Distributed Economic Model Predictive Control of Networks in Competitive Environments

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Abstract—This paper addresses the control of large-scale networks of dynamical systems with a certain global objective that needs to be pursued through the actions of individual competing agents. In particular, we consider the stabilization of a specific network output at zero. This is challenging as each agent is interested only in its own objectives such as the maximization of its economic profit. We introduce a control scheme based on the economic Model Predictive Control (MPC) theory to optimize economic performance of the competing agents. The theory is then modified to take global, network-wide objectives into account as well. Additionally, it is shown that the control scheme can be distributed between the agents using the dual decomposition technique such that exchange of confidential information among these competitors is not required. Simulation results for a power system example illustrate the potential of the control strategy in terms of ensuring stable and economical operation of networks in competitive environments.

I. INTRODUCTION

Control of large-scale networks of interconnected dynamical systems is a challenging problem due to their size, complexity and communication restrictions. This is the main motivation for research in the field of non-centralized control, in which the overall control action is formed by a set of local feedback laws. In many of these schemes the control actions are formed by sharing local measurements and information among all the controllers in the system, see for example [1]–[3]. However, in economic applications these controllers are often assigned to different market agents that are not willing to exchange their confidential information with competitors.

In this paper we focus on an application that involves $M$ competing agents that are dynamically coupled via a physical interconnection network, see Figure 1. The global objective of ensuring safe operation of the network has to be pursued via suitable actions of all the agents. However, each agent controls its own subsystem with the objective to maximize its own profit, which can have a negative impact on the performance and stability of the network as a whole. An independent network operator therefore interacts with the agents to obtain local, agent-specific control actions that support the global objective, in this case the stabilization of the output of the network at zero.

Examples of these applications are frequency control in liberalized electrical power networks [4], water level management in water supply infrastructures [5] and control of autonomous vehicles on automated highways [6]. The global network objectives in these applications are power supply and demand balancing, water level balancing, and congestion avoidance and traffic throughput maximization, respectively.

The purpose of this study is to design a control scheme for the $M$ competitive agents, that stabilizes the network, maximizes the total economic performance of all the agents, takes into account constraints on the system and ensures that the global objective is satisfied in steady state. The provided controller is based on the framework of Economic Model Predictive Control (MPC), see [7]–[9]. Economic MPC is a natural choice for the problem at hand because it is capable of maximizing economic performance while guaranteeing that constraints are satisfied. The scheme is modified in such a way that the network-wide objective of output stabilization is pursued. Dual decomposition [10], [11] is used to distribute the optimization problem of the MPC controller among the agents and the network operator. In the resulting distributed control scheme [12] the agents are not required to share confidential information such as their internal subsystem model and economic cost function with each other.

This paper is structured as follows. Section II describes the notation and recalls the stability theorems that are instrumental for the design of the control scheme provided in this paper. In Section III we describe how the network is modeled and introduce the formal control problem. A centralized control scheme is described in Section IV. This scheme underlies the distributed implementation provided in Section V. In Section VI the theory is applied to a power network to illustrate the potential of the proposed control method. Conclusions are drawn in Section VII.

II. PRELIMINARIES

A. Notation

Let $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{Z}^+$ be the sets of real, non-negative real and non-negative integer numbers respectively. For any time-
dependent vector \( x \in \mathbb{R}^n \), let \( x(k) \) denote the actual value at discrete time instant \( k \in \mathbb{Z}_+ \) and let \( x(i|k) \) denote the predicted future value at time \( k+i \) given \( x(k) \). \( ||x|| \) denotes an arbitrary norm on \( x \in \mathbb{R}^n \). Lastly, the distance of \( x \in \mathbb{R}^n \) to a set \( X \subseteq \mathbb{R}^n \) is defined as \( d(x, X) := \inf_{y \in X} ||x - y|| \).

