A Norm Optimal Iterative Learning Control Based Train Trajectory Tracking Approach*

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Abstract—A norm optimal iterative learning control (NOILC) is proposed and applied in train trajectory tracking problem, and it then is extended to the cases with traction/braking constraint. Rigorous theoretical analysis has shown that the proposed approach can guarantee the asymptotic convergence of train speed and position to desired profiles as iteration number goes infinity. Simulation results further demonstrate the effectiveness of the proposed NOILC approach.

I. INTRODUCTION

In recent decades, high-speed railway is developing with great achievements worldwide, and the automatic driving systems become an inevitable technique due to the high safety requirement of high-speed trains. Therefore, automatic train operation (ATO), together with automatic train protection (ATP) and automatic train supervision (ATS), has become the key technology for the train control and monitoring to ensure safety and comfort.

Speed regulation is one of the main tasks of ATO, along with programmed stopping, performance level regulation, etc. [1]. At present, various control methods have been developed to achieve this task, such as proportional integral derivative (PID) [2], fuzzy logic control [3], linear quadratic regulators (LQRs) [4], nonlinear output regulator [5], and H\(_2\)/H\(_\infty\) control [6]. Most of these methods are based on Newtonian mechanics model of train movement, and thus their control performances depend on the accuracy of train model. However, due to the complexity of train dynamics and various uncertainties, such as change of loaded mass and aerodynamic factors of high-speed train, it is quite difficult to obtain accurate physical model of the train. Therefore, model-based control methods cannot satisfy the high requirements of security, comfort and punctuality for high-speed trains. Thus control methods, less dependent on the system model, or even data-driven ones, are of great practice and desirable significance.

On the other hand, the operation process of a train from one station to the next possesses a high repetition pattern. Trains, especially high-speed trains and subway trains, operate according to their operation diagrams on the same track stringly every day, and the operating environments and transit tasks almost remain unchanged. The desired speed profile is pre-specified for each train, and the train is controlled to follow that profile each time running along the same track. This unique feature offers ATO an opportunity of improving its performance through error learning iteratively. However, the majority of existing ATO methods neglects this significant characteristic, and thus a large amount of valuable information of train operation, collected from various expensive equipment on-board or at the track side, is not well utilized. A limitation of the existing traffic control methods, whether feedback or feedforward dominant, is the lack of capability to learn and improve the control performance from the recurrent train operation pattern.

In control theory community, there is a control method, called iterative learning control (ILC), just proposed to address the control problem of the system repeating the same control task on a finite interval. After first proposed by Arimoto in 1984 [7], ILC has been well studied both in theory [8]-[10] and in applications [11]-[13]. Compared with other control methods, ILC possesses the capacity to learn and improve control performance from previous executions. Moreover, it requires less prior knowledge of system model, and hence it is an almost model-free method [9]. All these characteristics of ILC indicate that it is an ideal tool for train operation [14][15].

Although ILC can make good use of previous repetitive dynamics to achieve better control performance, its transient behavior along iteration axis sometimes cannot be acceptable [16], especially for real systems. In train operation, overshoot of train speed may cause excessive speed, which seriously threatens train operation safety. To solve this problem, many works have been done by combining optimal control with ILC [17]-[20]. These approaches achieve optimal control along iteration axis to improve the transient control performance and speed up the convergence by minimizing cost objectives of system performance. However, most works are based on the known accurate model of the linear systems, and very few theoretical researches are made on nonlinear systems [18].

In this paper, the norm optimal iterative learning control (NOILC) approach for input-affine nonlinear systems is first developed, and then is applied in train trajectory tracking. To make it applicable in practical train operation, traction/braking force constraint is also taken into account. Theoretical analysis and simulation results show that this approach can guarantee the optimal convergence of speed and position errors to zero as the iteration number goes infinity with little priori knowledge of the plant.

The organization of this paper is as follows. A train model is briefly described in Section 2. The NOILC approach for
train trajectory tracking is presented in Section 3. The case with traction/braking force constraint is discussed in Section 4. Simulation results are shown in section 5. Finally section 6 concludes the paper.

