Abstract—This paper studies some key technical issues of discrete-time adaptive control of a continuous-time nonlinear aircraft flight system with uncertain structural damage. A linearized discrete-time aircraft model with a dynamics offset is obtained from the discretized nonlinear aircraft model, which characterizes large unknown system parametric and structural changes caused by structural damage. The invariance of the discrete-time infinite zero structure before and after damage is derived for small sampling periods. A state feedback MRAC scheme is designed to ensure the signal boundedness of the closed-loop system and asymptotic output tracking in the presence of the dynamics and damage uncertainties. This linearization-based discrete-time adaptive control scheme is evaluated on a high-fidelity continuous-time nonlinear aircraft model—the NASA generic transport model (GTM) with uncertain damage. Simulation results show the desired system performance and demonstrate the effectiveness of the developed discrete-time adaptive control scheme for the continuous-time nonlinear aircraft system.

Keywords: Adaptive control, aircraft system, damage compensation, discrete-time design, system invariance.

I. INTRODUCTION

Uncertain structural and parametric changes of aircraft flight systems caused by airframe damage, such as loss of wing-tips, stabilizers, or flaps, can deteriorate the system performance significantly, which may lead to severe accidents. Considerable efforts have been devoted on developing effective and reliable control designs for the aircraft flight system, e.g., [1], [2], [3], [4], [7], [10], [12], [13], [15], [17]. Since digital control is widely used in aircraft flight systems due to some advantages over conventional analog control systems, such as capability of realizing complicated control algorithms and no degradation of performance due to wear or aging, discrete-time control designs are needed for the digital aircraft flight control systems.

In this paper, the continuous-time nonlinear aircraft model is discretized using the Taylor series expansion ([6], [18]). Then, a discrete-time adaptive control scheme is developed for the discrete-time nonlinear aircraft model to compensate the structural and parametric uncertainties caused by damage. To deal with complexity of the discrete-time nonlinear aircraft dynamics, we employ the linearization-based control design. Due to system parametric and structural uncertainties caused by damage, the equilibrium point is not available for linearization. Thus, an arbitrary operating point is chosen to linearize the discrete-time nonlinear aircraft model before and after damage occurs, which leads to a linearized discrete-time system with unknown system parameters and dynamics offset. Then, a discrete-time multivariable model reference adaptive control (MRAC) scheme is developed for the linearized discrete-time system to make all the signals of the closed-loop system bounded and the output signals track some given reference signals. Since the linearized discrete-time system is the approximation of the discrete-time nonlinear system around the chosen operating point, the developed discrete-time control design is applicable to the nonlinear aircraft system around a small neighborhood of the operating point.

For the multivariable MRAC scheme, an essential design condition—invariance of infinite zero structure is required for the linearized discrete-time aircraft system before and after damage occurs. Although, in [11], we have a conclusion about the invariance property, it is not applicable for the linearization-based nonlinear control design, since the discrete-time linear system in [11] is obtained by discretizing the continuous-time linear system (but not the nonlinear system). In this paper, for the linearization-based nonlinear discrete-time control design, the discrete-time linear system is obtained by linearizing the discretized nonlinear system at the given operating point. We can conclude that the infinite zero structure of the linearized discrete-time system is invariant for sufficiently small sampling periods, no matter how large the relative degree of the continuous-time nonlinear system is. This conclusion suggests that even if the damage changes the relative degree of the continuous-time nonlinear aircraft system, the infinite zero structure of the linearized discrete-time aircraft system is invariant before and after damage occurs. Based on such an invariance property of the linearized discrete-time aircraft system, we present the discrete-time multivariable MRAC scheme to compensate the unknown dynamics offset and parametric uncertainties and make the aircraft output signals track some given reference signals before and after damage occurs. The discrete-time multivariable MRAC scheme is applied to a continuous-time nonlinear aircraft system—the NASA generic transport model (GTM) [9] to assess the effectiveness of this linearization-
based discrete-time control design for the continuous-time nonlinear aircraft system.

The paper is organized as follows. In Section II, the discrete-time control problem of the nonlinear aircraft system with structural damage is formulated. The infinite zero structure of the linearized discrete-time system is investigated and the invariance of the infinite zero structure is concluded in Section III. In Section IV, we give the discrete-time state feedback for output tracking multivariable MRAC design to compensate damage. In Section V, the developed discrete-time control design is applied to the nonlinear GTM with damage to demonstrate the desired system performance.

