Economic MPC for a changing economic criterion

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Abstract—In the process industries it is often desirable that model predictive controllers (MPC) use a setpoint function that incorporates some types of economic criteria. In [1] it is proved that this kind of controller provides better economic performance than the standard setpoint-tracking MPC formulations. In [2] a Lyapunov function is provided for the economic MPC formulation. In [3], [4] an MPC for setpoint tracking is presented that ensures feasibility for a changing setpoint, enlarging the domain of attraction of the controller. In this paper, a new MPC controller is proposed, which is a hybrid of these two previous controllers, and inherits their best properties. Three examples are presented that demonstrate the advantages of the new formulation.

I. INTRODUCTION

Model predictive control (MPC) is one of the most successful techniques of advanced control in the process industries. This success is due to the control problem formulation: the use of the model to predict the expected evolution of the plant, the optimal character of the solution, and the explicit consideration of hard constraints in the optimization problem. Thanks to the recent developments of the underlying theoretical framework, MPC has become a mature control technique capable of providing controllers that ensure stability, robustness, constraint satisfaction, and tractable computation for linear and for nonlinear systems [5].

The control law is calculated by predicting the evolution of the system and computing the admissible sequence of future control inputs that make the predicted trajectory satisfy the constraints and minimize the predicted cost. This problem can be posed as a mathematical programming problem. To obtain a feedback law, the obtained sequence of control inputs is applied in a receding horizon manner, solving the optimization problem at each sample time. Considering a suitable penalty of the terminal state and an additional terminal constraint, asymptotic stability and constraints satisfaction of the closed loop system can be established [6], [7]. Moreover, if the terminal cost is the infinite-horizon optimal cost of the unconstrained system, then the MPC control law is optimal in a neighborhood of the steady state. This property is the so-called local optimality property and allows design of finite horizon MPC controllers for constrained system with local optimal closed-loop performance [8], [9].

Most of the results on MPC consider the regulation problem, that is steering the system to a fixed steady state (typically shifted to the origin), but when the target operating point changes, the feasibility of the controller may be lost and the controller may fail to track the reference [10], [11], [12], [13]. This loss of feasibility may be a consequence of one or both of the following: (i) the stabilizable terminal controller design depends on the steady state and stability might be lost when the operating point changes, and (ii) the terminal region at the new setpoint could be unreachable in N steps, which means that the optimization problem is infeasible. Therefore, this may require an on-line re-design of the controller for each setpoint, which can be computationally expensive. A novel MPC for tracking has been proposed in [3], [4] for linear systems and in [14] for nonlinear systems. This controller is able to lead the system to any admissible setpoint in an admissible way. The main characteristics of this controller are: an artificial steady state considered as a decision variable, a stage cost that penalizes the error with the artificial steady state, an additional term that penalizes the deviation between the artificial steady state and the target steady state (the offset cost function), an invariant set for tracking considered as an extended terminal constraint. This controller ensures that under any change of the steady-state target, the closed loop system maintains the feasibility of the controller, ensures the convergence to the target if admissible (and the offset cost function minimization if it is not), and inherits the optimality property of the MPC for regulation.

Recently, a new MPC formulation aimed to consider an economic performance stage cost instead of a tracking error stage cost, has been proposed in [1], [2]. In [1], [2] the authors show that this controller is stable and asymptotically steers the system to the economically optimal admissible steady state, and that the controlled system exhibits better performance with respect to the setpoint than standard target-tracking MPC formulations. In [2] the authors also show that the economic MPC schemes admit a Lyapunov function to establish stability properties. If the economic criterion changes, the economically optimal admissible steady state where the controller steers the system may change, and the feasibility of the controller may be lost. In this paper, an economic MPC for a changing economic criterion is presented. This controller inherits the feasibility guarantee of the MPC for tracking ([3], [4]) and the optimality of the economic MPC ([1], [2]).

