A parameter identification algorithm for the METANET model with a limited number of loop detectors

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Abstract—This paper analyzes the parameter identification of the macroscopic traffic flow model METANET. In previous papers [1] [2], this calibration has been done by minimizing numerically the difference between the data and the prediction of the model. Results indicate that this optimization usually falls in quite suboptimal local minima, especially when there are sensors only available in some segments. The authors propose an identification procedure where the calibration is done in 3 main steps: Firstly, the parameters of the fundamental diagram for each segment with data available are found. Subsequently, the parameters of the speed equation are computed using an optimization procedure considering the values of the fundamental diagram as known. Lastly, a global optimization is run in order to improve the final identification. Moreover, a new mathematical definition of the fundamental diagram is proposed. The identification algorithm was tested over a section of the I-210 West in Southern California, using several days of loop detector data collected during the morning rush-hour period. The results show a good estimation of both densities and speeds with a mean error of 10.95% and 11.35%, respectively.

I. INTRODUCTION

In model-based freeway traffic control is necessary to have an accurate but considerably faster than real-time model. The METANET model [3] is a macroscopic traffic model that provides a good trade-off between simulation speed and accuracy. It is a deterministic model for the simulation of the traffic system in motorway networks of arbitrary topology and characteristics, including motorway stretches, bifurcations, on-ramps, and off-ramps. This model allows to simulate almost all kinds of traffic conditions and events with prescribed characteristics. Moreover, METANET can handle control actions such as ramp metering, route guidance, and variable speed limits. The METANET model has been used in simulations for the formulation of Model-based Predictive Controllers [4] for freeway traffic control. Papers [5],[6] and [7] design MPC controllers for METANET in a global, local and distributed way, respectively.

The estimation of the unknown parameters of the METANET model is not a trivial issue because the dynamic equations are highly nonlinear. The most common way to identify the parameters is the global minimization of the discrepancy between the model calculations and the real process [1] [2]:

\[
J = \sum_{k=1}^{K} (y(k) - \hat{y}(k))^2
\]

where \(y\) is the output vector of the model, \(\hat{y}\) is the measured process output and \(K\) is the total number of measures. In [8] this parameter estimation is done for a real large-scale motorway network around Amsterdam. Unfortunately, this optimization usually falls in local minima, especially if a limited number of sensors are available. For example, if we were dealing with a network with 20 segments, 5 on-ramps and 5 off-ramps between two mainline loop detectors it would not be possible to compute so many variables (20*8=160) in a global optimization ensuring that the global minimum is reached. In fact, the research done for this paper was motivated by the possibility of identifying properly the METANET parameters for a specific real network using a global optimization. The network where these problems appeared is the same used for this paper. The easier solution for the identification problem could be to consider that the parameters have the same value for all the segments. With this supposition, it is possible to run a previous evaluation procedure in order to start the optimization with an initial point close to the global minimum. However, to consider that all the segment has the same value of the model parameters causes a considerable loss of accuracy. In previous identifications of the METANET model [2][8] heuristics are usually used in order to identify properly the model parameters.

An alternative parameter identification algorithm for the METANET model is proposed in [10]. In that paper the identification is also done by using a global optimization without a known initial point. Therefore, the approach is just well suited for the calibration task at hand.

The main contribution of this paper is the design of an algorithm which can estimate the parameters of METANET approaching the global minimum of the discrepancy between the prediction of the model and the data (1). With that purpose, a three step identification algorithm is proposed. In the first step, the fundamental diagram is computed for each segment with data available by a loop detector in the mainline. The fundamental diagram of the segments without loop detectors is interpolated between the closer segments with data.

In the second step, a minimization of the discrepancy between the model and the data is done by taking as decision...
variables the parameters of the speed equation. The values of the fundamental diagram are considered as known. In this step, the algorithm uses a pre-evaluation in order to start the optimization as close as possible to the global minimum. In the third step, a global optimization is run in order to improve the final performance of the system considering all the model parameters as decision variable. The second contribution of this paper is the proposal of a new mathematical definition of the Fundamental Diagram (FD) of traffic flow. The identification algorithm have been tested for a section of the I-210 in Pasadena, CA for several weekdays, over the interval 5AM-12PM. The upstream and downstream mainline data and the ramp flow data are assumed to be known and the density of the middle sensor is assumed to be missing, hence in need for estimation.