II. PROBLEM FORMULATION

A. Train Motion Dynamics Model and Discretization

The most widely used motion dynamics model of a train is firstly proposed by Davis[21], and then detailed by Hay [22]

\[
\begin{align*}
\frac{dv}{dt} & = u - w(v) - g(s), \\
\frac{ds}{dt} & = v,
\end{align*}
\]

(1)

(2)

\[
\begin{align*}
w(v) & = c_o v^2 + c_v v + c_0, \\
g(s) & = l \cdot \sin(\theta(s)),
\end{align*}
\]

(3)

(4)

where \(v\) (m/s) is the train speed, \(u\) (N/kg) is the traction or braking force on unit mass, \(s(t)\) is the position of the train, \(w(v(t))\) (N/kg) is the general resistance on unit mass, \(c_o\), \(c_v\), and \(c_0\) are its coefficients, depending on many factors, such as vehicle type and structure, \(g(s(t))\) (N/kg) is the additional resistance on unit mass caused by slope angle and curve degree, \(l\) is its coefficient, and \(\theta(s)\) is equivalent slope angle at position \(s\).

To make it easier to apply in ATO, above continuous-time differential equations (1)-(4) are transformed to discrete-time difference equations here. Consider the finite time interval \(T\), which describes the train operation from starting station to destination station according to its operation diagram, and set \(\Delta\) as the sampling period with \(T = \Delta \cdot K\). By using Euler Formula, (1)-(4) can be discretized as

\[
\begin{align*}
v(k+1) & = f(v(k)) + h(s(k)) + \Delta \cdot u(k), \\
s(k+1) & = s(k) + h(s(k)),
\end{align*}
\]

(5)

(6)

where

\[
f(v(k)) = -c_o \cdot \Delta \cdot v^2(k) - (c_\Delta - 1) \cdot v(k) - c_0 \Delta ,
\]

\[
h(s(k)) = -\Delta \cdot g(s(k)),
\]

\(k = 0, 1, \ldots, K\).

In the following, the structure of the dynamic model of (5) and (6) is assumed to be known, but all the parameters in this model are assumed to be unknown.

B. State Space Representation and Assumptions

Defining speed and position of the train as system states, i.e., \(x_n(k) = [v_n(k) \ s_n(k)]^T\), the train motion dynamics can be rewritten as the following state equation

\[
x_n(k+1) = f(x_n(k)) + b \cdot u_n(k),
\]

(7)

where \(f(x_n(k))\) is the training function, \(g(v, s)\) is uniform globally Lipschitz continuous on its compact set \(\Omega\) with respect to its arguments and position of train operation operation.

For convergence analysis, some assumptions are made.

Assumption 1. Function \(f(\cdot)\) is uniformly globally Lipschitz continuous on its compact set \(\Omega\) with respect to its arguments and position of train operation.

\[
\|f(x_1(k))-f(x_2(k))\| \leq k_f \cdot \|x_1(k)-x_2(k)\|
\]

(8)

where \(k_f\) is Lipschitz constant, compact set \(\Omega = \mathbb{V} \times \mathbb{S}\), \(\mathbb{V}\) and \(\mathbb{S}\) are range sets of speed and position of train operation.

Assumption 2. The re-initialization condition is satisfied throughout the repeated iterations, i.e.,

\[
v_n(0) = v_n(0) = s_n(0), \quad \forall n
\]

(9)

where \(v_n(0)\) and \(s_n(0)\) are initial values of the desired speed and position of the train.

Assumption 3. There exists an appropriate control \(u_n(k)\) which can drive the system state to track the desired state \(x_n(k+1)\) for system (7) over the finite interval \(k \in [0, K-1]\), i.e.,

\[
x_n(k+1) = f(x_n(k)) + b \cdot u_n(k).
\]

(10)

Remark 1. Assumption 1 requests the train motion dynamics to be globally Lipschitz continuous, which can be naturally satisfied, since the train motion dynamics (7) is bounded continuously differentiable on the bounded compact set \(\Omega\). Assumption 2 can be satisfied, since trains always depart from one station to the next according to the prescheduled timetable, i.e., \(v_n(0) = v_n(0) = s_n(0) = s_n(0) = 0\). Assumption 3 means that the control task assigned should be feasible.