The paper is organized as follows. In section II the problem is stated and in section III the economic MPC is presented.
In sections IV the new economic MPC is described. In section V the local optimality property is described. Finally, two illustrative examples and conclusions of this study are presented in sections VI and VII.

II. PROBLEM STATEMENT

Consider a system described by a nonlinear invariant discrete time model

\[ x^+ = f(x, u) \]  (1)

where \( x \in \mathbb{R}^n \) is the system state, \( u \in \mathbb{R}^m \) is the current control vector and \( x^+ \) is the successor state. We assume that the function model \( f(x, u) \) is continuous. The solution of this system for a given sequence of control inputs \( u \) and initial state \( x \) is denoted as \( x(j) = \phi(j, x, u) \) where \( x = \phi(0, x, u) \).

The system of the state and the control input applied at sampling time \( k \) are denoted as \( x(k) \) and \( u(k) \) respectively.

The system is subject to hard constraints on state and input:

\[ x(k) \in X, \quad u(k) \in U \]  (2)

for all \( k \geq 0 \), where \( X \subset \mathbb{R}^n \) and \( U \subset \mathbb{R}^m \) are closed sets.

The steady state, input and output of the plant \((x_s, u_s)\) are such that (1) is fulfilled, i.e. \( x_s = f(x_s, u_s) \).

We define the set of admissible equilibrium states as

\[ \mathcal{Z}_s = \{ (x_s, u_s) : x_s \in X, u_s \in U \text{ and } x_s = f(x_s, u_s) \} \]  (3)

\[ \mathcal{X}_s = \{ x_s \in X : \exists u_s \in U \text{ such that } x_s = f(x_s, u_s) \} \]  (4)

The controller design problem consists of deriving a control law that minimizes a given performance cost index

\[ J(x, u) = \sum_{j=0}^{N-1} l(x_j, u_j) \]

where \( l(x, u) \) defines the economic stage cost.

The model is assumed to satisfy the following:

**Assumption 1:**

1) The model function \( f(x, u) \) and the economic stage cost function \( l(x, u) \) are Lipschitz continuous in \((x, u)\); that is there exist Lipschitz constants \( L_f, L_l > 0 \) such that, for all \( x, x_0 \in X \) and \( u, u_0 \in U \):

\[ |f(x, u) - f(x_0, u_0)| \leq L_f |x - x_0| \]

\[ |l(x, u) - l(x_0, u_0)| \leq L_l |x - x_0| \]

2) The system is weakly controllable at any admissible equilibrium point in \( \mathcal{Z}_s \) such that there exists a \( K_\infty \)-function \( \gamma \) such that

\[ |u - \hat{u}| \leq \gamma(|x - \hat{x}|) \]

for all \((\hat{x}, \hat{u}) \in \mathcal{Z}_s\).

Given the economic stage cost, the economic controller should steer the system to the optimal reachable steady state, which is defined as follows:

**Definition 1:** The optimal reachable steady state and input, \((x_s, u_s)\), satisfy

\[ (x_s, u_s) = \arg \min_{x, u} J(x, u) \]

s.t. \( x = f(x, u) \)

\( x \in X, \quad u \in U \)

III. ECONOMIC MPC

In the economic MPC the stage cost is an arbitrary economic objective, which does not necessarily penalizes the tracking error to the optimal target \((x_s, u_s)\). The economic MPC control law is derived from the solution of the optimization problem \( P_N(x) \)

\[
\begin{align*}
\min_{u(0), \ldots, u(N-1)} & \sum_{j=0}^{N-1} l(x(j), u(j)) \\
\text{s.t.} & x(0) = x, \\
& x(j + 1) = f(x(j), u(j)), \\
& x(j) \in X, u(j) \in U, \quad j \in \mathbb{N}_{0:N-1} \\
& x(N) = x_s
\end{align*}
\]

The economic MPC control law is given by the receding horizon application of the optimal solution \( \kappa_N(x) = u^*(0; x) \).