**B. Stability**

Consider an arbitrary discrete-time time-invariant system
\[
x(k + 1) = f(x(k)), \quad k \in \mathbb{Z}_+ \tag{1}
\]
where \( x(k) \in \mathbb{R}^n \) is the state at discrete time instant \( k \) and \( f : \mathbb{R}^n \to \mathbb{R}^n \) is a continuous function. Let \( x_s \in \mathbb{R}^n \) be an equilibrium of (1), i.e., \( x_s = f(x_s) \). Let closed set \( X \subseteq \mathbb{R}^n \) and \( x_s \in \text{interior}(X) \).

**Definition 1** (Stability). (i) System (1) is Lyapunov stable at equilibrium \( x_s \) if \( \forall \epsilon > 0, \exists \delta(\epsilon) > 0 \) such that for each \( x(0) \) with \( ||x(0) - x_s|| < \delta(\epsilon) \), then \( ||x(k) - x_s|| < \epsilon \) for all \( k \in \mathbb{Z}_+ \). (ii) System (1) is asymptotically stable in \( X \) at equilibrium \( x_s \) if the system is Lyapunov stable and for any \( x(0) \in X \) it holds that \( \lim_{k \to \infty} ||x(k) - x_s|| = 0 \).

**Definition 2** (Lyapunov function). A function \( V : \mathbb{R}^n \to \mathbb{R}_+ \) is a Lyapunov function for (1) on \( X \) if (i) \( V \) is continuous and (ii) \( \Delta V(x) = V(f(x)) - V(x) \leq 0 \) for all \( x \in X \).

**Definition 3** (Positive (semi-)definite function). A function \( V : \mathbb{R}^n \to \mathbb{R}_+ \) is positive (semi-)definite with respect to \( x_s \) if (i) \( V(x_s) = 0 \) and (ii) \( V(x) \geq 0 \) for \( x \neq x_s \) and positive definite if in addition (iii) \( V(x) > 0 \) for \( x \neq x_s \).

**Definition 4** (Invariant set). A set \( Y \subseteq \mathbb{R}^n \) is positively invariant for system (1) if \( \forall x \in Y \) it holds that \( f(x) \in Y \).

Let \( V \) be a Lyapunov function for (1) on \( X \). Furthermore, let \( Y \) and \( \mathbb{W} \) be the largest positively invariant sets contained in \( \{ x \in X | V(x) = 0 \} \) and \( \{ x \in X | \Delta V(x) = 0 \} \) respectively. Then the following theorems hold

**Theorem 1** (Lyapunov stability, [13]). If (i) \( V \) is positive semi-definite with respect to \( x_s \), and (ii) (1) is asymptotically stable in \( Y \) at \( x_s \), then (1) is Lyapunov stable at \( x_s \).

**Theorem 2** (LaSalle’s invariance principle, [14]). If \( x(k) \in X \) and \( x(k) \) is bounded for \( k \in \mathbb{Z}_+ \), then \( d(x(k), \mathbb{W}) \to 0 \) for \( k \to \infty \).

**III. MODEL AND PROBLEM DESCRIPTION**

The system considered in this paper consists of \( M \) agent subsystems that are coupled via an interconnection network. A block diagram of the model is depicted in Figure 1.

**A. Network model**

The discrete-time time-invariant model of the interconnection network is described in state space as
\[
x_{\text{net}}(k + 1) = f_{\text{net}}(x_{\text{net}}(k), y_{\text{net}}(k), r_{\text{net}}(k)) \tag{2a}
\]
\[
y_{\text{net}}(k) = h_{\text{net}}(x_{\text{net}}(k)) \tag{2b}
\]
\[
w_{\text{net},m}(k) = C_{\text{net},m} x_{\text{net}}(k), \quad k \in \mathbb{Z}_+, \tag{2c}
\]
\( m = 1, \ldots, M \), with states \( x_{\text{net}} \), reference inputs \( r_{\text{net}} \), coupling inputs \( v_{\text{net}} = [v_{\text{net},1}^T \ldots v_{\text{net},M}^T]^T \), outputs \( y_{\text{net}} \) and coupling outputs \( w_{\text{net},m} \) for \( m = 1, \ldots, M \).