II. PROBLEM STATEMENT

Consider a nonlinear aircraft model

\[ \dot{x}(t) = f(x(t)) + g(x(t))u(t), y(t) = Cx(t), \]  

(1)

where \( x(t) \in \mathbb{R}^n \) and \( u(t) \in \mathbb{R}^M \) are the state and control input vector signals, and the output signal \( y(t) \in \mathbb{R}^M \) is chosen as a linear combination of the state signals. When airframe damage occurs, the system dynamics (1) undergoes uncertain parametric and structural variations ([1], [8]).

Discretization of the nonlinear model. To establish a digital control system for the nonlinear aircraft flight system, we first investigate the discrete-time nonlinear system model with zero-order holds (ZOH). For the digital control system with ZOH, elements of the control input signal \( u(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T \) are

\[ u_i(t) = u_i(kT), \quad kT \leq t < (k+1)T, \]  

(2)

for \( i = 1, 2, \ldots, M \), i.e. within each sampling interval, the control input signals remain constant. We expand the state input signal \( x(t) \) in a Taylor series about \( x(kT) \) within a sampling interval \( t \in [kT, (k+1)T) \), it follows that

\[ x((k+1)T) = x(kT) + \sum_{i=1}^{\infty} T^i \frac{\partial^i x}{\partial t^i} |_{t=kT} \]  

(3)

By taking successive partial derivatives of the right hand side of (1) with control input signal as a constant, we can obtain a discrete-time nonlinear model

\[ x(k+1) = f_d(x(k), u(k)), \]  

(4)

whose structure will be studied in the next section. Since damage will cause unknown structural and parametric variations for the aircraft system (1), the discretized nonlinear model (3) also undergoes uncertain changes.

Linearization-based control design. A discrete-time control scheme will be developed for the discretized nonlinear model (3) with structural and parametric uncertainties. Block diagram of the closed-loop digital control system is illustrated in Fig. 1, where the linearization-based control design is applied. We linearize the discrete-time nonlinear model (4) at a chosen operating point \((x_0, u_0)\), which may not be an equilibrium point for the damaged aircraft system due to system uncertainties caused by damage. Hence, we use a sequential discrete-time linear model with an unknown constant dynamics offset \( f_0 \) to characterize the linearized discrete-time aircraft system under damage conditions:

\[ \Delta x(k+1) = A\Delta x(k) + B\Delta u(k) + f_0, \quad \Delta y(k) = C\Delta x(k), \]  

(5)

where \( \Delta x(k) = x(k) - x_0, \Delta y(k) = y(k) - Cx_0, \) and \( \Delta u(k) = u(k) - u_0, \) and the system matrices and offset are \( A = \frac{\partial f_d}{\partial x}|_{(x_0,u_0)} \), \( B = \frac{\partial f_d}{\partial u}|_{(x_0,u_0)} \), and \( f_0 = f_d(x_0, u_0) \), which are unknown piecewise constants: \( (A,B,f_d) = (A_i,B_i,f_{d_i}), \) \( i=1,\ldots,N, \) for different damage conditions.

Control objective. The objective is to develop a discrete-time adaptive control law \( \Delta u(k) \) for the linearized discrete-time system (5) with uncertainties to make all the signals of the closed-loop system bounded and the system output signal \( \Delta y(k) \) track a given reference signal \( \Delta y_m(k) \):

\[ \Delta y_m(k) = W_m(z)[r](k), \]  

(6)

where \( r(k) \) is a bounded reference input signal.

Assumptions. To proceed the adaptive control design, we make the following assumptions: (A1) all zeros of \( G_i(z) = C(zI-A_i)^{-1}B_i, \) \( i=1,2,\ldots,N, \) lie within the unit circle in the z-plane; (A2) \( G_i(z), i=1,2,\ldots,N, \) have full rank, there exists a known modified left interactor matrix \( \xi_m(z) \) for all \( G_i(z), i=1,2,\ldots,N, \) and \( W_m(z)=\xi_m^{-1}(z); \) (A3) all leading principal minors \( \Delta_{ij}, i=1,2,\ldots,N, \) \( j=1,2,\ldots,M \) of high frequency gain matrices \( K_{pi} = \lim_{z \to \infty} \xi_m(z)G_i(z) \) are nonzero and their signs are known and invariant: \( \text{sign}[\Delta_{ij}] = \text{sign}[\Delta_{pq}], p,q = 1,2,\ldots,N, \) \( j=1,2,\ldots,M; \) (A4) \( (A_i, B_i) \) is controllable and \( (A_i, C) \) is observable.