The standard Lyapunov arguments to prove asymptotic stability of MPC cannot be used in this case because the optimal cost is not necessarily decreasing along the trajectory. In the recent paper [2], asymptotic stability of economic MPC is established using Lyapunov arguments. In order to find a suitable Lyapunov function, the following *rotated* stage cost function is defined:

**Assumption 2:** Let \( L_r(x, u) \) be the rotated stage cost function [2] given by

\[ L_r(x, u) = l(x, u) + \lambda^T (x - f(x, u)) - l(x, u) \]

where \( \lambda \) is a multiplier that ensures the rotated cost exhibits a unique minimum at \((x_s, u_s)\) for all \( x \in X \) and \( u \in U \) and moreover there exist two \( \mathcal{K} \) functions \( \alpha_1 \) and \( \alpha_2 \) such that

\[ L_r(x, u) \geq \alpha_1(|x - x_s|) + \alpha_2(|u - u_s|) \]

In [2] it is demonstrated that the predictive control law derived from the following optimization problem \( P_N^\kappa(x) \)

\[
\begin{align*}
\min_{u(0), \ldots, u(N-1)} & \sum_{j=0}^{N-1} L_r(x(j), u(j)) \\
\text{s.t.} & x(0) = x, \\
& x(j + 1) = f(x(j), u(j)), \\
& x(j) \in X, u(j) \in U, \quad j \in \mathbb{N}_{0:N-1} \\
& x(N) = x_s
\end{align*}
\]

is identical to the economic predictive control law, and that the optimal cost function is a Lyapunov function, which demonstrates asymptotic stability as in standard MPC.

When the economic objective of the controller changes, the optimal admissible steady state \((x_s, u_s)\) (to which the system should be steered by the controller) changes as well. This change may cause a loss of feasibility of the controller.

Recently in [14] an MPC for tracking constrained nonlinear systems has been presented. This controller provides stability and convergence to the setpoint under changing operating points.

The main objective of this paper is to combine the economic MPC proposed in [2] with the MPC for tracking proposed in [14] in such a way that the combined controller inherits the advantages of both formulations.
IV. ECONOMIC MPC FOR A CHANGING ECONOMIC CRITERION

As proposed in [2], a rotated stage cost function is used, but modified to include the tracking formulation. To this aim, a modified stage cost function is introduced as follows:

**Definition 2:**

\[ L_t(z, v) = L_r(z + x_s, v + u) \]

This stage cost function satisfies the following properties:

**Property 1:**

1. \( L_t(x - x_s, u - u_s) = L_r(x, u) \)
2. \( L_t(0, 0) = L_r(x_s, u_s) = 0 \)
3. \( L_t(z, v) \geq \alpha_1(|z|) + \alpha_2(|v|) \) for certain \( \alpha_1 \) and \( \alpha_2 \).

As in [3], [4], [14], an offset cost function is also considered in this problem and it is defined as follows:

**Definition 3:** Let \( V_O(x, u) \) be a positive definite function such that the unique minimizer of

\[
\min_{(x,u) \in Z_s} V_O(x, u)
\]

is \( (x_s, u_s) \).

The economic MPC for a changing economic criterion is then defined by the following optimization problem \( P_N^t(x) \)

\[
\min_{u(0), \ldots, u(N)} \sum_{k=0}^{N-1} L_t(x(j) - x(N), u(j) - u(N)) + V_O(x(N), u(N))
\]

s.t.

- \( x(0) = x \)
- \( x(j + 1) = f(x(j), u(j)) \), \( j \in \mathbb{I}_{0:N} \)
- \( x(j) \in X, u(j) \in U \), \( j \in \mathbb{I}_{0:N} \)
- \( x(N) = x(N + 1) \)

The control law is given by \( k_t^*(x) = u^*(0; x) \).

