Abstract— Our world consists of complex macro-systems of various natures. These systems are collection of various subsystems providing defined functions and interconnected by processes of forced dynamics interaction and exchange of power, matter and information. These macro-systems are mathematically nonlinear, multi-dimensional and multi-connected. In these systems there are complex transients and critical and chaotic modes has place. Problems of system synthesis, i.e. finding of common objective laws of control processes in a such dynamics system are much actual, complicated and, in many respects, practically inaccessible for present control theory. In the report we consider fundamental basis of nonlinear theory of system’s synthesis based on synergetic approach in modern control theory as well as its application.

I. INTRODUCTION

Directed self-organization implies control strategy that forms and keeps dynamic invariants either belonging to the system or external ones. Depending on the control aim, the invariants can be constant or variable causing stabilization or a new dynamic state respectively. Talking Biology, in the first case the system's invariants realize stabilizing selection, and in the second case – dynamic selection. In other words, purposeful forming of dynamic invariants allows performing purposive self-organization in the system.

In order to apply the ideas of Synergetics [1–5] in control theory, it is necessary to keep the conceptual correspondence to the main qualities of self-organization: nonlinearity – open systems – coherence. For control tasks the most important quality is that the system must be open.

II. THE PRINCIPLES

In the initial statement the control system is described by the differential equations of the object

$$\dot{x}(t) = F(x,u,q,M)$$

(1)

It includes state coordinates \(x(t)\) and some external forces consisting of sought controls \(u(t)\), setting actions \(q(t)\) and possibly the disturbing actions \(M(t)\). In order to move from the system “object–external forces” to forming the self-organization equations we must exclude these forces in an appropriate way. To do that we should extend the initial equations of the system “object–external forces” in such a way that excluded forces would become internal for the system. So for the new extended system its equations can become the self-organization equations. I.e. as a result of this extension we can move to organization of the system to its self-organization.

Such an extension takes place in the existing formulation of the control system synthesis problems, i.e. finding the control laws as a function of state coordinates of the extended system. These laws are the equations of the regulator and they should ensure the desired dynamic qualities of the closed-loop system “object–control law (regulator)”. Then we can apply relations characterizing the self-organization processes to the system (“object–regulator”) according to the qualities mentioned above.

So in order to apply the synergetic approach based on cooperative processes of self-organization to the control problems. It is necessary to move from initial control task including the object's equations and external forces (as control, setting and disturbing actions) to extended task statement, where the mentioned forces become internal interactions of the closed-loop system.

To do this we should present the external setting \(q(t)\) and disturbing \(M(t)\) actions as solutions of some additional differential equations describing the informational model. Doing this we perform their “submerging” into the general structure of the extended system. Then the control problem should be formed as a task of search for interaction laws between the components of the extended system ensuring appearing of self-organization processes. Specifically, this problem comes down to synthesis of appropriate closed-loop control laws \(u(x_1,...,x_n,w_1,...,w_\mu)\) as a function of state coordinates of the extended system. Here \(w_1,...,w_\mu\) - coordinates of the corresponding informational models of the setting and disturbing actions written as additional differential equations.

Then giving energy or matter to such system we can create an unbalanced situation necessary for emerging of directed self-organization processes. The mentioned extension of the initial system and forming the self-organization equations allows setting a connection between the ideas of Synergetics [1-6] and the problem of synthesis of nonlinear control systems basing on invariant relations. This means that synergetic control theory [6–10] is the theory of synthesis of closed-loop control systems based on forming the cohered cooperative processes in the systems of various natures.

According to the mentioned qualities of self-organization and to the state flow compression-decompression principle of the phase flow [6, 7] the basic statements of the synergetic approach to the synthesis of nonlinear dynamic systems are:
• firstly, forming the extended system of differential equations reflecting the processes of achieving the set values, suppressing the disturbances, optimization, coordinate observing etc.
• secondly, synthesis of such “external” controls that ensure the reduction of the extra degrees of freedom of the extended system with respect to the final manifold where the motion of the representing point is described by the equations of the system’s “internal” dynamics.
• thirdly, synthesis of “internal” controls by means of forming the links between the “internal” coordinates of the system. These links ensure the reaching of the control aim.