Apart from the above dynamics, the network model also includes input and state constraints given by
\[
g_{\text{net}}(x_{\text{net}}(k), r_{\text{net}}(k)) \leq b_{\text{net}}, \quad \forall k \in \mathbb{Z}_+. \tag{3}
\]

**B. Agent models**

The discrete-time time-invariant model of the subsystem of each agent \( m \) is described in state space as
\[
x_m(k + 1) = f_m(x_m(k), u_m(k), v_m(k), r_m(k)) \tag{4a}
\]
\[
w_m(k) = C_m x_m(k) \tag{4b}
\]
for \( k \in \mathbb{Z}_+ \) with controllable inputs \( u_m \). Applying inputs \( u_m \) results in costs for agent \( m \) which are given by the strictly convex economic cost function
\[
J_m(u_m). \tag{5}
\]

Apart from the above dynamics, the model also includes agent-specific input and state constraints given by
\[
g_m(x_m(k), u_m(k), r_m(k)) \leq b_m, \quad \forall k \in \mathbb{Z}_+. \tag{6}
\]

**C. Interconnected prediction model**

The subsystems of the agents are coupled to the interconnection network via the inputs \( v_{\text{net}} \) and \( v_m \) and outputs \( w_{\text{net}} \) and \( w_m \), for \( m = 1, \ldots, M \). The coupling is described by the following constraints
\[
v_{\text{net},m} = w_m \tag{7a}
\]
\[
v_m = w_{\text{net},m}. \tag{7b}
\]
for \( m = 1, \ldots, M \).

The prediction model of the total system is obtained by combining (7) with the models (2) and (4). The resulting model is given by
\[
x(k + 1) = f(x(k), u(k), r(k)) \tag{8a}
\]
\[
y(k) = y_{\text{net}}(k) = h(x(k)), \tag{8b}
\]
with states \( x = [x_{\text{net}}^T \ldots x_{M}^T]^T \), reference inputs \( r = [r_{\text{net}}^T \ldots r_{M}^T]^T \) and controllable inputs \( u = [u_1^T \ldots u_M^T]^T \).

Constraints (3) and (6) are combined resulting in the seperable constraints
\[
g(x(k), u(k), r(k)) \leq b, \quad \forall k \in \mathbb{Z}_+. \tag{9}
\]

**D. Problem description**

Let \( r(k) := \{r(0|k), \ldots, r(N-1|k)\} \) be a finite sequence of future reference inputs determined at time instant \( k \in \mathbb{Z}_+ \). Also, let the total economic costs be denoted by
\[
J(u(k)) = \sum_{m=1}^{M} J_m(u_m(k)). \tag{10}
\]

**Problem 1.** Given network dynamics (8), find a state feedback control law \( u(k) = \kappa(x(k), r(k)) \) such that the closed
loop system \( x(k + 1) = f(x(k), \kappa(x(k), r(k)), r(k)) \) is feasible with respect to (9) at all time instants \( k \in \mathbb{Z}_+ \). Moreover, for constant reference inputs \( r_s \) and \( r(k) = \{r_s, \ldots, r_s\}, \forall k \in \mathbb{Z}_+ \), the closed loop system must be Lyapunov stable and the control law must ensure that both the output \( y \) converges to zero, i.e., \( \lim_{k \to \infty} ||h(x(k))|| = 0 \) and the input converges to the economic optimum, i.e., \( \lim_{k \to \infty} h(x(k)) = 0 \) and \( \lim_{k \to \infty} u(k) = u_s \).

### IV. Control Scheme

We introduce a centralized control scheme that forms the basis for the distributed control law given in Section V. The control scheme is based on the standard economic MPC theory introduced in [7], [8] to optimize economic performance of the controlled system while guaranteeing closed loop stability. The theory is modified to suit our control problem, such that it no longer requires a unique steady state and guarantees convergence of the output \( y \) to zero.