Notice that the pair \( (x(N), u(N)) \) in \( P_N^t(x) \) defines an admissible equilibrium point, i.e. \( (x(N), u(N)) \) is an equilibrium point contained in \( Z_s \). Define the set of states that can be admissibly steered to \( x(N) \) in \( N \) steps, i.e.

\[
\mathcal{X}_N(x(N)) = \{ x \in X : \exists u = \{ u_0, \ldots, u_{N-1} \} \phi(k, x, u) \in X, u_k \in U, \forall k \in \mathbb{I}_{0:N-1}, \phi(N, x, u) = x(N) \}.
\]

The feasibility region of the optimization problem is a compact set given by

\[
\mathcal{X}_N = \bigcup_{x(N) \in X_s} \mathcal{X}_N(x(N))
\]

Consequently, the feasibility region of the proposed controller is (potentially) larger than that of the economic MPC [2].

We have the following theorem establishing the asymptotic stability of the proposed economic MPC for tracking.

**Theorem 1:** If assumptions 1 and 2 hold, then \((x_s, u_s)\) is an asymptotically stable equilibrium point for the controlled system and its domain of attraction is \( \mathcal{X}_N \).

**Proof:** (Sketch of the proof)
A complete proof of the theorem can be found in [15].

First, consider that \( x_k \in \mathcal{X}_N \) and define the optimal solution of \( P_N^t(x_k) \) as \( \bar{u}^0_s(x_k) \) and \( V_N^0(x_k) \) as the optimal cost function.

For the next state \( x_{k+1} = f(x_k, k_t^*(x_k)) \), let define a sequence of future control inputs as \( \bar{u} = \{ u^0(1; x_k), \ldots, u^0(N-1; x_k), u^0(N; x_k), u^0(N; x_k) \} \). It is easy to derive that \( \bar{u} \) is a feasible solution for \( P_N(x_{k+1}) \).

Therefore, \( \mathcal{X}_N \) is an admissible positive invariant set. Based on this feasible solution and the optimality property, the cost function ensures that

\[
V_N^0(x_{k+1}) - V_N^0(x_k) \leq -\alpha_1(|x_k - x^0(N; x_k)|) - \alpha_2(|u_k - u^0(N; x_k)|)
\]

Since the optimal cost function is a positive definite function (that is, \( V_N^0(x_k) \geq \alpha_1(|x_k - x^0(N; x_k)|) \)) then

\[
\lim_{k \to \infty} |x_k - x^0(N; x_k)| = 0, \quad \lim_{k \to \infty} |u_k - u^0(N; x_k)| = 0
\]

Then, resorting on the continuity of the model, it can be proved that the system converges to an equilibrium point \((x_{\infty}, u_{\infty}) \in Z_s \) such that \( x_{\infty} = x^0(N; x_{\infty}) \) and \( u_{\infty} = u^0(N; x_{\infty}) \).

Secondly, using optimality arguments, it can be proved that the equilibrium point of the closed-loop system \((x_{\infty}, u_{\infty}) \) is \((x_s, u_s)\).

Finally, using assumption 1 it is proved that that \((x_s, u_s)\) is a stable equilibrium point (in the Lyapunov sense) for the closed-loop system.

**Property 2 (Changing economic criteria):** Since the set of constraints of \( P_N^t(x) \) does not depend on \((x_s, u_s)\), the proposed controller is able to guarantee the recursive feasibility, admissibility and asymptotic stability for any change on-line of the economic criterion.

**Remark 1:** The optimization problem \( P_N^t(x) \) could also be written following the notation introduced in [3], [4], [14] where an artificial equilibrium point is added as decision variables. In \( P_N^t(x) \) \((x(N), u(N)) \) plays the role of the artificial equilibrium point. This new notation has been adopted for the sake of clarity of the presentation, but notice that if the equality terminal constraint is not used, then this notation can not be used.