II. METANET MODEL

The METANET [3] model is a second-order model that is discrete in both space and time. This model discretizes the freeway in consecutive sections as can be seen on Fig. 1.

METANET model represents a network as a graph where the links (m) correspond to freeway stretches. Each link m is divided into Nm segments (i) of length Lm with $\lambda_m$ lanes. Each segment is characterized dynamically by the traffic density $\rho_{m,i}(k)$ and the mean speed $v_{m,i}(k)$ where k is the time instant $t = kT$ and T is the simulation time step. In this paper $T = 5s$. $q_{m,i}(k)$ is the traffic flow which can be computed for each time step using:

$$q_{m,i}(k) = \rho_{m,i}(k)v_{m,i}(k)\lambda_m$$

[9] considers the possibility of using the traffic flow instead of the density as the state variable for traffic control purpose. The reason to do so is that flow measurement with a point sensor, such as inductive loop detector, with good density estimation is very costly if not impossible. This equivalent model may be a good option in practical cases but it still needs to be validated.

In the METANET model, two equations describe the system dynamics. The first one (3) expresses the conservation of vehicles:

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) +$$

$$\frac{T}{L_m} (q_{m,i-1}(k) - q_{m,i}(k) + q_{r_m,i}(k) - S_{m,i}(k)q_{m,i-1}(k))$$

where $q_{r_m,i}(k)$ is the traffic flow that enters the freeway from an on-ramp and $S_{m,i}(k)$ is the split ratio of an off-ramp (i.e. the percentage of vehicles that leave the freeway to an off-ramp). In this paper, $q_{r_m,i}(k)$ and $S_{m,i}(k)$ are considered as measurable (or estimable) disturbances of the system and are taken from real data. $L_m$ is the length of each segment.

The second equation (4) expresses the mean speed as a sum of the previous mean speed, a relaxation term, a convection term and an anticipation term:

$$v_{m,i}(k+1) = v_{m,i}(k) +$$

$$\frac{T}{\rho_{m,i}(k)}(V(\rho_{m,i}(k)) - v_{m,i}(k)) +$$

$$\frac{T}{L_m} v_{m,i}(k)(v_{m,i-1}(k) - v_{m,i}(k)) -$$

$$\frac{\mu T}{\tau L_m} \rho_{m,i}(k+1) - \rho_{m,i}(k) + K$$

where K, $\tau$ and $\mu$ are model parameters that have to be estimated for each segment and $V(\rho_{m,i}(k))$ is the desired speed by the drivers (8). If there is an on-ramp, a penalization in the speed equation is added in order to model the speed drop caused by merging phenomena:

$$v_{m,i}(k+1) = v_{m,i}(k) + \cdots - \frac{\delta T q_{ramp}(k)}{L_m \lambda_m (\rho_{m,i}(k) + K)}$$

where $\mu$ is a model parameter. As proposed in [5], the model can take different values for $\mu$, depending on whether the downstream density is higher or lower than the density in the actual segment:

$$\mu_{m,i}(k) = \begin{cases} \mu_h & \text{if } \rho_{m,i-1}(k) \leq \rho_{m,i}(k) \\ \mu_i & \text{otherwise} \end{cases}$$

[14] suggests improvements on the model proposing several alternatives for the convection term of the speed equation. The desired speed is defined by the desired flow:

$$V(\rho_{m,i}(k)) = \frac{Q(\rho_{m,i}(k))}{4\rho_{m,i}(k)}$$

The Fundamental Diagram (FD) of traffic shows the relation between the desired flow $Q(\rho_{m,i}(k))$ and the density modeling the static characteristic of the traffic system. Different functions can model the fundamental diagram. The following exponential form is used in the majority of the references [10] [3] in absence of variable speed limits:

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \exp\left(-\frac{1}{a_m} \frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}}\right)$$

Where $a_m$ is a model parameter, $v_{\text{free},m}$ is the free flow speed that the cars reach in steady state and $\rho_{\text{crit},m}$ is the critical density (the density corresponding to the maximum flow in the FD). This paper uses a different function for the fundamental diagram. Section IV.A goes into the function selection for the fundamental diagram and defines the FD used in this study.

