Adaptive Control of Inter-Area Oscillations in Wind-Integrated Power Systems using Distributed Parameter Control Methods

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Abstract—This paper presents a preliminary study using adaptive control theory to damp inter-area oscillations in power systems through controlling wind farm that is injecting power into the system. The power system is modeled as a distributed parameter system using a first order hyperbolic wave equation, which represents the dynamics of an aggregate rotor model for a system of coupled swing equations. A direct adaptive controller is used to stabilize the power swing in the face of disturbances using power injected from an alternate source like a wind farm.

I. INTRODUCTION

The amount of electricity generated from wind turbines is growing steadily and hence a large amount of electrical power from wind power plants is being injected onto existing transmission lines. A number of studies have examined how the intermittent nature of power produced by wind turbines will impact on the power grid [1] - [3]. A number of recent studies have also investigated the use of wind farm control to support the existing transmission system or to mitigate the effects of adverse conditions. For example for the provision of frequency support/regulation [4], power system stabilization after sudden changes in load or power generation [5], and controlling power output of a wind farm to damp the inter-area oscillations [6].

Existing power systems are interconnected via transmission lines which carry bulk electrical power. Generators in close geographic proximity tend to tightly synchronized (swing together) but these coherent groups of generators (areas) tend to be weakly connected to one another across long power transfer pathways [7]. This leads a situation whereby two or more of these tightly coupled areas swing against one another. The low frequency [0.1 – 1] Hz oscillations between two areas are known as inter-area oscillations. These inter-area oscillations, if left undamped, can destabilize the power system which can ultimately lead to load shedding.

Many conventional techniques based on Flexible AC Transmission Systems (FACTS) are used to stabilize the power systems and hence damp the inter-area oscillations [8] - [10]. With increased penetration of wind energy in power systems, studies have been done to utilize the wind farm power output to alter spectrum of rotor angle swing [11] - [13]. Most of these techniques use the idea of controlling the Doubly Fed Induction Generators (DFIG) of a wind turbine to support the power system. Such control techniques do not consider the location of the wind power plant in existing power systems.

In [14], it is shown that the location (i.e. the electrical distance along the transfer path) of a wind farm greatly affects the spectrum of inter-area oscillation in power systems and it is possible to have an optimal location for wind farm installation to damp a given oscillation mode. Since installing a wind farm at particular location to damp a power system swing mode is not a practical idea, the idea of controlling the wind farm power output to alter spectrum of rotor angle swing to match required profile is proposed in [6].

In this paper, the power system framework proposed in [14] is used to study the effectiveness of an adaptive controller to damp the oscillations due to disturbances in a large radial power system.

II. POWER SYSTEM AS A DISTRIBUTED PARAMETER SYSTEM

A. Power System Modeling

In this section the rotor dynamics of large power system is modeled as a continuous function of rotor angle spread over spatial location.

Fig. 1. A power system model with disturbance at y and wind farm power injection at α

The power system under study is depicted in Fig. 1 where the generators in two different regions (Gₐ₁ and Gₐ₂) are connected with a transmission line and series of generators. The generation Gᵢ and rotor angle change δᵢ of each generator are continuously distributed over the length u. Assuming xᵢ and ΔL to be the line reactance and...
spacing between generators \( G_i \) and \( G_{i+1} \) respectively, the rotor dynamics of \( i^{th} \) generator can be expressed as:

\[
(2H_i \Omega_i) G_i \dot{\delta}_i + \xi \dot{\delta}_i = P_i \quad \forall i = 1, 2, \ldots, n \tag{1}
\]

where \( H_i \) is the inertia constant, \( \Omega_i = 2\pi 60 \text{ rad/sec} \) is the electrical frequency with 60 Hz base, \( P_i \) is the net real power flowing out of \( i^{th} \) machine with \( P_0 = 0 \) and \( \xi \) is the damping coefficient. Now, the real power flow from node \( i \) to \( i+1 \) over a lossless line is expressed as:

\[
P_{i,i+1} = \frac{E_i E_{i+1} \sin(\delta_i - \delta_{i+1})}{x_i} \tag{2}
\]