V. LOCAL ECONOMIC OPTIMALITY

The proposed economic MPC for a changing economic criterion may be considered as a suboptimal controller (with respect to the setpoint) due to the stage cost to minimize. As demonstrated in the following property, however, under mild conditions on the offset cost function \( V_O(\cdot) \), the proposed controller ensures the economic optimality property as in [2].

**Assumption 3:** There exist two positive constants \( \alpha_1, \alpha_2 \) such that

\[
\alpha_1(|x - x_s| + |u - u_s|) \leq V_O(x, u) \leq \alpha_2(|x - x_s| + |u - u_s|)
\]

for all \((x, u) \in Z_s \).

**Property 3 (Local optimality):** Consider that assumptions 1, 2 and 3 hold and assume that
\( x_0 \in X_N \). Then there exists a \( \alpha^0 > 0 \) such that for all \( \alpha_1 \geq \alpha^0 \) and for all \( x_k \in X_N(x_s) \) the proposed economic MPC for tracking is equal to the economic MPC, i.e. \( \kappa^*_N(x_k) \).

**Proof:** First, notice that \( P_N^0(x) \) can be rewritten as follows:

\[
\min_{u(0), \ldots, u(N-1), u(N)} \sum_{j=0}^{N-1} L_t(x(j) - x(N), u(j) - u(N)) + V_0(x(N), u(N))
\]

s.t.
\[
x(0) = x,
\]
\[
x(j + 1) = f(x(j), u(j)), \quad j \in \mathbb{I}_{0:N}
\]
\[
x(j) \in X, \quad u(j) \in U, \quad j \in \mathbb{I}_{0:N}
\]
\[
x(N) = x(N + 1), \quad |x(N) - x_s| + |u(N) - u_s| = 0
\]

Let \( \nu(x) \) be the Lagrange multiplier of the equality constraint \( |x(N) - x_s| + |u(N) - u_s| = 0 \) of the optimization problem \( P_N^0(x) \). We define the following constant \( \alpha^0 \)
\[
\alpha^0 = \max_{x \in X_N(x_s)} |\nu(x)|
\]

Define the optimization problem \( P_N^0(x) \) as a particular case of \( P_N^0(x) \) with \( V_0(x, u) \triangleq \alpha^0(|x - x_s| + |u - u_s|) \). This optimization problem results from \( P_N^0(x) \) with the last constraint posed as an exact penalty function. Therefore, in virtue of the well-known result on the exact penalty functions [16], taking any \( \alpha \geq \alpha^0 \) we have that \( V_{N,\alpha}(x) = V_{N,\alpha}^0(x) \) for all \( x \in X_N(x_s) \).

From assumption 3 and the optimality property, we have that
\[
V_{N,\alpha_1}^0(x) \leq V_{N,\alpha}^0(x) \leq V_{N,\alpha_2}^0(x)
\]

Since \( \alpha_2 \geq \alpha_1 \geq \alpha^0 \), we have that
\[
V_{N,\alpha}^0(x) \leq V_{N}^0(x) \leq V_{N,\alpha_2}^0(x)
\]

and both optimization problems provide the same optimal solution, i.e. \( \kappa^*_N(x) = \kappa^*_N(x) \), for all \( x \in X_N(x_s) \).

VI. ILLUSTRATIVE EXAMPLES

In this section three examples are presented. The first one shows that the economic MPC for a changing economic criterion inherits the large feasible set associated with MPC for tracking ([3] and [4]). In the second example, the local optimality property is demonstrated. The third example shows that the economic MPC for a changing economic criterion inherits the economic optimality property of the economic MPC ([11] and [2]).