Finally, in order to fulfill the model, is necessary to know the upstream speed of the first segment, the downstream density of the last segment and the flow that enters the mainline. These variables are considered as inputs (as the ramp flows) of the system and taken from real data.
III. NETWORK ANALYZED

Fig. 2 shows the mainline segment that has been simulated. The image comes from the tool “NetworkEditor” of the TOPL Project [9]. The freeway is a subsection of the I-210 West located in Pasadena (Los Angeles County, California, United States). The freeway stretch has four mainline lanes and a length of 2.81 miles. The freeway is partitioned using NetworEditor tool in 10 segments with lengths:

\[ L = [0.19, 0.34, 0.24, 0.38, 0.20, 0.35, 0.37, 0.16, 0.35, 0.23] \]
miles.

Fig. 3 shows schematically the freeway section used to identify and test the traffic model.

As can be seen in the figure, the section has three mainline loop detector stations labeled (Mountain ML 34.90, Huntington ML 33.05, Santa Anita ML 32.20), and additional detector stations on each ramp. ML stands for mainline, and the numbers, e.g. 34.05, are the absolute postmile indices of the detector stations. The section has three on-ramps and three off-ramps located in the junctions with the avenues Santa Anita, Huntington, Myrtle and Mountain. The middle density \( \rho_m \) is the measurement of the detector loop in the station Huntington. The seventh segment of the model correspond to this detector and, therefore, \( \rho_7 \) is an estimation of \( \rho_m \). The data of this loop detector is assumed to be missing, hence in need for estimation.

This simulation uses loop detector data over the 5AM:12PM time range for ten different days. The data was obtained from the Performance Measurement System (PeMS) [12] using the tool Data ClearingHouse. The morning rush-hour congestion usually occurs at this time. Each loop detector provides measurements of volume (veh/timestep) and percent occupancy every 5 minutes. Densities are computed according to the PeMS algorithm [13].

The choice of the network and the data set has been done based on [11] where Munoz et al. estimated the parameters of a first order traffic model (Cell Transmission Model) in a part of this network with a similar time range.

For stability, the segment length and the simulation time step should satisfy for every link \( i \):

\[ L_i > v_{free,i} T \]

Therefore, the model sample time has to be largely smaller than data sample time (5 minutes). As this paper uses a model sample time \( T \) of 5 seconds, a zeroth-order interpolation was applied to the PeMS data.

The METANET simulation of the network has been programmed in MATLAB. The average time needed for the simulation of one day (7 hours) using a Intel Core i3 CPU is 1.4 seconds. The optimizations involved in the identification algorithm were computed using the “fmincon” function of the Optimization Toolbox of Matlab.

IV. OPTIMIZATION PROCEDURE

A. Optimization First Step

The first step of the optimization is the estimation of the parameters of the FD for each segment with a detector available in the mainline. [15] describes an equivalent procedure for the CTM model using a triangular form of the FD. As explained in section II, different forms of the fundamental diagram can be used. This paper considers 3 possible choices for the fundamental diagram:

- **FD 1. Typical Fundamental Diagram**:

\[
V(\rho_i(k)) = v_{free,i} \exp\left(-\frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{crit,i}}\right)^{a_i}\right)
\]

- **FD 2. Typical Fundamental Diagram with a different value for “\( a \)” for densities higher than the critical density**:

\[
V(\rho_i(k)) =
\begin{cases}
  v_{free,i} \exp\left(-\frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{crit,i}}\right)^{a_i}\right) & \text{for } \rho_i(k) \leq \rho_{crit,i} \\
  \exp\left(-\frac{1}{b_i} \right) \exp\left(-\frac{\rho_i(k)}{\rho_{crit,i}} b_i\right) & \text{for } \rho_i(k) > \rho_{crit,i}
\end{cases}
\]

where \( b_i \) is a model parameter equivalent to \( a \) but only defined for the congested part.