C. Control Objective

The control objective is to seek an appropriate control profile \(u_n(k)\) \((k = 0, 1, \ldots, K-1)\), i.e., the traction or braking force of the train, which drives system states to converge to the desired ones \(x_n(k+1)\) \((k = 0, 1, \ldots, K-1)\), despite the inaccuracy and large uncertainties of the train dynamics.

III. NORM OPTIMAL ITERATIVE LEARNING CONTROL BASED TRAIN TRAJECTORY TRACKING

A. Controller Design

According to the demand of train operation system, the NOILC law is constructed by minimizing the following cost function

\[
J_n(k) = \delta x_n^T(k) \delta x_n(k) + \lambda (u_n(k) - u_{n-1}(k))^2
\]

(11)

where \(\delta x_n(k) = x_n(k) - x_n(k)\) denotes the tracking error of system state at the n-th train operation, \(x_n(k)\) stands for the desired state of the train, and \(\lambda\) is a positive weighting factor.

Differentiating (7) along iteration axis gives

\[
x_n(k+1) = x_{n-1}(k+1) + f(x_n(k)) - f(x_{n-1}(k)) + b \cdot (u_n(k) - u_{n-1}(k)).
\]

(12)

By substituting (12) into (11) and using the optimality condition \(\frac{1}{2} \frac{\partial J_n(k)}{\partial u_n(k)} = 0\) of optimal problem \(\min J_n(k)\), we can get the NOILC law as follows:

\[
\frac{1}{2} \frac{\partial J_n(k)}{\partial u_n(k)} = 0
\]

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\[ u_n(k) = u_{n+1}(k) + \frac{b^r}{b^r b + \lambda} \cdot \{ \delta x_{n+1}(k+1) - \delta x_n(k) \} \]  

(13)

It should be noted that, the accurate value of function \( f(x_n(k)) \) cannot be obtained since unknown parameters are included in train motion model. Thus, we will design an algorithm to estimate the value of \( \{ f(x_n(k)) - f(x_{n-1}(k)) \} \) as a whole.

According to Assumption 1 and differential mean value theorem, there exists a matrix
\[
A_n(k) = \left[ \frac{\partial f(\cdot)}{\partial x} \right]_k \left[ \begin{array}{c} \partial f(\cdot) \\ \partial s \\ \Delta \\ 1 \end{array} \right]_{k=\xi(k)},
\]

satisfying the following equation
\[
f(x_n(k)) - f(x_{n-1}(k)) = A_n(k) (x_n(k) - x_{n-1}(k)),
\]

(14)

where \( x^\ast_n(k) \) denotes the differential mean point within interval \( [x_{n-1}(k), x_n(k)] \). Since the second row \( f' = [\Delta 1] \) is known, where sampling period \( \Delta \) is available, only elements in the first row are needed to be estimated. There are various methods to achieve this task. Here we take the projection algorithm along iteration axis [18] as an example and get the following estimation law
\[
\hat{f}_{n+1}(k) = \hat{f}_{n+1}(k) + \frac{\alpha (x_{n+1}(k) - x_{n+1}(k))}{\mu + \| x_n(k) - x_{n-1}(k) \|} \\
\times \left[ \begin{array}{c} 1 \\ 0 \\ x_n(k+1) - x_{n-1}(k+1) \\ b (u_n(k) - u_{n+1}(k)) \\ -(x_n(k) - x_{n+1}(k))' \end{array} \right] \]
\]

(15)

\[
\hat{f}_{n+1}(k) = \begin{cases} \hat{f}_{\max} \\ \hat{f}_{n+1}(k) \geq \hat{f}_{\max} \\ \hat{f}_{\min} < \hat{f}_{n+1}(k) < \hat{f}_{\max} \\ \hat{f}_{n+1}(k) \leq \hat{f}_{\min} \\ \end{cases}
\]

(16)

where \( \hat{f}_{\max} \) and \( \hat{f}_{\min} \) are the upper and lower bound for the estimated parameter \( \hat{f}_{n+1}(k) \), which could be determined by the train mechanics and the power system constraints in practical applications.

