Control of Energy Systems as Distributed Parameter Systems with Software Support by Virtual Software Environments

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Abstract — Thanks to development of information technology, the so-called virtual software environments offer wide possibilities for the estimation of time-space dynamical characteristics of energy systems, including modeling, control and design of distributed parameter systems. Based on these advances, we present a novel approach to control energy systems as lumped-input and distributed-parameter-output systems. An adaptive-predictive controller is deployed to control temperature fields to ensure optimal conditions for the desulphurization process in a coal-burning fluidized bed furnace, demonstrating the potential of the proposed methodology.

I. INTRODUCTION

The significant development of information technology opened a wide space for the development of numerical methods for solving nonlinear partial differential equations (PDE) and for building virtual software environments based on the finite element method (FEM) or computational fluid dynamics (CFD). These tools serve the numerical analysis of the dynamics of energy systems (ES) in temporal and spatial relations. Numerical solutions of PDE defined on complex shape definition domains in 3D initiated new directions in the development of engineering methods for the control of distributed parameter systems (DPS) [1, 6, 10]. Generally, in engineering practice distributed-parameter objects are being controlled by lumped, discrete actuators and together they form lumped-input and distributed-parameter-output systems (LDS) (Fig. 1). In recent works [2–5], engineering methods for the control of LDS have been developed, along with software support in the MATLAB & Simulink environment. The aim of this paper is to demonstrate the potential of these results for realizing innovations in ES control methodology and development of software support for control of ES as LDS.

Figure 1. Lumpied-input and distributed-parameter-output system – LDS

In the framework of the proposed approach we shall demonstrate:

- The general decomposition of dynamics of ES to time and space components, using the LDS formulation
- The methodology for controlling ES as LDS, by separating the control synthesis to time-domain and space-domain
- Possibilities for generating distributed-parameter dynamic characteristics of ES in virtual software environments
- Capabilities of the software tool developed for the design of control systems of ES as LDS in the MATLAB & Simulink software environment
- Results of adaptive-predictive control of the bed temperature field in a fluidized bed desulphurization process [11, 12].

In engineering practice ES are usually found as dynamic systems of considerable geometrical dimensions, whose state and/or output quantities are given as quantity fields. From the viewpoint of systems and control theory, these systems are DPS. Analysis of the structural organization of these systems shows that in input-output relation these systems are very often lumped-input and distributed-parameter-output systems. For example, in a fluidized bed combustor (Fig. 2.),

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the controlled variable, the distributed quantity, is the temperature field of the bed, while the fuel feed rates of each fuel feeder are lumped input parameters. Similarly, in a nuclear reactor, (Fig. 3.), the neutron flow density, system state, has a distributed nature, while the position of each control bar is a discrete (lumped) input to the system. A further example would be the Tokamak. The spatially distributed magnetic field keeping the hot plasma in place is controlled by currents flowing through individual coils (Fig. 4).

II. DECOMPOSITION OF DYNAMICS

Let us examine the dynamics of ES as LDS, given on complex shape definition domain in 3D. By utilizing FEM based virtual software environments, transient responses of the system to a step change in each individual manipulating variable, i.e. input quantity, are obtained in the form of a discrete time sequence. Normalizing these data we obtain the system’s distributed parameter step responses

\[
\{\mathcal{H}_i(x,k)\}_{i,k}.
\]  

(1)

By subtracting the shifted distributed-parameter step characteristics distributed-parameter impulse characteristics of LDS with zero order hold units H: HLDS are obtained

\[
\{\mathcal{G}_iH_i(x,k)\}_{i,k} = \{\mathcal{H}_i(x,k) - \mathcal{H}_i(x,k-1)\}_{i,k}.
\]  

(2)

Then the system’s dynamics is described by the sum

\[
YH(x,k) = \sum_{i=1}^{n} YH_i(x,k) = \sum_{i=1}^{n} \sum_{q=0}^{i-1} \mathcal{G}_iH_i(x,q)U_i(k-q),
\]  

where \(\{YH_i(x,k)\}_{i=1,n}\) are distributed-parameter responses to individual input quantities, which are expressed in the form of a convolution sum.

For simplicity, in the following figures, the system is considered on the one-dimensional interval \([0,L]\), nevertheless, this result is generally valid for continuous as well as discrete DPS given on complex 3D definition domains. Let us introduce the term partial distributed-parameter impulse response, which is defined as the system’s response in the point with the highest gain, which is generally the one closest to the actuator, (Fig. 5).