III. THE METHOD

The stated basic principles of the synergetic approach [6–10] lead to its stages shown below. First we write the initial differential equations of the object, e.g. in the following form:

\[
\begin{align*}
\dot{x}_k(t) &= f_k(x_1, \ldots, x_n) + M_k(t); k = 1, 2, \ldots, m-1, \ m \leq n, \\
\dot{x}_{k+1}(t) &= f_{k+1}(x_1, \ldots, x_n) + u_{k+1} + M_{k+1}(t); \\
\dot{x}_n(t) &= f_n(x_1, \ldots, x_n) + u_n + M_n(t),
\end{align*}
\]

(2)

where:
- \(x_1, \ldots, x_n\) are object’s state coordinates,
- \(u_{k+1}, \ldots, u_n\) are controls,
- \(M_k(t), \ldots, M_n(t)\) are disturbances,
- \(k = 1, 2, \ldots, m-1, \ m \leq n\).

Then we add \(\mu\) equations connected to the problem of prediction and suppression of disturbances in the system (2).

\[
\dot{w}_j(t) = g_j(w_1, \ldots, w_{\mu}, x_1, \ldots, x_n), \quad j = 1, \ldots, \mu
\]

Then, the task of forming the links between the equations of the initial object (1) and the equations of the disturbances.

After the selection of the links equations we get the extended system of differential equations

\[
\begin{align*}
\dot{w}_j(t) &= g_j(w_1, \ldots, w_{\mu}, x_1, \ldots, x_n); \\
\dot{x}_i(t) &= f_i(x_1, \ldots, x_n) + w_i; \\
\dot{x}_{i+1}(t) &= f_{i+1}(x_1, \ldots, x_n) + w_{i+1} + u_{i+1}; \\
\dot{x}_n(t) &= f_n(x_1, \ldots, x_n) + w_n + u_n,
\end{align*}
\]

(3)

where \(j = 1, \ldots, \mu; \ i = \mu + 1, \ldots, m - 1\).

Equations (3) allow to set the synthesis task of control laws \(u_{\mu+1}, \ldots, u_n\) allowing to suppress the disturbances \(M_k(t), \ldots, M_n(t)\) and ensuring the set dynamic qualities of the closed-loop system. It is necessary to synthesize such a control vector \(u(\mu+1, \ldots, u_n)\) that would ensure motion of the representing point (RP) of the extended object (3) from and arbitrary initial state (in some allowed area), first, to some manifolds \(\psi_S(x_1, \ldots, x_n, w_1, \ldots, w_{\mu}) = 0\) and then to the set state, e.g. origin point of the extended state space. On the trajectories of the closed-loop system motion minimum of some optimizing functional can be reached or some prime performance criteria can be satisfied. Asymptotic stability in some area or in the large must be ensured.

In the report we mention that RP motion of the synthesized system must satisfy the following system of functional equations:

\[
T_S\psi_S(t) + \phi_S(\psi_S) = 0, \quad s = 1, 2, \ldots, m. \tag{4}
\]

Functions \(\phi_S(\psi_S)\) in (4) are selected in such a way that: asymptotic stability in the large with respect to \(\psi_S = 0\) and \(\psi_S = B\) is ensured in the system (4); the desired performance criteria of RP’s movement to the attracting manifolds

\[
\psi_S(x_1, \ldots, x_n, w_1, \ldots, w_{\mu}) = 0, \quad s = 1, 2, \ldots, m
\]

where \(\psi_S\) are some aggregated variables.

There are no special limitations put on the selection of functions (4). It is important to underline that macro-variables \(\psi_S\) reflect synergetic (cooperative, coherent) qualities of the synthesized multi-level systems. This means that it would be rather reasonable to use the known laws of the natural systems, which manifest the qualities of coherence and cooperative activities.

We will be using synergetic ideology in order to solve the stated task. This method means that the RP of the system gets to the intersection of manifolds \(\psi_1 = 0, \ldots, \psi_m = 0\) as a result of action performed by “external” controls \(u_{\mu+1}, \ldots, u_n\). Moving along the intersection is described by the following equations of “internal” dynamics

\[
\begin{align*}
\dot{v}_j(t) &= g_j(w_{\mu+1}, \ldots, w_{\mu+m-1}, x_1, \ldots, x_{m-1}); \\
\dot{x}_i(t) &= f_i(x_1, \ldots, x_{m-1}, v_{i+1}, \ldots, v_n),
\end{align*}
\]

(5)

where \(v_{\mu+1}, \ldots, v_n\) are “internal” controls, \(j = 1, \ldots, \mu, \ i = \mu + 1, \ldots, m - 1\).

Considering the decomposed system (5) having the order of \(n + \mu - m\), we synthesize “internal” controls \(v_{\mu+1}, \ldots, v_n\) ensuring the dynamic qualities of RP’s motion along the
intersection of manifolds $\psi=0,...,\psi_m=0$. Synthesis of controls $v_{i+1},...,v_n$ is a separate task of controlling the sub-object (5). Consecutive-series totality of invariant manifolds is used for this purpose.