By using the estimated value \( \hat{A}_n(k) = [\hat{f}_{n+1}^T(k) \hat{f}'_{n+1}^T(k) \hat{f}_n^T(k)] \) of \( A_n(k) \), the NOILC update law (13) can be revised as follows
\[
u_n(k) = u_{n+1}(k) + \frac{b^r}{b^r b + \lambda} \cdot \{ \delta x_{n+1}(k+1) - \hat{A}_n(k) (x_n(k) - x_{n+1}(k)) \} \]

(17)

Remark 2. Rewriting (17) yields
\[
u_n(k) = u_{n+1}(k) + \frac{b^r}{b^r b + \lambda} \cdot \{ \delta x_{n+1}(k+1) + \hat{A}_n(k) (\delta x_n(k) - \delta x_{n+1}(k)) \}
\]

(18)

Above equation shows that the proposed control law includes both feedforward error information \( \delta x_{n+1}(k+1) \) and feedback error information \( \delta x_n(k) \) and \( \delta x_{n+1}(k) \). Thus the NOILC law can not only enjoy the control performance learning improvement from the feedforward ILC, but also retain the merit of feedback controller.

B. Convergence Analysis

Theorem 1. For system (7) satisfying Assumptions 1-3, then the NOILC law (17), together with its estimation algorithm (15) and (16), can guarantee that the state converges to the desired one along iteration axis, i.e., \( \lim_{n \to \infty} x_n(k) = x_d(k) \) for any \( k \in [0, K] \).

Proof. Similar to Theorem 2.

Remark 3. From Theorem 1, we can see that state errors are guaranteed to converge to zero when iteration number goes to infinity, whatever its nonlinear \( f(x_n(k)) \) is known or not. The only priori knowledge need to be known is \( b \), which is sampled period \( \Delta \) in train operation, and the other system parameters, such as \( c_0, c_v, c_a \) and \( l \), need not to be known. In other words, the unknown part or parameters do not affect the asymptotic convergence. This feature demonstrates that the proposed NOILC approach is suitable for the train trajectory tracking in practice.

IV. NORM OPTIMAL ITERATIVE LEARNING CONTROL WITH TRACTION/BRAKING FORCE CONSTRAINT

In practice, traction and braking force of the train are constrained by mechanical feature and safety factor of the train. Hence, even if Assumption 3 holds with \( u_d(k) \) satisfying traction/braking constraint, there is no guarantee that \( u_\bullet(k) \) calculated by above control law would satisfy it. Therefore, it is natural and practical to extend above NOILC approach to the case with input constraint. Thus, state equation (7) with input constraint can be described as
\[
x_n(k+1) = f(x_n(k)) + b \cdot u_n(k)
\]

(19)

where \( \tilde{u}_n(k) = \text{sat}(u_n(k)) \) is saturated control input defined by the maximum traction \( u_{\max}(k) > 0 \) and the maximum braking force \( u_{\max}(k) > 0 \) with following formulation,
\[
\text{sat}(u(k)) = \begin{cases} -u_{\min}(k) & u(k) \leq -u_{\min}(k), \\ u(k) - u_{\min}(k) & u_{\min}(k) < u(k) < u_{\max}(k), \\ u_{\max}(k) & u(k) \geq u_{\max}(k). \end{cases}
\]

(20)

Assumption 4. There exists an appropriate control \( u_s(k) \in [u_{\min}(k), u_{\max}(k)] \), which can drive the system state to track \( x_d(k) \) for system (19) over the whole finite interval \( k \in [0, K] \), i.e.,
\[
x_d(k+1) = f(x_d(k)) + b \cdot u_d(k) = f(x_d(k)) + b \cdot u_d(k).
\]

(21)