Substituting (3) into (1) and dividing by \( \Delta L \) we get:

\[
\frac{2}{\Omega_i} \frac{H_i}{\Delta L} \delta_i + \frac{\xi}{\Delta L} \dot{\delta}_i = \frac{1}{\Delta L} \frac{(\delta_i - \delta_{i-1}) - (\delta_i - \delta_{i+1})}{(\Delta L)^2}
\]

taking the limit as \( \Delta L \to 0 \) results in \( x_i \to x_i \) and

\[
\frac{2}{\Omega_i} \frac{dH(u)}{du} \frac{dG(u)}{du} \delta + \frac{d\xi(u)}{du} \dot{\delta} = \frac{1}{\Delta L} \frac{(\delta_i - 2(\delta_i + \delta_{i-1})}{(\Delta L)^2}
\]

\[
= \frac{1}{\Delta x} \frac{d^2H(u)}{du} \delta_{uu}
\]

where \( \frac{dG(u)}{du}, \frac{dH(u)}{du}, \frac{d\xi(u)}{du} \) and \( \frac{dx(u)}{du} \) are respectively the generation, inertia, damping and reactance densities over the string of generators. Considering an \( n \) link system each with infinitesimal length \( \Delta u_i \), \( i = 1, 2, \ldots, n \), the average inertia density can be defined as:

\[
H_T := \frac{1}{n} \sum_{i=1}^{n} \frac{dH(u)}{du} \Delta u
\]

the average reactance density is:

\[
\gamma := \frac{1}{n} \sum_{i=1}^{n} \frac{dx(u)}{du} \Delta u \tag{5}
\]

and the average damping density is:

\[
\eta := \frac{1}{n} \sum_{i=1}^{n} \frac{d\xi(u)}{du} \Delta u \tag{6}
\]

The generation density is approximated by the total generation and is denoted by \( G_T \). In the continuum as \( n \to \infty \), the densities in (5) - (7) can be expressed as:

\[
H_T = \frac{1}{L} \int_0^L dH(u) = \frac{H(L)}{L}; \quad \gamma = \frac{x(L)}{L}; \quad \eta = \frac{\xi(L)}{L}.
\]

Substituting these expressions into (4) yields a damped hyperbolic wave equation in terms of the aggregate generator angle \( \delta(u,t) \)

\[
\frac{\partial^2 \delta(u,t)}{\partial t^2} + \frac{\xi}{\gamma} \frac{\partial \delta(u,t)}{\partial t} = \nu^2 \frac{\partial^2 \delta(u,t)}{\partial u^2} \tag{8}
\]

with wave speed \( \nu = \sqrt{\frac{\gamma}{2H_T\Omega_T}} \).

The corresponding power flow is:

\[
P(u,t) = -\frac{1}{\gamma} \frac{\partial \delta(u,t)}{\partial u} \tag{9}
\]

The system (8) represents an unforced system. Now adding a power injection (from a wind farm) in (8):

\[
\frac{\partial^2 \delta(u,t)}{\partial t^2} + \frac{\xi}{\gamma} \frac{\partial \delta(u,t)}{\partial t} - \nu^2 \frac{\partial^2 \delta(u,t)}{\partial u^2} = W(u,t) \tag{10}
\]

where \( W(u,t) \) is the net power injection. Assuming the power is injected at a distance of \( \alpha \) as in Fig. 1, the net power injection can be expressed as:

\[
W(u,t) = P_g(t) \delta(u - \alpha) \tag{11}
\]

where \( \delta(u - \alpha) \) is the Dirac delta function modeling a point source injection and \( P_g(t) \) is the net power injected.