#### A. Steady state problem

At time instant \( k \), given reference input \( r(N - 1 | k) \), the economically optimal steady state and input \( \{x_s(k), u_s(k)\} \) are the optimizers of the following optimization problem

\[
\min_{x,u} \ J(u) \quad \text{subject to} \quad x = f(x, u, r(N - 1 | k)) \quad \text{and} \quad g(x, u, r(N - 1 | k)) \leq b \quad \text{and} \quad h(x) = 0.
\]

In contrast to the steady state solution in [8], the solution to (11) is dependent on the time-varying reference input and must therefore be calculated at each time instant. Moreover, the steady state \( x_s(k) \) is not necessarily unique and the constraint (11d) is added to guarantee that a steady state is selected for which the output \( y \) is zero. Problem 1 is only solvable if constraint (11d) can be satisfied.

#### B. Transient optimization problem

Let \( x(k) := \{x(0 | k), \ldots, x(N | k)\} \) and \( u(k) := \{u(0 | k), \ldots, u(N - 1 | k)\} \) denote the horizon-N predicted state and input trajectories of the system respectively.

At time instant \( k \) with state \( x(k) \), optimal steady state \( x_s(k) \) and reference input sequence \( r(k) \), the optimal future state and input trajectories \( \{x^*(k), u^*(k)\} \) are the optimizers of the following optimization problem

\[
\min_{x(k), u(k)} \sum_{j=0}^{N-1} \ell(x(j | k), u(j | k)) \quad \text{subject to} \quad x(0 | k) = x(k) \quad x(i + 1 | k) = f(x(i | k), u(i | k), r(i | k)) \quad g(x(i | k), u(i | k), r(i | k)) \leq b \quad x(N | k) = x_s(k),
\]

for \( i = 0, \ldots, N - 1 \) and some stage cost function \( \ell(x, u) \).

The state feedback control law \( u(k) = \kappa(x(k), r(k)) \) is summarized by the following algorithm

**Algorithm 1.** At each time instant \( k \in \mathbb{Z}_+ \),

1. Obtain \( x(k) \) and \( r(k) \).
2. Compute \( \{x_s(k), u_s(k)\} \) with (11) using \( r(N - 1 | k) \).
3. Compute \( u^*(k) \) with (12) using \( x(k), r(k) \) and \( x_s(k) \).
4. Apply \( u(k) = \kappa(x(k), r(k)) := u^*(0 | k) \).

#### C. Stability and convergence

We now address stability, economic cost minimization in steady state and output convergence given the following assumptions.

**Assumption 1** (Reference input known exactly). The reference input is known exactly for the entire prediction horizon.

**Assumption 2** (Strong duality). Strong duality holds for steady state optimization problem (11).

Under Assumptions 2 and with \( r_s = r(N - 1 | k) \), optimization problem (11) can be written as

\[
\min_{x,u} \ J(u) + (x - f(x, u, r_s))^T \lambda_s^x + h(x)^T \lambda_s^h \quad \text{subject to} \quad g(x, u, r_s) \leq b,
\]

where \( \lambda_s^x \) and \( \lambda_s^h \) are the optimal Lagrangian multipliers obtained from the dual problem of (11)

\[
\max_{\lambda_s^x, \lambda_s^h} \min_{x,u} \ J(u) + (x - f(x, u, r_s))^T \lambda_s^x + h(x)^T \lambda_s^h \quad \text{subject to} \quad g(x, u, r_s) \leq b.
\]

In what follows, we consider the following stage cost function for use in optimization problem (12).

\[
\ell(x, u) = J(u) + h(x)^T \lambda_s^x + \beta(h(x)),
\]

where \( \lambda_s^x \) is computed each time instant in problem (11) and \( \beta(x) : \mathbb{R}^n \to \mathbb{R}_+ \) is a positive definite continuous function.

Stage cost function (15) does not satisfy Assumption 2 in [8] since this would require penalizing the full state. In the control problem considered here the actual state values are not of interest and are therefore not penalized. Instead the outputs \( y \) are penalized as they are the only variables required to converge to zero. Because the assumptions in [8] are not satisfied, the theorem that ensures asymptotic stability of the closed loop system is not applicable. However, Lyapunov stability is guaranteed by the following theorem.