A. Feasibility: the CSTR case

The system considered is a continuous stirred tank reactor (CSTR), [17], [18]. Assuming constant liquid volume, the CSTR for an exothermic, irreversible reaction, \( A \rightarrow B \), is described by the following model:

\[
\dot{C}_A = \frac{q}{V}(C_{Af} - C_A) - k_{ce}(\frac{C_A}{RT})C_A
\]

\[
\dot{T} = \frac{q}{V}(T_f - T) - \frac{\Delta H}{\rho C_p}k_{ce}(\frac{C_A}{RT})C_A + \frac{UA}{V \rho C_p}(T_c - T)
\]

where \( C_A \) is the concentration of \( A \) in the reactor, \( T \) is the reactor temperature and \( T_c \) is the temperature of the coolant stream. The nominal operating conditions are: \( q = 100 \) l/min, \( T_f = 350 \) K, \( V = 100 \) l, \( \rho = 1000 \) g/l, \( C_p = 0.239 \) J/g K, \( \Delta H = -5 \times 10^4 \) J/mol, \( E/R = 8750 \) K, \( k_0 = 7.2 \times 10^{10} \) min\(^{-1} \), \( UA = 5 \times 10^3 \) J/min K and \( C_{Af} = 1 \) mol/l.

The objective is to regulate \( y = x_2 = T \) and \( x_1 = C_A \) by manipulating \( u = T_c \). The constraints are \( 0 \leq C_A \leq 1 \) mol/l, \( 280 \leq T \leq 370 \) K and \( 280 \leq T_c \leq 370 \) K. The nonlinear discrete time model of system (5) is obtained by discretizing equation (5) using a 5-th order Runge-Kutta method and taking as sampling time 0.03 min. The set of reachable output is given by 304.17K \( \leq T \leq 370 \) K. The outputs in this range are all controllable.

The economic stage cost function is
\[
\|x - x_s\|_Q + \|u - u_{sp}\|_R^2
\]

with \( Q = diag(1, 1/100) \) and \( R = 1/100 \) as weighting matrices. The function \( V_O = \alpha|x - x_s|_\infty \) has been chosen as the offset cost function. The controller has been implemented in MATLAB 7.8 and the function \texttt{fmincon} has been used to solve the optimization problem.

In Figure 1 the the evolution of the system for a change of setpoint is plotted. The system has been considered to be steered from \( x_0 = (0.7950, 332) \), \( u_0 = 302.8986 \), to \( u_{sp} = 400 \), and then to \( y_{sp} = 300 \). Both setpoints are unreachable. The optimal equilibrium points are \( x_s = (0.2057, 370) \), and \( x_s = (0.9774, 304.17) \). A horizon \( N = 3 \) has been used. The evolution of the system (solid line), the artificial reference\(^1\) \( x^0(N; x_s) \) (dashed-dotted line), and the real reference (dashed line) are shown. Notice that the controller steers the system to the extremes of the reachable range, even with the short horizon.

\[\text{Fig. 1. Time evolution of } T \text{ and } C_A.\]

\[\text{Fig. 2. Time evolution of } T \text{ and } C_A.\]

In figure 2(a), 2(b) and 3(a) the feasible sets are shown for \( N = 2, N = 10 \) and \( N = 17 \), respectively. These regions have been estimated solving a Phase I problem [19] in a grid. The dashed line represents the set of equilibrium point of the system. The controller has been compared with the economic

\[\text{As stated in [14], the terminal state and input } (x^0(N; x_s), u^0(N; x_s)) \text{ is an equilibrium point that can be understood as an artificial reference that the system could reach with a feasible evolution.}\]
MPC presented in [2]. The resulting feasible set for \(N = 17\) is shown in figure 3(b). Notice that this set is smaller than those obtained with the economic MPC for tracking, thus showing that the new formulation has increased the domain of attraction of the controller.

\[V_N^x(x; u) = \sum_{i=0}^{N-1} \|x(i)-x_{sp}\|^2_Q + ||u(i)-u_{sp}\|^2_R + V_0(x(N), u(N))\]

\[V_N^o(x) = \sum_{i=0}^{N-1} \|x(i)-x(N)\|^2_Q + ||u(i)-u(N)\|^2_R + V_0(x(N), u(N))\]

The offset cost function considered for the MPC for tracking and the economic MPC for tracking is \(V_0 = \alpha \|x_s-x_{sp}\|\infty\).