According to the principle of control preservation [7, 9], the internal controls $v_k$ have a constant dimension $\dim v_k = m$ that coincides with the dimension of the external controls. Internal controls influence the sub-object (5) decomposing it to the object of dimension $n-2m$ with its own controls. After that the process of consecutive decomposition continues until the RP of the object reaches the preset final manifold. As a result of the described procedure, we find the internal controls connected recursively. Knowing the controls $v_{i+1},...,v_n$, we can introduce the desired macro variables that can have, for example, the following linear form:

$$\psi_s = \gamma_{s1}(x_{i+1} - v_1) + ... + \gamma_{sm}(x_n - v_n), \quad s = 1,...,m \quad (6)$$

Basing on the functional equations of form (4) and on the desired macro variables $\psi_s$ (6) and with respect to the equations of the extended system (6) we find the external controls using the method of analytical design of aggregated regulators (ADAR) [6–15]

$$u_{i+1} = -f_{i+1}(x_1,...,x_n) - w_{i+1} - \frac{D_1}{D},$$

$$u_n = -f_n(x_1,...,x_n) - w_n - \frac{D_n}{D}, \quad (7)$$

where

$$D = \begin{bmatrix} \gamma_{11} & \gamma_{12} & ... & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & ... & \gamma_{2m} \\ ... & ... & ... & ... \\ \gamma_{m1} & \gamma_{m2} & ... & \gamma_{mm} \end{bmatrix} \neq 0, \quad (8)$$

$$D_1 = \begin{bmatrix} \Phi_1 & \gamma_{12} & ... & \gamma_{1m} \\ \Phi_2 & \gamma_{22} & ... & \gamma_{2m} \\ ... & ... & ... & ... \\ \Phi_m & \gamma_{m2} & ... & \gamma_{mm} \end{bmatrix} \neq 0, \quad (9)$$

for $\Phi_s \neq 0$, where

$$\Phi_s = \gamma_{s1}\dot{v}_1(t) + \gamma_{s2}\dot{v}_2(t) + ... + \gamma_{sn}\dot{v}_n(t) - \frac{1}{T_s}\varphi_s(\psi_s) \quad (11)$$

The relations (7)-(9) allow to find the vector control laws (7), which move the RP to the vicinity of the manifold's intersection $\psi_1 = 0,...,\psi_m = 0$. Motion along it is determined by the equations of internal dynamics (5). The control laws (7) together with link equations form the dynamic equations of an aggregated regulator that ensures the selective invariance of the closed loop system (3)–(7) to the disturbances $M_k(t),...,M_n(t)$. It also provides asymptotic motion stability and the desired qualities of transients.

IV. CONCLUSION

From the point of view of the problem of synthesis of control laws, the differences of the developed new approach are as follows. First, shift of the main attention to the behavior of the system on the attractors. This allows to decompose the system and therefore to simplify it. It lets us focus our attention on the stable asymptotic motion modes. Second difference is in cascade synthesis of parallel-consecutive group of internal controls, i.e. dynamically tied links of the synthesized system. When we use a synergetic approach, there is an internal process of self-control in the synthesized system, when the cascade sequence of internal controls compressing the volume of the phase flow is formed. This is performed starting from the external maximal possible area and going through the enclosed one into the other internal areas until the RP gets to the desired state of the system.

The new synergetic approach allowed to perform a certain breakthrough in the area of synthesis of multiply connected systems of continuous, discontinuous, discrete-time, selective-invariant, multi-criteria, terminal and adaptive control for nonlinear dynamic objects of various nature [6–9]. This approach found a specific application for solution of complex problems of control of nonlinear technical objects (flying apparatus, turbo-generators, robots, electric drives, technological aggregates etc.) and in the control tasks of ecology, biotechnology etc.

The developed ADAR method allowed to decide for the first time the famous problem of multivariable control for multidimensional nonlinear dynamic objects of an arbitrary nature. This method also allowed to solve the problem of synthesis of N. Wiener’s self-organizing nonlinear dynamical systems. On the basis of this method have been synthesized common objective laws [10] for control of various engineering objects, i.e. for energy [11, 12], electromechanical [13], aerospace [14, 15], and other systems. These laws provide the asymptotic stability of the closed-loop system as a whole, increased robustness and invariance of the worst external disturbances. This method by properties greatly exceeds the currently known methods.
for the synthesis of nonlinear control systems. The developed method was recognition of Russian and international public.

REFERENCES