**Theorem 3** (Lyapunov stability and convergence). Let system (8) be stabilizable and Assumptions 1 and 2 hold. If the reference input is constant such that the optimal steady state and input are constant, i.e., \( r(k) = r_s \) and \( \{x_s(k), u_s(k)\} = \{x_s, u_s\} \) for \( k \in \mathbb{Z}_+ \), and if optimization problems (11) and (12) are feasible for initial state \( x(k) \) and reference input sequence \( r = \{r_s, \ldots, r_s\} \), then the closed loop system \( x(k + 1) = f(x(k), \kappa(x(k), r), r_s) \) with stage cost function (15) is Lyapunov stable at steady state \( x_s \). Furthermore, the output \( y = h(x) \) converges to zero and the input converges to the economic optimum, i.e., \( \lim_{k \to \infty} h(x(k)) = 0 \) and \( \lim_{k \to \infty} u(k) = u_s \).
The proof can be found in the Appendix.

V. DISTRIBUTED IMPLEMENTATION

Straightforward implementation of Algorithm 1 is not feasible in competitive environments because the algorithm requires knowledge of the internal models and cost functions of all the $M$ agents, which are usually not willing to share this confidential information with their rivals.

This section shows how the requirement to share confidential information can be eliminated by decomposing the optimization problems (11) and (12) in Algorithm 1. We use the decomposition method proposed in [12], which is based on the dual decomposition method, see, e.g., [11]. We decompose steady state optimization problem (11), but a similar decomposition can be performed on optimization problem (12).

Using (7), problem (11) can be rewritten as

$$\begin{align}
\min_{x,m} & \quad J(u)
\text{s.t.} & \quad x_m = f_m(x_m, v_m, r_m(N - 1|k)) \quad (16a)
& \quad g_m(x_m, v_m, r_m(N - 1|k)) \leq b_m \quad (16c)
& \quad r_h(x_m) = 0 \quad (16d)
& \quad x_m = f_m(x_m, u_m, v_m, r_m(N - 1|k)) \quad (16e)
& \quad g_m(x_m, v_m, r_m(N - 1|k)) \leq b_m \quad (16f)
& \quad v_{net,m} = C_m x_m \quad (16g)
& \quad v_m = C_{net,m} x_{net} \quad (16h)
\end{align}$$

for $m = 1, \ldots, M$.

Let $\lambda_{net} = \{\lambda_{net,1}, \ldots, \lambda_{net,M}\}$ and $\lambda = \{\lambda_1, \ldots, \lambda_M\}$.

The partial dual of (16) with dualized constraints (16g)-(16h) is given by

$$\begin{align}
\max_{\lambda_{net}, \lambda} & \quad V_{net}(\lambda_{net}, \lambda) + \sum_{m=1}^{M} V_m(\lambda_{net,m}, \lambda_m) \quad (17)
\end{align}$$

where $V_{net}$ can be calculated by the network operator without requiring the agent’s internal models, states or cost functions and is defined as

$$V_{net}(\lambda_{net}, \lambda) := \min_{x_{net}, v_{net}} \sum_{m=1}^{M} \left[ v_{net,m}^T \lambda_{net,m} + (C_{net,m} x_{net})^T \lambda_m \right]$$

subject to (16b)–(16d). Problem $V_m$ can be calculated by agent $m$ using only its own internal model and cost function and is defined as

$$V_m(\lambda_{net,m}, \lambda_m) := \min_{x_m, u_m, v_m} J_m(u_m) - v_m^T \lambda_m - (C_m x_m)^T \lambda_{net,m} \quad (19)$$

subject to (16e)–(16f). In many applications, the variables $v_m$ and $u_m = C_m x_m$ represent shared resources between the agents and the interconnection network and $\lambda_m$ and $\lambda_{net,m}$ can be interpreted as the prices for these resources respectively. The term $v_{net,m}^T \lambda_{net,m} + (C_{net,m} x_{net})^T \lambda_{net,m}$ can be interpreted as the reward for agent $m$ and the objective function in (19) then represents the negative of the profit of the agent. In accordance with (19) each agent thus selects its inputs in such a way that its profit is maximized.