6985
- FD 3. Linear Freeflow and Exponential Congestion:

\[
V(\rho_i(k)) = \begin{cases} 
  v_{\text{free},i} & \text{for } \rho_i(k) \leq \rho_{\text{crit},i} \\
  v_{\text{free},i} \exp\left(-\frac{1}{a_i} \frac{\rho_i(k)}{\rho_{\text{crit},i}}\right) & \text{for } \rho_i(k) > \rho_{\text{crit},i}
\end{cases}
\] (11)

1) Capacity Estimation: Firstly, the capacity (i.e. the maximum flow that can go through a point of the freeway) of the segments with detectors in the mainline are approximated. The algorithm takes the mean of the five measures with the higher flows among all the dates available for a detector in the segment.

Knowing the capacity \(C_i\), it is possible to define the critical density as combination of capacity, freeflow velocity and variable “a”.

- For Fundamental Diagrams 1 and 2:

\[
\rho_{\text{crit},i} = \frac{C_i}{4V_{\text{free},i}} \exp\left(-\frac{1}{a_i}\right)
\]

- For Fundamental Diagrams 3:

\[
\rho_{\text{crit},i} = \frac{C_i}{4V_{\text{free},i}}
\]

2) Fundamental Diagram Estimation: Once the capacity is obtained, a SQP optimization algorithm finds the value of the independent parameters of the fundamental diagram minimizing the quadratic error:

\[
J_{\text{part1}} = \sum_{k=1}^{K} (Q(\hat{\rho}(k)) - \hat{q}(k))^2
\] (12)

where \(\hat{\rho}(k)\) and \(\hat{q}(k)\) are the measured density and flow of the corresponding segment.

Fig. 4 shows that the typical fundamental diagram (FD 1) for METANET is not able to model properly the freeflow part accumulating an error of \(J_{\text{part1}} = 4.68 \times 10^8\). Because of that, the authors decided to use a fundamental diagram with different values for freeflow and congested parts (FD 2). As can be seen on Fig. 4, the FD 2 (10) models better the freeflow part. The accumulated error is now \(J_{\text{part1}} = 2.75 \times 10^8\); almost the half than for the typical FD.

However, it can be seen that the freeflow part in this diagram is almost straight with \(a=196\). Therefore, it is interesting to eliminate the parameter “a” simplifying the computation. The accumulated error for the FD 3 (11) is almost the same \(J_{\text{part1}} = 2.79 \times 10^8\) that for FD2.

Defining the fundamental diagram in piecewise form does not increase substantially the computational power for traffic control because, when variable speed limits and ramp metering are considered, the model is inherently piecewise.

B. Optimization Second Step

1) Interpolation of the Fundamental Diagram: After the estimation of the fundamental diagram for the segments with a sensor, it is necessary to set the fundamental diagram of the rest of the network. The values of the FD variables of the segments without a detector are computed according to:

\[
V_{\text{free},i} = \frac{L_d}{L_d + L_u} V_{\text{free},u} + \frac{L_u}{L_d + L_u} V_{\text{free},d}
\] (13)

\[
a_i = \frac{L_d}{L_d + L_u} a_u + \frac{L_u}{L_d + L_u} a_d
\] (14)

\[
\rho_{\text{crit},i} = \frac{L_d}{L_d + L_u} \rho_{\text{crit},u} + \frac{L_u}{L_d + L_u} \rho_{\text{crit},d}
\] (15)

