1cm} (4)
where, \( b \) is a vector of parameters which are uncertain, or subject to change due to wear and tear. The order of the system is \( n \) and the system has \( n_x \) number of exogenous outputs, \( n_y \) number of exogenous inputs, \( n_u \) number of output channels and \( n_v \) number of input channels are used in the feedback. We assume differentiability of system matrices with respect to the parameters \( b \). Given the controller dynamics in state space form -
\[
A(b)_{n \times n} = \begin{bmatrix}
A(b) + B_2(b)D_cC_2(b) & B_2(b)C_c \\
B_1C_2(b) & A_c
\end{bmatrix}
\]
\[
B(b)_{n \times n} = \begin{bmatrix}
B_1(b) + B_2(b)D_cD_{21}(b) \\
B_cD_{21}(b)
\end{bmatrix}
\]
\[
C(b)_{n_z \times n} = \begin{bmatrix}
C_1(b) + D_{12}(b)D_cC_2(b) \\
D_{11}(b) + D_{12}(b)D_cD_{21}(b)
\end{bmatrix}
\]
\[
D(b)_{n_z \times n_w} = D_{11}(b) + D_{12}(b)D_cD_{21}(b)
\]
\[ (5) \]

Differentiating the closed loop system in (5) with respect to parameter \( b \) gives closed-loop parameter sensitivity dynamics. The augmented closed-loop system with this sensitivity dynamics can be written as:
\[
\begin{bmatrix}
\dot{X} \\
\dot{X}_b
\end{bmatrix} = \begin{bmatrix}
\dot{A} & 0 \\
\frac{dA}{db} & \dot{A}
\end{bmatrix}
\begin{bmatrix}
X \\
X_b
\end{bmatrix} + \begin{bmatrix}
\dot{B} \\
\frac{dB}{db}
\end{bmatrix}
\begin{bmatrix}
X \\
X_b
\end{bmatrix}
\begin{bmatrix}
\dot{C} & 0 \\
\frac{dC}{db} & \dot{C}
\end{bmatrix}
\begin{bmatrix}
X \\
X_b
\end{bmatrix}
\begin{bmatrix}
\dot{D} \\
\frac{dD}{db}
\end{bmatrix}
\begin{bmatrix}
X \\
X_b
\end{bmatrix}
\begin{bmatrix}
u
\end{bmatrix}
\]
\[ (6) \]

(The dependence of system matrices on parameter \( b \) is not explicitly written hereon, but it is assumed.) where, \( X \in \mathbb{R}^{n \times n} \) is a vector of closed loop states with \( X = [x; x_c] \), \( x_c \) being the controller dynamics states. \( X_b \) are the closed loop sensitivity states with \( X_b = [x_b; x_{cb}] \).

**Notes:**
- The matrix \( \hat{A} \) is lower triangular, with the same blocks on diagonal. Hence, the augmented system has repeated Eigen values.
- The exogenous inputs of the system are parameter independent, hence derivative of input \( dv/db \) is zero.
- The two outputs of the system are the nominal system output and the sensitivity of output of the system.

### B. \( \mathcal{H}_\infty \)-Norm Bound Objective

If the objective is to find only the optimal controller that minimizes the \( \mathcal{H}_\infty \)-norm bound of system in (6), then the constraint in the \( \mathcal{H}_\infty \)-norm minimization problem can be written as a BMI, due to bounded real lemma.[24]
\[
\begin{bmatrix}
sym(P_1\hat{A}) & P_1\hat{B} & \hat{C}^T \\
* & -\gamma I & \hat{D}^T \\
* & * & -\gamma I
\end{bmatrix} < 0
\]
\[ (7) \]

where,

The BMI constraint problem in (8) cannot be converted into equivalent LMI constrained problem using linearization change of variables or by projection lemma, but the algorithm presented in [33] to solve BMI constrained problem can be simplified further for this specific class of sensitivity augmented systems due to the special lower block triangular structure. The algorithm is presented in [32].

### III. COMBINE HARVESTOR MODEL

Schematic diagrams of combine machine and header link are shown in Fig. 1 and Fig. 2 respectively.