Problem (17) can be solved iteratively using a subgradient optimization method [10]. Let $\lambda_{net}^{(\gamma)}$ and $\lambda_m^{(\gamma)}$ denote the $\gamma$-th iterate and $\alpha^{(\gamma)} \in \mathbb{R}_+$ the $\gamma$-th step size, where $\gamma \in \mathbb{Z}_+$. The method for solving steady state optimization problem (11) using dual decomposition is given by the following algorithm.

**Algorithm 2.** Given some initial values $\lambda_{net}^{(0)}$ and $\lambda_m^{(0)}$ for iteration $\gamma = 0$, repeat:

1. Solve subproblems: for $m = 1, \ldots, M$
   - $V_m(\lambda_{net}^{(\gamma)}, \lambda_m^{(\gamma)})$ with optimizers $x_m^{(\gamma)}(\lambda_{net}^{(\gamma)}, \lambda_m^{(\gamma)})$ and $u_m^{(\gamma)}(\lambda_{net}^{(\gamma)}, \lambda_m^{(\gamma)})$
   - $\lambda_{net,m}^{(\gamma+1)} = \lambda_{net,m}^{(\gamma)} + \alpha^{(\gamma)} (\lambda_{net,m}^{(\gamma)} - C_m x_m^{(\gamma)})$
   - $\lambda_m^{(\gamma+1)} = \lambda_m^{(\gamma)} - \alpha^{(\gamma)} (v_m^{(\gamma)} - C_{net,m} x_{net}^{(\gamma)})$
2. Update the Lagrangian multipliers: for $m = 1, \ldots, M$
   - $\lambda_{net,m}^{(\gamma+1)} = \lambda_{net,m}^{(\gamma)} + \alpha^{(\gamma)} (v_m^{(\gamma)} - C_{net,m} x_{net}^{(\gamma)})$
   - $\lambda_m^{(\gamma+1)} = \lambda_m^{(\gamma)} - \alpha^{(\gamma)} (v_m^{(\gamma)} - C_{net,m} x_{net}^{(\gamma)})$
3. Increase the iteration number: $\gamma := \gamma + 1$

Now consider the following theorem.

**Theorem 4** (Convergence to optimum). If the sequence of step sizes $\alpha^{(\gamma)}$ is chosen such that $\lim_{\gamma \to \infty} \alpha^{(\gamma)} = 0$ and $\sum_{\gamma=1}^{\infty} \alpha^{(\gamma)} = \infty$, then the solution of the subgradient method converges to the optimum [10, Theorem 2.3]. Thus, in this case Algorithm 2 converges to the optimizers $u_s = [u_1^{T,s}, \ldots, u_M^{T,s}]$ and $x_s = [x_1^{T,s}, x_1^{T,s}, \ldots, x_M^{T,s}]$ of (11), i.e., $\lim_{\gamma \to \infty} x_{net}^{(\gamma)} = x_{net,s}$, $\lim_{\gamma \to \infty} x_m^{(\gamma)} = x_{m,s}$ and $\lim_{\gamma \to \infty} u_m^{(\gamma)} = u_{m,s}$.

Given Theorem 4, Algorithm 2 provides a method to calculate the optimization problems (11) and (12) in Algorithm 1 in a distributed manner.

VI. RESULTS

We apply the described theory to a frequency control problem for a small electrical power network [4], [15] to illustrate the potential of the control method provided in Algorithm 1. The simulated power network consists of three zones interconnected by limited capacity power lines. Each zone contains one generator as shown in Figure 2.