In the following section, we will investigate the infinite zero structure of the linearized discrete-time system (5) to see whether the invariant conditions (A2) and (A3) can hold for the aircraft system under damage conditions.

III. INFINITE ZERO STRUCTURE OF LINEARIZED DISCRETE-TIME SYSTEM

To investigate the infinite zero structure of the linearized discrete-time model (5), we first examine the generic structure of the discretized nonlinear model (3). By taking successive partial derivatives of the right hand side of (1), the discretized nonlinear model (3) can be expressed as

\[ x(k+1) = f_{d0}(x(k)) + \sum_{i=1}^{M} g_{di}(x(k))u_i(k) \]

\[ + \sum_{l=2}^{\infty} \left( \sum_{i_1=1}^{M} \cdots \sum_{i_l=1}^{M} \left( g_{d_{i_1} \cdots i_l}(x(k)) \prod_{j=i_1}^{i_l} u_j(k) \right) \right), \]  

(7)

where...
\[ \begin{align*}
\dot{x}(k) &= x(k) + \sum_{i=1}^{\infty} \frac{T_i}{i!} L^{i-1}_f f(x(k)), \\
\dot{y}_i(k) &= L_{g_i} L^{i-2}_f f(x(k)) + \sum_{i=3}^{\infty} \frac{T_i}{i!} L_{g_i} L^{i-2}_f f(x(k)),
\end{align*} \]

with the Lie derivative as \( L^{i+1}_f f = \frac{\partial f}{\partial x} \frac{\partial x}{\partial y} \) and \( L^0_f f = f \).

We choose the operating point as \( (x_0, u_0) = (x_0, 0) \).

It follows that \((A, B, f_0)\) of the linearized discrete-time system (5) are

\[ f_0 = f_0(x_0) - x_0 = \sum_{i=1}^{\infty} \frac{T_i}{i!} L^{i-1}_f f(x_0), \]

\[ A = \frac{\partial f_0}{\partial x} |_{x=x_0} = I_n + \sum_{i=1}^{\infty} \frac{T_i}{i!} \frac{\partial}{\partial x} (L^{i-1}_f f) |_{x=x_0}, \]

\[ B = [g_{a1}(x_0), g_{a2}(x_0), \ldots, g_{aM}(x_0)]. \]

We use the following calculation of \( G(z) = C(zI_n - A)^{-1} B \) to determine the infinite zero structure of the linearized discrete-time system (5):

\[ G(z) = \frac{1}{\det(zI_n - A)} \left( E_{n-1} z^{n-1} + \cdots + E_1 z + E_0 \right), \]

(11) For the ith row \( E_{(n-1),i} = [C_i B_1, C_i B_2, \ldots, C_i B_M] \) with \( B_j \) as in (10), we obtain \( C_i B_j \) as

\[ C_i B_j = \frac{T_{p_i}}{\rho_i!} C_i L_{g_i} L^{p_i-2}_f f(x_0) + \sum_{l=p_i+1}^{\infty} \frac{T_l}{l!} C_i L_{g_i} L^{l-2}_f f(x_0) \]

\[ + \sum_{l=3}^{\infty} \frac{T_l}{l!} C_i \left( L^{l-1}_f g_i(x_0) + \sum_{p=1}^{l-2} L^p_f g_i L^{p-2}_f f(x_0) \right). \]

From (16), there exist small \( T_0 > 0 \) and \( f_n > 0 \), such that, if \( T < T_0 \) and \( ||f(x_0)|| < f_n \), \( C_i B_j \) can be approximated as

\[ C_i B_j \approx \frac{T_{p_i}}{\rho_i!} C_i L_{g_i} L^{p_i-2}_f f(x_0). \]

From (15) and (17), it follows that

\[ E_{n-1} = C B \approx P \alpha(x_0), \]

(18) where \( P = \text{diag} \left( \frac{T_{p_1}}{\rho_1!}, \frac{T_{p_2}}{\rho_2!}, \ldots, \frac{T_{p_M}}{\rho_M!} \right) \), which is non-singular.