Assumption 4 indicates that the control task is feasible within the input constraints. Otherwise, the control task is not achievable. For ATO systems, the desired profile is obtained by experimental methods, which synthesizes all the factors, such as energy conservation, ride comfort, punctuality, and safe requirement. Therefore, it can be accomplished by the constrained input naturally.
A. Controller Design

Similar to (11), the cost objective with traction/braking force constraint is defined as

\[
\hat{J}_n(k) = \delta x_n'(k+1)\delta x_n(k+1) + \lambda (\hat{u}_n(k) - \tilde{u}_{n-1}(k))^2
\]  
(22)

By optimizing above cost objective, the NOILC law with input constraint can be got as

\[
u_n(k) = \hat{u}_{n-1}(k) + \frac{b^r}{b^r + \lambda} \cdot \left(\delta x_{n-1}(k+1) + \hat{A}_n(k) (\delta x_n(k) - \delta x_{n-1}(k))\right)
\] 
(23)

where \( \hat{A}_n(k) \) shares the same definition with (17).

\[
\rho_n(k) = \text{sat}[\nu_n(k)]
\] 
(24)

Remark 5. Similar to (18), it derives from (23)

\[
u_n(k) = \hat{u}_{n-1}(k) + \frac{b^r}{b^r + \lambda} \cdot \left(\delta x_{n-1}(k+1) + \hat{A}_n(k) (\delta x_n(k) - \delta x_{n-1}(k))\right)
\] 
(25)

Above equation shows that NOILC law with traction/braking force constraint also has a feedforward-feedback structure.

B. Convergence Analysis

Under input constraints, we can immediately obtain the following important lemma.

Lemma. \[ z_d(k) = \text{sat}[z(k)] \leq z_d(k) - z(k) \], where \[ z_d(k) \in \left[ z_{\min}(k), z_{\max}(k) \right] \], and

\[
\text{sat}[z(k)] = \begin{cases} 
  z_{\max}(k) & z(k) \leq z_{\min}(k), \\
  z(k) & z_{\min}(k) < z(k) < z_{\max}(k), \\
  z_{\max}(k) & z(k) \geq z_{\max}(k).
\end{cases}
\]


Theorem 2. For system (19) satisfying Assumption 1, 2, 4, then the NOILC law with traction/braking force constraint (23) and (24), together with its estimation algorithm (15) and (16), can guarantee that the state converges to the desired one along iteration axis, i.e., \( \lim_{n \to \infty} x_n(k) = x_d(k) \) for any \( k \in [0, K] \).

Proof. From control law (23), we can get

\[
\delta u_n(k) = \delta \hat{u}_{n-1}(k) - \frac{b^r}{b^r + \lambda} \cdot \left(\delta x_{n-1}(k+1) + \hat{A}_n(k) (\delta x_n(k) - \delta x_{n-1}(k))\right)
\] 
(26)

where \( \delta u_n(k) = u_n(k) - u_{n-1}(k) \) and \( \delta \hat{u}_n(k) = u_n(k) - \hat{u}_n(k) \).

By subtracting (19) from (21), and using differential mean value theorem and Assumption 1, the error dynamics is got

\[
\delta x_n(k+1) = \mathbf{f}(x_n(k)) - \mathbf{f}(x_d(k)) + \mathbf{b} \cdot \delta \hat{u}_n(k)
\] 
(27)

From (27), the following equation can be derived recursively

\[
\delta x_n(k+1) = \prod_{i=0}^{k-1} \Phi_n(i) \cdot \delta x_n(0) + \sum_{j=0}^{k-1} \prod_{i=j+1}^{k-1} \Phi_n(i) \cdot \delta \hat{u}_n(j) + \mathbf{b} \cdot \delta \hat{u}_n(k)
\] 
(28)

According to Assumption 2, \( \delta x_n(0) = 0 \) holds, and (28) renders

\[
\delta x_n(k+1) = \sum_{j=0}^{k-1} \prod_{i=1}^{k} \Phi_n(i) \cdot \mathbf{b} \cdot \delta \hat{u}_n(j) + \mathbf{b} \cdot \delta \hat{u}_n(k).
\] 
(29)