Where \(L_d\) and \(L_u\) are the distances to the closer downstream and upstream detectors and \(V_{\text{free},d}, a_d, \rho_{\text{crit},d}, V_{\text{free},u}, a_u, \rho_{\text{crit},u}\) are the values of the fundamental diagram parameters (obtained in the first step) of the closer downstream and upstream detectors.
2) Pre-evaluation of the speed equation parameters: With an initial value of the fundamental diagrams defined, the next step is to obtain an estimation of the parameters of the speed equation: $\tau, K, \mu_h, \mu_l$, and $\delta$. In this case, these variables are considered equal for all the segment in the network. Papageorgiou et al. [8] demonstrated that the model is most sensitive with respect to the values of the parameters used in the fundamental diagram equation. This justify to consider equals the speed equation parameters for the full network. These values will be set in order to minimize the following error function:

$$J_{part2} = \sum (\rho_l(k) - \rho_h(k))^2 + 0.1(\rho_l(k) - \rho_m(k))^2 +$$

$$+ 0.1(\rho_l(0) - \rho_m(0))^2 + 0.1(\tau_l(k) - \tau_h(k))^2 +$$

$$+ 0.1(\tau_l(0) - \tau_m(0))^2 + 0.1(\delta_l(k) - \delta_h(k))^2 +$$

$$+ 0.1(\delta_l(0) - \delta_m(0))^2$$

(16)

where $\rho_l(k)$, $\rho_m(k)$ and $\rho_m(a)$ are the measured densities on sensor Huntington, Mountain and Santa Anita respectively. $\tau_l(k)$, $\tau_m(k)$ and $\tau_m(a(k))$ are the measured speed for these detectors. The weights are higher for the segment corresponding to the “missing” density of the detector Huntington. Because we are using only five decision variables, it is possible to run a “evaluation” before the optimization so this optimization starts as close as possible to the global minimum. Therefore, $J_{part2}$ is evaluated for a “net” of values for the speed equation parameters. The point with lower error cost function is chosen as initial point for the next step.

3) Optimization of the speed equation parameters: Taking the fundamental diagrams defined in IV.A.1) and the initial point obtained in IV.A.2), an SQP optimization algorithm (minimizing $J_{part2}$) can be run. The optimization considers the fundamental diagram parameters as known.

C. Optimization Third Step

Step 1 and 2 give a estimation of the parameters that is, in theory, close to the global minimum. This set of variables can be used as initial point for a global optimization that considers as decision variables all the parameters of the model. The total number of variables for the network proposed in section III is 14: 5 speed variables + 3*3 FD variables. Step 3 runs a SPQ optimization algorithm in order to minimize $J_{part3} = J_{part2}$ in a global way.

V. NUMERICAL RESULTS

A. Optimization First Step

The results given in Table I were obtained at the middle sensor (“Huntington”) when running the algorithm proposed in IV.A for the analyzed network. It can be seen how the accumulated error using the typical fundamental diagram of METANET almost doubles the other two options.

B. Optimization Second Step

As explained in IV.B, a pre-evaluation of the cost function for a "net of values" was done before the optimization. The value of the parameters evaluated with lower cost function was:

$$x_{Ev} = [\tau, \mu_l, \mu_h, \delta, K] = [40, 30, 45, 1, 45]$$

With an associated cost function of: $J_{part2} = 5.8687e+004$ Taking xEv as initial point, the SQP optimization algorithm converges to:

$$x_{part2} = [38.9041, 30.9310, 46.3964, 1.45, 46.593]$$

With an associated cost function of: $J_{part2} = 4.7263e+004$

C. Optimization Third Step

Table II shows the final values obtained after the final step of the optimization. In the table, $V_{free\_j}$, $a_j$ and $\rho_{crit\_j}$ with $j = m, h, sa$ are the free flow speed, the parameter “a” and the critical density associated to the segment with the loop detectors Mountain, Huntington and Santa Anita, respectively.