Since the matrix \( \alpha(x_0) \) is non-singular, we can conclude that the coefficient \( E_{n-1} = C B \) is non-singular, when the sampling period \( T < T_0 \) and \( ||f(x_0)|| < f_n \), i.e., the operating point \((x_0, 0)\) is close to the equilibrium point.

Hence, we choose the interactor matrix \( \xi_m(z) \) as

\[ \xi_m(z) = \text{diag} \{ z + a_1, z + a_2, \ldots, z + a_M \}, \]

(19) with \( |a_i| < 1, i=1, 2, \ldots, M \), such that \( \lim_{z \to \infty} \xi_m(z) G(z) = \nabla \).

From the proof of Theorem 1, we can see that if the relative degree \( \rho_i \leq 2 \), the condition on the operating point \((x_0, 0)\), i.e., \( ||f(x_0)|| < f_n \), can be relaxed.

**Corollary 1.** For the continuous-time nonlinear system (12) with the relative degree \( \{\rho_1, \rho_2, \ldots, \rho_M\} \) at the point \( x_0 \), where \( \rho_i \leq 2 \), \( i=1, 2, \ldots, M \), there exists a small \( T_0 > 0 \), such that, if the sampling period \( T < T_0 \), the linearized discrete-time system (13) at the operating point \((x_0, 0)\) with \( A, B \), and \( f_0 \) as in (8)-(10), has a diagonal left modified interactor matrix: \( \xi_m(z) = \text{diag} \{ z + a_1, z + a_2, \ldots, z + a_M \} \), with \( |a_i| < 1 \), for \( i=1, 2, \ldots, M \).
It follows that
\[ E_{n-1} = CB \approx P\alpha(x_0), \]  
(24)
where \( P \) is a diagonal matrix with the diagonal elements are \( T \) (if \( \rho_i = 1 \)) or \( \frac{1}{T} \) (if \( \rho_i = 2 \)). Hence, the first coefficient \( E_{n-1} \) is non-singular. So we choose the interactor matrix as
\[ \xi_m(z) = \text{diag}\{z + a_1, z + a_2, \ldots, z + a_M\}, \]
(25)
with \( |a_i| < 1 \), for \( i = 1, \ldots, M \), such that \( \lim_{z \to \infty} \xi_m(z)G(z) = CB \) is non-singular.

**Invariance of interactor matrix.** In this paper, for the aircraft flight system, we choose pitch angle \( \theta \) and yaw angle \( \psi \) as output signals: \( y = Cx = [\theta, \psi]^T \), and elevator \( d_e \) and rudder \( d_r \) as control input signals: \( u = [d_e, d_r]^T \).

From the continuous-time nonlinear aircraft model in [8], we can conclude that, before and after damage occurs, the relative degree is \( \{\rho_1, \rho_2\} = \{2, 2\} \). Therefore, based on Corollary 1, for a small sampling interval \( T \), the linearized discrete-time aircraft model (5) has a common interactor matrix \( \xi_m(z) \) as
\[ \xi_m(z) = \text{diag}\{z + a_1, z + a_2\}, \]
(26)
with \( |a_1| < 1 \) and \( |a_2| < 1 \), before and after damage occurs.

**Invariance of high frequency gain matrix.** From (24), we obtain the high frequency gain matrix before and after damage occurs as:
\[ K_p \approx \begin{bmatrix} \frac{T}{2} & 0 \\ 0 & \frac{T}{2} \end{bmatrix} \begin{bmatrix} C_1A_cB_{c1} & C_1A_cB_{c2} \\ C_2A_cB_{c1} & C_2A_cB_{c2} \end{bmatrix}, \]
(27)
where \( A_c \triangleq \frac{d}{T}\frac{\partial f}{\partial x}|_{x_0} \) and \( [B_{c1}, B_{c2}] \triangleq \{g_1(x_0), g_2(x_0)\} \).

It has been shown in [8] that, when operating at a wings-level flight condition, signs of leading principal minors of
\[ K_{pc} = \begin{bmatrix} C_1A_cB_{c1} & C_1A_cB_{c2} \\ C_2A_cB_{c1} & C_2A_cB_{c2} \end{bmatrix}, \]
(28)
are invariant before and after damage occurs. Hence, from (27), we conclude that for the linearized discrete-time aircraft model (5), signs of leading principal minors of \( K_p \) are invariant before and after damage occurs, when operating at the wings-level flight condition.