The interconnection network model is composed of the combined dynamics of the zones and the power lines. A zone is modeled as a ‘copper-plate’ in which all transmission constraints and internal losses are neglected [4]. A block diagram of a zone $z$ is depicted in Figure 3a, where $v_{net,z}[W]$ is the total power generation in zone $z$, $r_{net,z}[W]$ is the power consumption in zone $z$, $p_{z,[W]}$ is the power flow over the power line between zone $z$ and $\tilde{z}$ and $\Delta \omega_{z}[\text{rad/s}]$ is the
The output of the interconnection network is the frequency deviation at zero. The system-wide control objective is to stabilize the frequency deviation with respect to the nominal frequency (50Hz in Europe). Power flows \( p_{z,\tilde{z}} \) are modeled using the linear DC power flow model [16] and are given by

\[
p_{z,\tilde{z}} = (\delta_z - \delta_{\tilde{z}}) b_{z,\tilde{z}}.
\]  

(20)

The three generators are operated by three competing agents. The internal models of the agents are given by the dynamics of the generators and therefore based on the linearized model of a steam turbine given in [15] and depicted in Figure 3b, where \( w_m \) is the actual power generated by generator \( m \) and \( r_m \) the power setpoint of generator \( m \).

A list of all parameter values can be found in [17].

We simulate an unpredicted stepwise increase in power consumption with 25 MW in zone 3 at time instant \( k = 10 \). The generator inputs \( u_m \) are limited to \(-15 \leq u_m \leq 15\) MW and the capacity of the power lines is limited to 10 MW each. The simulation results of the centralized control scheme, i.e., Algorithm 1, are presented in Figure 4.

The simulation results confirm that the control scheme satisfies the requirements in Problem 1. Firstly, the control law ensures that the constraints are satisfied at all time instants as shown in Figure 4b and Figure 4c for the input constraints and power line power flow constraints respectively. Finally, the controller drives the frequency deviations to zero as shown in Figure 4a and minimizes the economic costs \( J(u) \) in steady state as shown in Figure 4b where the optimal inputs \( u_s \) are indicated by the small dotted lines.

VII. CONCLUSION

This paper focused on the control of large-scale networks of interconnected dynamical systems with input and state constraints that are operated in an economic environment. This means that the global objectives of the network, in our case output stabilization at zero, need to be pursued through the actions of multiple competing agents. This is challenging as each agent is interested only in its own objectives such as maximization of its economic profit. By modifying the existing theory on economic MPC, a control solution was obtained that guarantees stabilization of the output and satisfaction of the constraints, while maximizing economic performance of the agents. Furthermore, the control scheme was formulated in a way that allowed for distributed implementation among the competing agents based on a dual decomposition technique. In contrast to the centralized implementation, the distributed control scheme does not require exchange of confidential information among competitors. Simulation results for a power system example illustrated the potential of the control strategy in terms of ensuring stable and economical operation of networks in competitive environments.

APPENDIX I

PROOF OF THEOREM 3

In this appendix we give the proof of Theorem 3, which follows along similar steps as in [8, Theorem 1].

Proof: Consider the rotated stage cost function

\[
L(x, u) := \ell(x, u) + (x - f(x, u, r_s))^\top \lambda_s^x - \ell(x_s, u_s).
\]  

(21)

From Assumption 2, (15) and strict convexity of \( J(u) \), it follows that \( L(x, u) \) satisfies the following properties

\[
L(x_s, u_s) = 0 \tag{22a}
\]

\[
L(x, u) \geq 0, \quad u = u_s, \tag{22b}
\]

\[
L(x, u) > 0, \quad u \neq u_s, \tag{22c}
\]

\[
L(x, u) \geq \beta(h(x)), \tag{22d}
\]