**Figure 3.** Nuclear Reactor

**Figure 4.** Tokamak

\[
\{\mathcal{G}_HR_i(x,k)\}_{i,k} = \left\{ \max_{x_i} \{\mathcal{G}_iH_i(x,k)\} \right\}_{i,k}.
\]  

(4)

These will represent time components of the system’s dynamics. Furthermore, we introduce the reduced partial distributed-parameter impulse responses, which represent the system’s dynamics in space

\[
\{\mathcal{G}_HR_i(x,k)\}_{i,k} = \left\{ \frac{\mathcal{G}_iH_i(x,k)}{\mathcal{G}_iH_i(x_0,k)} \right\}_{i,k},
\]  

(5)

for \(\{\mathcal{G}_iH_i(x,k) \neq 0\}_{i,k}\). Then the system output can be expressed as

\[
YH(x,k) = \sum_{i=1}^{n} \sum_{q=0}^{i} \mathcal{G}_iH_i(x,q)\mathcal{G}_HR_i(x,q)U_i(k-q).
\]  

(6)

Analogously, for the distributed-parameter step responses we introduce reduced steady-state values of distributed-parameter step responses

\[
\{\mathcal{H}_HR_i(x,\infty)\}_{i,k} = \left\{ \frac{\mathcal{H}_iH_i(x,\infty)}{\mathcal{H}_iH_i(x_0,\infty)} \right\}_{i,k}.
\]  

(7)

**Figure 5.** \(i\)-th discrete distributed-parameter impulse characteristics of LDS and zero order holds H: HLDS
where $\{\mathcal{H}_i(x, \infty) \neq 0\}_i$ are transient responses in steady-state in points with the highest gain. The system’s steady-state output becomes

$$Y_H(x, \infty) = \sum_{i=1}^{n} Y_{iH}(x, \infty) \mathcal{H}_i \mathcal{H}_i(x, \infty), \quad (8)$$

where $\{Y_{iH}(x, k)\}_{i,k}$ are partial distributed output quantities in time domain in the highest-gain points and their corresponding transfer functions are $\{\mathcal{SH}_i(x, z)\}_i$, (Fig. 6).

### III. CONTROL SYNTHESIS

The dynamics of controlled ES as LDS is decomposed into time components $\{\mathcal{GH}_i(x, k)\}_{i,k}$, $\{\mathcal{SH}_i(x, z)\}_i$ and space components $\{\mathcal{GH}_i(x, k)\}_{i,k}$, $\{\mathcal{HR}_i(x, \infty)\}_i$ thus allowing the control synthesis problem to be solved separately in space domain and time domain. Figure 7. shows the typical control loop arrangement.

#### A. Space-domain Synthesis

Based on (8), the output/state of the controlled system is expressed as a weighted sum of partial distributed-parameter responses. In fact, these weights express the contribution of each actuating member to the overall output/state. The Space Synthesis block solves the optimal approximation of the system’s output, reference or control error, depending on the control loop configuration, by the base $\{\mathcal{HR}_i(x, \infty)\}_i$.

$$\{\hat{E}_i(k)\}_i = \arg \min_{\{E_i(k)\}_i} \left\| E(x, k) - \sum_{i=1}^{n} \hat{E}_i(k) \mathcal{HR}_i(x, \infty) \right\|, \quad (9)$$

thus finding the optimal approximation coefficients, which are a transformation of the distributed control error to a vector of lumped quantities. This transformed control error is processed by the time domain control system.

#### B. Time-domain Synthesis

The control problem is solved in time domain by means of a group of SISO regulators, $\{R_i(z)\}_i$ (Fig. 8), which are tuned to the time components of the system’s dynamics $\{\mathcal{GH}_i(x, k)\}_{i,k}$ or the corresponding transfer functions $\{\mathcal{SH}_i(x, z)\}_i$, (Fig. 9). Thus the control problem is reduced to a group of SISO control problems, since the individual controllers are tracking the transformed quantities, instead of
solving a full-scale DPS optimization problem or a MIMO controller synthesis. This enables to use any control synthesis approach developed for SISO systems, without the need of special adaptation to DPS.

IV. DPS BLOCKSET FOR MATLAB & SIMULINK

For the design of control systems for ES as LDS, a dedicated software support is provided by the Distributed Parameter Systems Blockset for MATLAB & Simulink (DPS Blockset), (Fig. 10).

The HLDS and RHLDs blocks model the controlled DPS dynamics, described by numerical structures as LDS with zero-order holds H. DPS Control Synthesis provides feedback to distributed parameter controlled systems in control loops with blocks for discrete-time PID, Algebraic, State-Space and Robust Synthesis. The block DPS Input generates distributed quantities, which can be used as distributed reference quantities or distributed disturbances, etc. DPS Display presents distributed quantities with many options including export to AVI files. The block DPS Space Synthesis performs the aforementioned space synthesis approximation problem.