**IV. MULTIVARIABLE MRAC SCHEME**

In this section, we will present a multivariable state feedback model reference adaptive control design for the linearized discrete-time aircraft model (5) with damage.

We choose the state feedback controller structure as
\[ \Delta u(k) = K_1^T(k)\Delta x(k) + K_2(k)r(k) + k_3(k), \]
(29)
where \( k_3(k) \in \mathbb{R}^M \) is the adaptive estimate of an unknown constant compensation term \( k_3 \) for canceling the effect of the piecewise constant offset \( f_0 \), and \( K_1(k) \) and \( K_2(k) \) are the estimates of the nominal \( K_1^* \) and \( K_2^* \), which satisfy the matching conditions [5]
\[ C(zI - A - BK_1^T)\text{inv}_z BK_2^* = W_m(z), \quad K_2^* - 1 = K_p. \]
(30)
To derive \( k_3^* \), we apply the nominal controller
\[ \Delta u(k) = K_1^T(k)\Delta x(k) + K_2^*r(k) + k_3^* \]
(31)
to the system (5). Considering a particular set of the piecewise system matrices \( (A, B, f_0) \), we have the closed-loop system in the \( z \)-domain as
\[ \Delta y(z) = C(zI - A - BK_1^T)^{-1}BK_2^*r(z) + \Delta(z), \]
(32)
with \( \Delta(z) = C(zI - A - BK_1^T)^{-1}(B\bar{x}_M(z) + \bar{f}_2(z)). \)

From the reference system (6) and the matching conditions (30), we have the output tracking error \( \Delta e(k) \) in the \( z \)-domain as
\[ \Delta e(z) = \Delta y(z) - \Delta y_m(z) = \Delta(z). \]
(33)
Applying the \( z \)-domain final value theorem, we obtain
\[ \lim_{k \to \infty} \Delta e(k) = \lim_{z \to 1} \Delta(z) = W_m(1)K_{p}k_3^* + d \]
(34)
with \( d = C(I - A - BK_2^*)^{-1}f_0 \). Then we set
\[ k_3^* = -K_p^{-1}\xi_m(1)d, \]
(35)
which follows that \( \lim_{k \to \infty} \Delta y(k) - \Delta y_m(k) = 0 \).

**Remark 1:** For the linearized discrete-time aircraft system with damage, although the system matrices \( A \) and \( B \) are of uncertainties before and after damage occurs, the interactor matrix is invariant and \( \xi_m(z) = \text{diag}\{z + a_1, z + a_2\} \) with \( |a_1| < 1 \) and \( |a_2| < 1 \). Hence, from the matching conditions (30) and (35), we conclude that there exists a nominal controller (31) with piecewise constant parameters \( K_1^*, K_2^* \), and \( k_3^* \) to make the output signals track an invariant reference system \( W_m(z) = \xi_m^{-1}(z) \), which ensures that the aircraft maintains desired system performance when uncertain damage occurs at an unknown instant.

Since the nominal parameters \( K_1^*, K_2^* \), and \( k_3^* \) are unknown, we employ the adaptively updated control law (29).

**Tracking error equation.** Substituting the control law (29) in the plant (5), and from the reference model (6), matching conditions (30) and (35), we obtain the output tracking error as
\[ \Delta e(k) = W_m(k)K_p[\tilde{\Theta}^T(k)\omega(k)], \]
(36)
where \( \omega(k) = [\Delta x^T(k), r^T(k), 1]^T \), \( \tilde{\Theta}(k) = \Theta(k) - \Theta^* \), \( \Theta(k) = [K_1^T(k), K_2^T(k), k_3^T(k)]^T \), and \( \Theta^* = [K_1^*T, K_2^*T, k_3^*T]^T \).