D. Simulation Results

Table III shows the mean relative error for the density of each day (17) and the mean error for all the days.

$$\text{DensityError} = \frac{\sum_{k=1}^{K} ||\rho_l(k) - \rho_h(k)||}{\rho_h(k)}$$

(17)

where $K$ is the total number of measures. In this paper, $K = 84$ because there are available measures every 5 minutes during 7 hours:

$$K = 7\text{hours} / 5\text{minutes} = 84.$$  

The mean error of the speeds is 11.35% for the 10 days. The identification days were 22th and 29th of January, 26th of

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**TABLE I**

**Step 1 Results**

<table>
<thead>
<tr>
<th>FD</th>
<th>$V_{free}$</th>
<th>$\rho_{crit}$</th>
<th>$a$</th>
<th>$b$</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>FD 1</td>
<td>72.07</td>
<td>42.77</td>
<td>2.17</td>
<td>4.68*10^8</td>
<td></td>
</tr>
<tr>
<td>FD 2</td>
<td>63.12</td>
<td>31.11</td>
<td>196</td>
<td>0.91</td>
<td>2.75*10^8</td>
</tr>
<tr>
<td>FD 3</td>
<td>63.04</td>
<td>30.83</td>
<td>0.89</td>
<td>2.79*10^8</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE II**

**Final parameter estimation**

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\mu_l$</th>
<th>$\mu_h$</th>
<th>$\delta$</th>
<th>$K$</th>
<th>$a_m$</th>
<th>$V_{free_m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.07</td>
<td>22.60</td>
<td>99.96</td>
<td>4.19</td>
<td>21.63</td>
<td>1.57</td>
<td>57.72</td>
</tr>
<tr>
<td>$\rho_{crit_m}$</td>
<td>$a_h$</td>
<td>$V_{free_h}$</td>
<td>$\rho_{crit_h}$</td>
<td>$a_s$</td>
<td>$V_{free_sa}$</td>
<td>$\rho_{crit_sa}$</td>
</tr>
<tr>
<td>34.93</td>
<td>0.90</td>
<td>63.12</td>
<td>31.10</td>
<td>1.11</td>
<td>58.08</td>
<td>34.72</td>
</tr>
</tbody>
</table>

**TABLE III**

**Simulation Results**

<table>
<thead>
<tr>
<th>Day</th>
<th>Simulation Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22th January</td>
<td>9.66</td>
</tr>
<tr>
<td>29th January</td>
<td>7.49</td>
</tr>
<tr>
<td>6th February</td>
<td>11.04</td>
</tr>
<tr>
<td>12th February</td>
<td>14.10</td>
</tr>
<tr>
<td>26th February</td>
<td>8.2</td>
</tr>
<tr>
<td>2th April</td>
<td>8.85</td>
</tr>
<tr>
<td>9th April</td>
<td>12.62</td>
</tr>
<tr>
<td>20th April</td>
<td>9.07</td>
</tr>
<tr>
<td>17th May</td>
<td>16.04</td>
</tr>
<tr>
<td>4th June</td>
<td>12.88</td>
</tr>
<tr>
<td>Mean</td>
<td>10.95</td>
</tr>
</tbody>
</table>
February and 2nd and 20th of April. The rest of the days are used for the simulation but not for the identification in order to validate the data.

Fig. 5 shows the densities and velocities measured in the sensor ML 33.05 and the values estimated by the model. It is possible to see how the identified model shows a good estimation in both density and speed, validating the identification algorithm.

VI. CONCLUSIONS

This paper proposes an identification algorithm for the macroscopic traffic model METANET that tries to avoid undesirable local minima. The algorithm was tested with real data from the I-210 freeway in Pasadena, California. The results show a good estimation of the traffic densities and speeds validating the identification algorithm with a mean error of 10.95% and 11.35%, respectively. Moreover, a new form of the fundamental diagram is proposed. This definition allows to improve the match between the fundamental diagram and the data without an increase in the computational power needed for the simulation. The identification algorithm proposed may be especially useful for traffic control using Model Predictive Control, where the accuracy of the model and the possibility of executing the identification in real time are key issues.

VII. ACKNOWLEDGMENTS

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REFERENCES