The objective of this case study is to synthesize a robust controller and optimize a header parameter to maintain constant header height above the sinusoidal terrain, minimize the sensitivity of performance with respect to changes in the same plant parameter and reduce control power. The plant design variable is chosen to be location of a pin joint (LB), between feeder house and hydraulic piston, in body centered coordinate system as shown in Fig.2. Nominal value of LB is 1.541m. The uncertain parameter was chosen to be \( LB \) because this parameter has wider range due to less physical constraints. The header height control problem is important because even small improvement in tracking, is
directly related to higher crop yield, better speed of combine harvester which then relates to leveraged profit for farmer. In this study we assumed the speed of combine machine to be 10 mph (16.1 km/h) and designed two controllers using two methods namely, robust $H_\infty$ design using LMI and integrated robust optimal design (IROD) method. The plant parameter $LB$ was re-designed using IROD method. The parameters of the linkage are for actual combine machine of John Deere and company. The rough terrain is the reference input for tracking (Input 1) and the header height is controlled through hydraulic actuator (Input 2). The hydraulic actuator is assumed as a perfect source of force. The four tires are modeled as a spring-mass-damper system. The height sensor is attached at the tip of the header, and there is a natural delay between header and front tires and rear tires. Velocity of the combine is assumed to be a constant at 10 mph (4.4707m/s) and since the distance between the header height sensor and the tires is known, the delay between header height sensor and front tires is 0.5810s and the delay between header height sensor and rear tires is 1.37s. These delays are modeled using first order Pade approximation. The model of the header, feeder-house linkage along with the vehicle and tires is built in SimMechanics. The dynamics incorporated in the SimMechanics model is shown in Fig. 3.

A. Sequential $H_\infty$ Design

In order to design robust $H_\infty$ optimal controller, parameter $LB$ was varied in the range of $\pm 0.1 m$ from nominal value with step size of 0.005m. At every discrete value of $LB$, the harvester model was linearized to obtain a family of plant models with uncertainty in the parameter $LB$. Uncertainty in the model is modeled as polytopic uncertainty. Bode magnitude diagrams of normalized uncertain plants and upper bound on uncertainty are shown in Fig. 4.

Terrain tracking controller does not require high frequency tracking. The weighting function $w_T$ on reference input is shown in (9).

$$w_T = \frac{(s + 10)}{(s + 10000)}$$  \hspace{1cm} (9)

$H_\infty$ optimal control is synthesized for the set of uncertain plant models $uG$ using LMI method. Another controller without considering uncertainty was designed using the same method. For nominal controller, the same weighting function is used as shown in (9).

B. Integrated Robust Optimal Design (IROD)

The IROD theory is used to derive controller and system parameter $LB$ such that the sensitivity w.r.t. the same parameter $LB$, the tracking error and control power are minimized. The SimMechanics model of combine harvester is linearized at operating point such that the header is raised to desired height. The linear sensitivity dynamics is obtained by numerical differentiation of linearized SimMechanics model. The minimum $H_\infty$ norm bound problem for the augmented system and sensitivity dynamics is solved using IROD method explained in the second section. The system is linearized at every step of the optimization since, when $LB$ is changed, the operating point as well as linearization of nonlinear system changes. The IROD method proposed the piston joint location to be moved closer to the header pivot. The proposed change in the position is $\delta LB = -6.36 cm$. Integrated design shows 14% improvement in $H_\infty$ norm from $\gamma_{\text{nominal}} = 11.64$ to $\gamma_{\text{integrated}} = 9.9123$.

Figure 5 shows comparison between performance of controller obtained from robust $H_\infty$ control synthesis and closed loop system obtained from IROD. All the three (nominal $H_\infty$, robust $H_\infty$, and IROD) controllers are used in the loop with full non linear system developed in SimMechanics. Terrain shape is chosen to be sinusoidal with amplitude 1m and frequency 0.1Hz (0.2πrad/sec) i.e. 0.1406rad/m if combine velocity is 4.4707m/s (10mi/hr). The sensitivity of the closed loop system in all three cases is computed as a function of angle of header, using numerical differentiation. Figure 6 compares the maximum sensitivities of the closed loop systems obtained from different approaches. Table I shows that the integrated design method gave better results in terms of all the three design criteria. All the three measures (RMS error, control power, and sensitivity) are computed for steady state performance.
IV. CONCLUSION

A new methodology for integrated controller/plant robust optimal design is applied to Combine harvester header height control problem. This method is compared with the nominal and robust $H_{\infty}$ controllers. It is demonstrated, that the IROD method provides better overall closed loop system than the sequential design methods, by comparing the tracking performance, sensitivity, and control power. This example also shows that the IROD method is an effective alternative to existing integrated design methods.

REFERENCES


TABLE I

<table>
<thead>
<tr>
<th>Comparison Between $H_{\infty}$ Controller and IROD</th>
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<tbody>
<tr>
<td>RMS Tracking Error (m)</td>
</tr>
<tr>
<td>$H_{\infty}$</td>
</tr>
<tr>
<td>0.144934</td>
</tr>
<tr>
<td>Maximum sensitivity</td>
</tr>
<tr>
<td>1.1047</td>
</tr>
<tr>
<td>Total Control Power (J)</td>
</tr>
<tr>
<td>$2.264 \times 10^6$</td>
</tr>
<tr>
<td>$2.1167 \times 10^6$</td>
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<td>$1.9480 \times 10^6$</td>
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