To deal with the uncertainty of \( K_p \), we use its LDS decomposition:
\[ K_p = L_sD_sS, \]
where \( S = St^* > 0 \), \( L_s \) is a unit lower triangular matrix, and \( D_s = \text{diag}\{\text{sign}[\Delta_1], \ldots, \text{sign}[\Delta_M]\} \) with \( \gamma_i > 0 \), \( i = 1, \ldots, M \). Substituting the LDS decomposition of \( K_p \) (with a common \( D_s \)) in (36), we have
\[ L_s^{-1}\xi_m(z)[\Delta e(k)] = D_sS\tilde{\Theta}(k)\omega(k). \]
(37)
Operating both sides of (37) by \( h(z) = 1/f_n(z) \), where \( f_n(z) \) is a stable and monic polynomial of degree equals to the degree of \( \xi_m(z) \), and introducing \( \Theta_0^* = L_s^{-1}I - \Theta_{ij}^* \) with \( \Theta_{ij}^* = 0 \) for \( i = 1, 2, \ldots, M \) and \( j \geq i \), the equation (37) is parameterized as
\[ \tilde{e}(k)[0, \Theta_2^T, \eta_1, \ldots, \Theta_M^T, \eta_M]^T = D_sS\tilde{\Theta}(k)\omega(k), \]
(38)
where \( \tilde{e}(k) = \xi_m(z)h(z)[e(k)] = [\tilde{e}_1(k), \ldots, \tilde{e}_M(k)]^T \), \( \eta_i(k) = [\tilde{e}_1(k), \ldots, \tilde{e}_{i-1}(k)]^T \), \( i = 2, \ldots, M \), \( \Theta_i^* = [\theta_{i1}^*, \ldots, \theta_{ii-1}^*]^T \), and \( i = 2, \ldots, M \).
Estimation error. Introduce the estimation error signal
\[ \epsilon(k) = [0, \theta_2^T(k)\eta_2, \ldots, \theta_M^T(k)\eta_M]^T + \Psi(k)\xi(k) + \bar{e}(k), \]
where \( \theta_i(k), i = 2, 3, \ldots, M \) are the estimates of \( \theta^*_i \), \( \Psi(k) \) is the estimate of \( \Psi^* = D_sS \), and
\[ \xi(k) = \Theta^T(k)\zeta - h(z)(\Theta^T\omega)(k), \quad \zeta(k) = h(z)\omega(k). \]

From (38)–(40), we derive that
\[ \epsilon(k) = [0, \theta_2^T(k)\eta_2, \ldots, \theta_M^T(k)\eta_M]^T + \Psi(k)\xi(k) + \bar{e}(k), \]
where \( \theta_i(k) = \theta_i(k) - \theta^*_i, i = 2, 3, \ldots, M \), and \( \Psi(k) = \psi(k) - \psi^* \) are the related parameter errors.

Adaptive laws. With the estimation error model (41), we choose the adaptive laws as
\[ \theta_i(k+1) = \theta_i(k) - \frac{\Gamma_i\epsilon_i(k)\eta_i(k)}{m_i^2(k)}, i = 2, 3, \ldots, M, \]
\[ \Theta^T(k+1) = \Theta^T(k) - \frac{\Gamma\epsilon^T(k)}{m^2(k)}, \]
where the signal \( \epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_M]^T \) is computed from (39), \( 0 < \Gamma_{\theta_1} = \Gamma_{\theta_2} < 2I_m \), \( i = 2, 3, \ldots, M \), \( 0 < \Gamma = \Gamma^T < 2I_M \), \( D_s \) is chosen to satisfy \( 0 < D_sS < 2I_M \), and
\[ m(k) = (1 + \zeta^T(k)\zeta(k) + \epsilon^T(k)\xi(k) + \sum_{i=2}^{M} \eta_i^T(k)\eta_i(k))^{1/2}. \]

By considering a positive definite function
\[ V = \sum_{i=2}^{M} \theta_i^T(k)\theta_i + \text{tr}[\Theta^T(k)\Theta - \epsilon^T(k)\epsilon] + \text{tr}[\Theta S\Theta^T], \]
we can derive that \( \theta_i, \Theta, \Psi \in L^\infty, \frac{\epsilon(k)}{m(k)} \in L^2 \cap L^\infty \), and
\[ \theta_i(k+1) - \theta_i(k), \Theta(k+1) - \Theta(k), \Psi(k+1) - \Psi(k) \in L^2 \cap L^\infty, \]
which allow us to prove the following theorem.

Theorem 2: The multivariable MRAC scheme with the state feedback control law (29) updated by the adaptive laws (42)–(43), when applied to the plant (5), guarantees the closed-loop signal boundedness and asymptotic output tracking: \( \lim_{k \to \infty} (\Delta y(k) - \Delta y_m(k)) = 0 \), for any initial conditions.