B. Approximation of Hyperbolic PDE

To solve the hyperbolic PDE of (10), the power angle \( \delta(u,t) \) is first expressed as the Fourier series:

\[
\delta(u,t) = \frac{1}{2} A_0(t) + \sum_{n=1}^\infty [A_n(t) \cos(k_n u) + B_n(t) \sin(k_n u)]
\]

(12a)

\[
W(u,t) = \frac{1}{2} P_0(t) + \sum_{n=1}^\infty [F_n(t) \cos(k_n u) + G_n(t) \sin(k_n u)]
\]

(12b)

where \( k_n \) are the wave numbers for each mode \( \lambda_k \). Now, assuming \( G_{A1} \) and \( G_{A2} \) produce a constant power, the power flow at the boundaries are constant in time. Then, the power angle \( \delta(u,t) \) is a standing wave with zero slope at \( u = 0 \) and \( u = 1 \) which yields the boundary conditions:

\[
P(0,t) = P(1,t) = 0 \tag{13}
\]

Now, using this boundary conditions (12a) and (12b) results in:

\[
\delta(u,t) = \sum_{n=1}^\infty A_n(t) \cos(k_n u) = \sum_{n=1}^\infty A_n(t) \phi_n(u) \tag{14a}
\]

\[
W(u,t) = \sum_{n=1}^\infty F_n(t) \cos(k_n u) = \sum_{n=1}^\infty F_n(t) \phi_n(u) \tag{14b}
\]

where \( \phi_n(u) = \cos(k_n u) \) are the spatial modes. Substituting (14a) and (14b) in (10) we get,

\[
\frac{\partial^2 A_n(t)}{\partial t^2} + \frac{\xi}{\gamma} \frac{\partial A_n(t)}{\partial t} + \nu^2 k_n^2 A_n(t) = F_n(t) \tag{15}
\]

where \( F_n(t) \) is obtained from the expression:

\[
F_n(t) = \frac{2}{L} \int_0^1 \cos(k_n u) \delta(u - \alpha) du = 2P_g \cos(k_n \alpha) \tag{16}
\]
C. Adaptive Control using Selective Modes

From (14a) it can be observed that the expression for power angle \( \alpha \) combination of spatial and temporal modes. The spatial modes are obtained using the boundary conditions whereas the temporal modes are dictated by the expression in (14). In this paper, the boundary condition (13) is considered which results in the approximate spatial modes discussed in [14]:

\[
\phi_k(u) = \cos(k_n u) = \cos \left( \frac{n\pi u}{L} \right)
\]  

(17)

Fig. 2 shows the plots of the first five spatial modes (17) for the normalized distance of \( L = 1 \).

Now based on the system configurations (14a), (16), (18), (19), and (20), we select an adaptive controller [15] that takes the rotor angle rate at specified spatial point as input and controls the injected power at that point as:

\[
u_p = G_c \dot{\delta} + G_D \dot{\gamma}_D
\]

(21)

where \( G_c \) and \( G_D \) are the adaptive gains based on the gain adaption laws:

\[
\dot{G_c} = -\delta \dot{\phi}_D^T \gamma_c
\]

\[
\dot{G_D} = -\delta \dot{\phi}_D^T \gamma_D
\]

(22)

In these expressions \( \gamma_c \) and \( \gamma_D \) are the positive definite gain matrices, \( \phi_D \) is the basis functions for the disturbance which is assumed to arise from a process having a form of:

\[
\left\{ \begin{array}{c}
u_D = \Theta z_D; \\
z_D = L_D \dot{\gamma}_D
\end{array} \right.
\]

(23)

where \( \Theta, L_D \) and \( F \) are the matrices that parameterize the known disturbance waveforms. For this particular problem, a step disturbance is considered which resulted in \( F = 0 \), \( \Theta = 1 \), and \( \phi_D = 1 \).