The controllers performances have been assessed using the following closed-loop control performance measure:

\[\Phi = \sum_{k=0}^{T} \|x(k)-x_{sp}\|^2_Q + ||u(k)-u_{sp}\|^2_R + (\|x_s-x_{sp}\|^2_Q + \|u_s-u_{sp}\|^2_R)\]

where \(T\) is the simulation time.

The system considered is the double integrator:

\[A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 & 0.5 \\ 1.0 & 0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\]

Hard constraints on state and input have been considered: \(\|x\|\infty \leq 5\) and \(\|u\|\infty \leq 0.3\).

The setpoint considered is \(x_{sp} = (4.85; 3)\), which is unreachable. The optimal steady state, for the considered set of constraints, is \(x^0 = (4.85; 0.15)\). The MPC parameters are \(Q = I_2\) and \(R = 1.1I_2\). The simulation time is \(T = 50\).

The initial condition considered is \(x_0 = (-4; 2)\). A horizon \(N = 10\) has been used.

The closed-loop performances of the three controllers are shown in table I.

The performance of the new formulation is equal to the economic MPC performance, while the MPC for tracking...
TABLE I
COMPARISON OF CONTROLLER PERFORMANCE

<table>
<thead>
<tr>
<th>Measure</th>
<th>E-MPC</th>
<th>MPC</th>
<th>E-MPCT</th>
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<tbody>
<tr>
<td>Φ</td>
<td>139.7150</td>
<td>150.3770</td>
<td>139.7150</td>
</tr>
</tbody>
</table>

has significantly worse performance. This shows how the new formulation inherits the optimality with respect to the setpoint from the economic MPC.

If a shorter horizon is considered, i.e. N = 5, the initial state is infeasible for the economic MPC controller, but remains feasible for the other two controllers. The closed-loop performances for this case are shown in table II.

TABLE II
COMPARISON OF CONTROLLER PERFORMANCE

<table>
<thead>
<tr>
<th>Measure</th>
<th>E-MPC</th>
<th>MPC</th>
<th>E-MPCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ</td>
<td>164.1020</td>
<td>152.5934</td>
<td></td>
</tr>
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Notice that the new formulation again gives better closed-loop performance compared to MPC for tracking.

In figures 5(a) and 5(b) the state trajectories and the feasible sets are shown for the two cases. The E-MPCT feasible set (solid line) and economic MPC feasible set (dashed line) are shown. Notice that the E-MPCT provides a larger feasible set than economic MPC. The state evolution of the E-MPCT and of the economic MPC are drawn respectively in dashed-dotted and dashed lines. The MPC for tracking state evolution is drawn in dotted line. The (unreachable) setpoint \(x_{sp}\) and the optimal steady state \(x_s\) are drawn respectively as a dot and a star. Notice that the cost of E-MPCT is identical to the economic MPC evolution, establishing that the two controllers have the same performance with respect to the setpoint.

![Feasible sets for N = 5.](image)
![Feasible sets for N = 10.](image)

This example illustrates the two main properties of the economic MPC for tracking: it provides optimality with respect to the setpoint as in economic MPC, and a large feasible regions as in the MPC tracking formulation.

VII. CONCLUSIONS

In this paper, an MPC that handles a changing economic criterion has been presented, which is a hybrid of the MPC for setpoint tracking ([3], [4]) and the economic MPC ([1], [2]). This paper has shown how the new formulation inherits the main properties of the two other controllers: the feasibility guaranty of the MPC for tracking and the optimality with respect to the setpoint of the economic MPC. Asymptotic stability of the controller has been established. Three illustrative examples have been presented showing the advantages of the new controller.

REFERENCES