The proof can be carried out in a similar way to that for multivariable MRAC using output feedback [16]. The key step in the procedure is to express a filtered version of the output \( \Delta y(k) \) in a feedback framework which has a small gain due to the \( L^2 \) properties of \( \Theta(k+1) - \Theta(k) \), \( \theta_i(k+1) - \theta_i(k) \), and \( \epsilon(k)/m(k) \), which is done by using \( \Delta x(k+1) = (A - LC)\Delta x(k) + B\Delta u(k) + L\Delta y(k) + f_0 \) for a gain matrix \( L \) such that \( A - LC \) is stable. Then, the analysis procedure in [16] can be used to conclude the closed-loop signal boundedness and asymptotic output tracking.

V. SIMULATION STUDY FOR GTM WITH DAMAGE

In this section, the discrete-time adaptive control scheme will be applied to the NASA generic transport model (GTM) with damage to assess its effectiveness for control of the nonlinear continuous-time aircraft flight system. The GTM is a 5.5% dynamically scaled twin-turbine powered test aircraft used to test flight research control laws in adverse flight conditions such as upsets, damage, and failures [9].

High-fidelity simulation model. We use the high-fidelity Matlab Simulink model of the GTM developed by the NASA, which contains actuator dynamics, sensor dynamics, aerodynamics, etc., to test the control design. Hence, the nonlinear GTM simulation will offer a realistic representation of the aircraft and the simulation results can provide a credible assessment of the proposed design.

Damage scenarios. The GTM simulation model contains several damage scenarios. In this study, we consider two damage conditions: (i) loss of outboard left wing tip, which is approximately 25% semi-span of the left wing; (ii) loss of entire left stabilizer.

Digital control system of GTM. The block diagram of the digital control system is shown in Fig. 1. The operating point \((x_0, u_0)\) is chosen as a wings-level flight condition obtained by trimming the nominal GTM with the equivalent airspeed as 90 knots. The output signals are chosen as the pitch angle \( \theta \) and the yaw angle \( \psi: y(kT) = [\theta(kT), \psi(kT)]^T \), and the control inputs are chosen as elevator \( d_e \) and rudder \( d_r \): \( u(kT) = [d_e(kT), d_r(kT)] \). From the analysis in Section III, we have the interactor matrix is invariant before and after damage occurs with a small sampling interval \( T \), which is \( \xi_m(z) = \text{diag}\{z + a_1, z + a_2\} \) with \( |a_1| < 1 \) and \( |a_2| < 1 \). Hence, we choose the reference system as
\[ W_m(z) = \text{diag}\{1/z, 1/z\}, \]
for the simulation study.

By applying the discrete-time control law \( u(k) = \Delta u(k) + u_0 \), where \( \Delta u(k) \) is the adaptive controller (29) with the adaptive laws (42)–(43), to the continuous-time GTM via the ZOH (illustrated in Fig. 1), we can obtain the desired system performance of the nonlinear GTM around the chosen operating point \((x_0, u_0)\), before and after damage occurs.

Simulation results. In addition to show the output signal \( y(kT) = \Delta y(kT) + Cx_0 = [\Delta \theta(kT) + \theta_0, \Delta \psi(kT) + \psi_0]^T \), another state signal–roll angle \( \phi(kT) = \Delta \phi(kT) + \phi_0 \) will be illustrated to verify that the aircraft can execute the maneuvers around the chosen operating condition \((x_0, u_0)\). We consider two damage cases: the loss of the outboard left wing-tip and the loss of the entire left stabilizer.

Case I. We choose the sampling interval \( T = 0.02 \) seconds and the reference input as \( r(kT) = [4\pi/180 \sin(0.1kT), -8\pi/180 \sin(0.15kT)]^T \). The wing-tip damage occurs at 30 seconds. From Fig. 2, it can be seen that the output signals (solid)–pitch angle \( \theta(kT) \) and yaw angle \( \psi(kT) \) track the reference output signals (dotted) and the GTM state signal–roll angle \( \phi(kT) \) is bounded before and after damage occurs.

Case II. To further demonstrate the effectiveness of the discrete-time control design, the damage is chosen as the loss of the entire left stabilizer, which occurs at 20 seconds. In the simulation, we choose the sampling interval \( T = 0.05 \) seconds. Fig. 3 shows the GTM output signals \( \theta(kT) \) and \( \psi(kT) \), which follow the desired reference signals, and
discrete-time control design for the nonlinear NASA generic transport model (GTM) have shown the desired system performance, which demonstrates that the linearization-based discrete-time adaptive control scheme is effective for the continuous-time nonlinear aircraft system around a small neighborhood of the operating point.

REFERENCES