For the guaranteed stability and convergence of the adaptive controller (20) with gain adaption laws (21), the system under study has to be Almost Strict Positive Real (ASPR), i.e. the plant transfer function has to be minimum phase and the high frequency gains (product \( CB \)) are also scalars. The values of these gains are obtained as 0.5 and 0.000001 respectively by trial and error. The value of \( \gamma_D \) is comparatively smaller because we have not included a persistent disturbance for this particular problem.

III. SIMULATION

For the preliminary study examining the effectiveness of the adaptive controller in damping the inter-area oscillations (aggregate rotor angle swing), the system is setup according to the description in previous section. For the simulation, a total generation \( \dot{G}_T \) of 75000 MW with an inertia constant \( \Theta \) of 5 MWs/MVA for generators and 1 MWs/MVA for loads were considered. The impedance density is assumed to be \( 3.13 \times 10^{-3} \) and the propagation velocity is found to be 40 percent per second. These power system parameters were taken from [16]. Only the first five modes are used to illustrate the theory presented in previous sections. Since the measured rotor angle rate \( \dot{\delta} \) and \( \phi_D \) are scalars, both of the gains \( \gamma_c \) and \( \gamma_D \) are also scalars. The values of these gains are obtained as 0.5 and 0.000001 respectively by trial and error. The value of \( \gamma_D \) is comparatively smaller because we have not included a persistent disturbance for this particular problem.

A step power of magnitude 0.25 p.u. is injected at distance of 0.5 whereas the rotor angle rate is assumed to be measured at the distance of 0.15 and the control power is assumed to be injected at the same place where the rotor angle rate is measured.

The rotor angle profile for uncontrolled as well as controlled conditions with inclusion of first five modes is in Fig. 3. In this figure, the rotor angle is observed at normalized distance of 0.15.

Fig. 4 shows the zoom-in of Fig. 3. Here, it can be observed that the rotor angle is steady when the first three
modes are included. The addition of fourth mode has minimal effect but adding fifth modes introduces an oscillation in aggregate angle trajectory.

This change in aggregate rotor angle trajectory can be best described with the help of the spatial mode profiles shown in Fig. 2. As described in the previous section the adaptive controller needs the plant to be non-minimum phase to ensure stability and convergence of the closed loop system. On the other hand, the location of zero in the plant is affected by the sign of spatial modes at a particular distance when rotor angle rate is assumed to be measured. From Fig. 2 it can be observed that, the first three modes have positive signs when they are if the rotor angle rate is measured at normalized distance of 0.15, which leads to minimum phase zeros for the system. But, the fourth and fifth modes add non-minimum phase zeros which violate requirement of that the system have minimum phase zeros in order to guarantee convergence of the adaptive controller. It may well be the case that the overall bandwidth of the power system would limit this concern to only the first few modes. However, appropriate filtering included in the control loop can also be used to mitigate the problem, extending the controller for this purpose is a direction of future work.

IV. CONCLUSION AND FUTURE WORK

This paper investigated the use of an adaptive controller for the damping inter-area oscillation (modeled as the swing of an aggregate rotor angle)- in power system with wind power injection at a given spatial location. The control design assumed that the rotor angle rate at a particular location is available for the measurement. Moreover, it is observed that the location of measurement as well as where the wind power injection is applied of power greatly affects the particular modes present in the system. Certain locations can lead to modes with non-minimum phase zeros, which introduces a new implementation problem for the adaptive controller.

Future research direction involve investigating the efficacy of increasing the number of modes used to design the controller as well as using Residual Mode Filters (RMF) to filter out the modes that introduce non-minimum phase zeros. In addition, since the inter ? area oscillations only occur within a small range of low frequencies, a low pass filter might be used to filter out the high frequency modes, which would greatly simplify the analysis of control design.

Finally, since the control is actually done via the dynamics of a wind turbine farm future work will involve extending the analysis to include one of the low order wind farm models that exist in the literature.

REFERENCES