for all \( \{x, u\} \) such that \( g(x, u, r_s) \leq b \).
The rotated transient optimization problem is given by
\[ V^*(x(k)) := \min_{x(k),u(k)} \sum_{j=0}^{N-1} L(x(j|k),u(j|k)) \]
subject to (12b)-(12e) for \( i = 0, \ldots, N - 1 \) with \( x_s(k) = x_s \). Optimization problems (12) and (23) have the same optimizer \( u^*(k) \) since the objective functions and optimal costs of both problems only differ by a constant term. Using \( x_j \) instead of \( x(j|k) \) to simplify notation, the objective function of (23) can be rewritten as
\[
\sum_{j=0}^{N-1} \left[ L(x_j, u_j) \right]
\]
\[ = \sum_{j=0}^{N-1} \left[ \ell(x_j, u_j) + (x_j - f(x_j, u_j, r_s))^\top \lambda_s^x - \ell(x_s, u_s) \right]
\]
\[ + (x_0 - x_N)^\top \lambda_s^x \]
\[ = \sum_{j=0}^{N-1} \left[ \ell(x_j, u_j) - \ell(x_s, u_s) \right] + (x_0 - x_s)^\top \lambda_s^x + (x_s - x_N)^\top \lambda_s^x \]
where we used constraints (12b)-(12e) to eliminate terms. The terms \( (x(k) - x_s)^\top \lambda_s^x \) and \( N\ell(x_s, u_s) \) are constant and therefore optimization problems (12) and (23) have the same optimizer \( u^*(k) \).

Next, we prove that \( V^*(x(k)) \) is a Lyapunov function for the closed loop system \( x(k+1) = f(x(k), \nu(x(k), r), r_s) \).

Consider the optimal input sequence \( u^*(k) \) obtained with either (12) or (23) at time instant \( k \). If Assumption 1 holds, then the following input sequence is feasible at time instant \( k+1 \)
\[
\hat{u}(k+1) = \{u^*(1|k), \ldots, u^*(N-1|k), u_s\}. \tag{24}
\]

Let
\[
V(x(k), u(k)) = \sum_{j=0}^{N-1} L(x(j|k),u(j|k)) \text{ where } \begin{cases} x(0|k) = x(k) \text{ and } f(x(i|k),u(i|k),r_s) = x(i+1|k) \text{ for } i = 0, \ldots, N - 1 \end{cases}
\]
It follows that
\[
V^*(x(k+1)) = V(x(k+1), u^*(k+1)) \tag{25a}
\]
\[
\leq V(x(k+1), \hat{u}(k+1)) \tag{25b}
\]
\[
= V(x(k), u^*(k)) + L(x_s, u_s) - L(x(k), u^*(0|k)) \tag{25c}
\]
\[
= V^*(x(k)) - L(x(k), u^*(0|k)), \tag{25d}
\]
where the last equality follows from (22a). Properties (22) ensure that \( V^*(x(k+1)) - V^*(x(k)) = \Delta V^*(x(k)) \leq 0 \). This shows that \( V^*(x(k)) \) is a Lyapunov function conform Definition 2. Moreover, the properties show that \( V^*(x(k)) \geq 0 \) and if \( x(k) = x_s \) then the input sequence \( u = \{u_s, \ldots, u_s\} \) is feasible and consequently \( V^*(x_s) = 0 \), which shows that \( V^*(x(k)) \) is positive semi-definite with respect to \( x_s \). Finally, \( V^*(x(k)) = 0 \) only holds if \( u^*(k) = u \), \( h(x(i|k)) = 0 \) for \( i = 0, \ldots, N - 1 \) and the system converges to \( x_s \) within the prediction horizon. If \( x(k) \in \mathcal{Y} = \{x|V^*(x) = 0\} \), then by (25) it follows that \( x(k+1) \in \mathcal{Y} \). The controller will repeatedly apply the input \( \nu(x(k), r) = u_s \), and hence the system converges to \( x_s \) within the prediction horizon. The closed loop system is asymptotically stable for all initial states \( x(k) \in \mathcal{Y} \).

Theorem 1 ensures that the closed loop system \( x(k+1) = f(x(k), \nu(x(k), r), r_s) \) is Lyapunov stable and Theorem 2 ensures convergence of the state to a subset of set \( \mathcal{W} = \{x|\Delta V^*(x) = 0\} \). This proves convergence of the input \( u \) and output \( y \), since (22) and (25) ensure that \( \forall x \in \mathcal{W} \) we have \( u = u_s \) and \( y = h(x) = 0 \).