The block Tutorial presents methodological framework for formulation and solution of DPS control problems. The block Show contains motivation examples, such as: Control of temperature field of 3D metal body (the controlled system was modeled in COMSOL Multiphysics); Control of 3D beam of “smart” structure (the controlled system was modeled in ANSYS); Adaptive control of glass melting furnace (the controlled system was modeled by the Partial Differential Equations Toolbox of MATLAB), and Groundwater remediation control (the controlled system was modeled in MODFLOW). The block Demos contains examples oriented at the methodology of modeling and control synthesis. The DPS Wizard gives an automatized guide for arrangement and setting distributed parameter control loops in a step-by-step operation.

Along with this blockset, an Interactive Control service is available at http://www.dpscontrol.sk for the formulation and solution of DPS model control problems via the Internet.

V. ADAPTIVE PREDICTIVE CONTROL OF A FLUIDIZED BED COMBUSTION PROCESS

Capabilities of the proposed approach to control energy systems are demonstrated by select results of an adaptive predictive control of the temperature field of a coal-burning fluidized bed combustion, ensuring optimal conditions for the desulphurization process. Controlling the bed temperature field is a challenging problem, especially in the view of significant input disturbances caused by fluctuations in the fuel calorific value and the narrow allowable fluctuation of the bed temperature, so to keep the flue gas sulphur content in specified limits. The controlled system is a 4x6 m base fluidized bed of a coal burning furnace with four fuel feeders each supplying one zone, (Fig. 2). The aim of the control task is to keep the bed temperature throughout its extent at 1173±5K, whereas the fuel combustion heat fluctuation is about 30%. The predictive control system was built in MATLAB – Simulink using the aforementioned DPS Blockset, (Fig. 11).
Around the operating point $T=1173\,\text{K}$ the system’s dynamics is described in the form of LDS, where the input quantities are coal feed rates and the output is the bed temperature field. Initial system characteristics were obtained by simulation in ANSYS [13] in the form of discrete time transient responses, recorded in selected nodes of the FEM mesh, (Fig. 12).

From this data, reduced steady-state values of distributed-parameter step responses \(\mathcal{H}_i(x,k)\), (Fig. 13), and nominal partial distributed-parameter impulse responses are extracted.

\[ \mathcal{G}_i(x,k) = \mathcal{H}_i(x,k) - \mathcal{H}_i(x,k-1). \]  

(10)

The space components of dynamics are considered constant during the process as the set-up is given by the furnace construction.

During the control process, partial distributed impulse responses are being identified on-line from the system inputs \(\{U_i(k)\}\), and the responses in maximum gain points, whose locations have been identified off-line as described before [7-9]. Since the system arrangement is not subject to changes during the process, these points remain the same as well.

In each time step “\(k\)” the control error is evaluated

\[ E(x,k) = T_w(x) - \bar{Y}H(x,k), \]  

(11)

where \(T_w(x)\) is the reference temperature field, in this case a uniform 1173K for all \(x\). Next, the space synthesis approximation problem (9) solved and the individual SISO predictive controllers calculate the control actions.

However, controller horizon settings and weights for the individual SISO controllers have to observe some rules to get the best possible control quality. Each SISO controller should work with the same prediction and control horizons and their responses should be equally fast.

Finally, in steady state we obtain the controlled system output

\[ \bar{Y}H(x,k) = \sum_{i=1}^{n} \bar{Y}H_i(x,\infty) \mathcal{H}HR_i(x,\infty), \]  

(12)

For optimal desulphurization, the bed temperature has to be within \(1173\pm5\,\text{K}\). This restriction results in the condition for steady state

\[ |E(x,\infty)| = \left| T_w(x) - \bar{Y}H(x,\infty) \right| \leq \delta, \]  

(13)

where \(\delta=5\,\text{K}\) in this case. Expanding (13) we get

\[ \left| T_w(x) - \bar{Y}H_i(x,\infty) \mathcal{H}HR_i(x,\infty) \right| \leq \delta, \]  

(14)

which is the basic equation for optimization of the number, shape and location of fuel feeders. The result are steady-state values of distributed-parameter step responses, space components, corresponding to the physical arrangement of the fluidized bed furnace for given reference temperature field \(T_w(x)\) and error tolerance \(\delta\).
In the case shown here, the furnace has four fuel feeders, with steady-state characteristics according to Fig. 13. This arrangement has proven sufficient for a uniform reference temperature field and tolerated temperature deviation of ±5K. The control process is shown on Fig. 14 and Fig. 15.

![Figure 14. Control variables – coal feed rates in time.](image)

![Figure 15. Controlled bed temperature courses in selected nodes.](image)

REFERENCES