Substituting (29) into (26) yields

\[
\delta u_n(k) = \left[1 - (b^r b^r + \lambda)^{-1} b^r b^r \right] (b^r + \lambda)^{-1} \mathbf{b} \cdot \delta \hat{u}_n(k)
\]

(30)

By taking absolute value of both sides of (30), it yields

\[
\left|\delta u_n(k)\right| \leq \sum_{j=0}^{k-1} \prod_{i=1}^{k-1} \Phi_n(i) \left|\delta \hat{u}_n(j)\right| + \sum_{j=0}^{k-1} \prod_{i=1}^{k-1} \Phi_n(i) \left|\delta \hat{u}_n(j)\right|
\] 
(31)

where

\[
\eta_{n,k} = \left|\mathbf{b}^r \cdot \mathbf{b}^r + \lambda\right|^{-1} \mathbf{b}^r \cdot \hat{A}_n(k) \cdot \mathbf{b}
\]

(32)

By definition \( \delta \hat{u}_n = \left[\delta \hat{u}_n(0), \ldots, \delta \hat{u}_n(K-1)\right]^T \),

\[
H_n = \begin{bmatrix} 1 & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}, \quad \text{and} \quad K_n = \begin{bmatrix} k^{0,0} & 0 \\ \vdots & \ddots & \vdots \\ k^{K-1,K-1} & \cdots & k^{K-1,K-1} \\ 0 & \cdots & k^{K-1,K-1} \end{bmatrix}
\]

the following compact form can be got according to (32),

\[
\delta \hat{u}_n \leq H_n K_n \delta \hat{u}_{n-1}
\] 
(33)

where symbol ‘\( \leq \)’ for vectors is defined as each element of the vector on the left side is less than or equal to its counterpart on the right side.

Note that \( H_n \) is a lower triangular matrix with unity as its diagonal element, thus \( H_n^{-1} \) has the same structure with \( H_n \), and \( H_n^{-1} K_{n-1} \) shares the same eigenvalues with \( K_{n-1} \).
Obviously, lower triangular matrix $K_{n-1}$ has a single repeated
eigenvalue $1 - \frac{b^T b}{b^T b + \lambda}$.

According to the property of Projection Algorithm, $\hat{A}_n(k)$
is bounded. From Assumption 1, $\Phi_n(k)$ is also bounded for
any $k = 0,1,\cdots, K$ and $n = 1,2,\cdots$. Therefore, there exists a
lower triangular matrix $\tilde{Z}=
\begin{bmatrix}
\kappa^{0,0} & 0 \\
\vdots & \ddots \\
\kappa^{K-1,1} & \cdots & \kappa^{K-1,K-2} \\
\end{bmatrix}$
satisfying the following inequality

$$H_n \tilde{K}_{n-1} \leq \tilde{Z}, \quad \forall \ n = 1,2,\cdots.$$  

Combining (33) and (34), it derives

$$\delta u_n \leq \tilde{Z} \delta u_{n-1}. \quad (35)$$

Note that $1 - \frac{b^T b}{b^T b + \lambda} < 1$ can always be satisfied, since
$b^T b \geq 0$ and $\lambda$ is a positive weighting factor. Therefore, $\tilde{Z}$
is a stable matrix, and the asymptotic convergence of
$[\delta u_n(k)] \ (k = 0,1,\cdots, K-1)$ is guaranteed along iteration
axis. In terms of Assumption 4, $x_n(k) \rightarrow x_n(k)$ can be assured
when $n \rightarrow \infty$ for all $k \in [0,K]$.

Remark 6. Note that, despite the presence of input constraint,
the proposed NOILC law can still guarantee its iteratively
asymptotic convergent tracking performance with the same
learning condition.

Remark 7. Some works about NOILC have made [19][20].
The approach proposed in this paper has a similar motivation,
but gets totally different results by using the different analysis
and design method. First, input-affine nonlinear systems are
studied here, rather than linear systems or linearized nonlinear
systems. Secondly, lifting technique is used in cost objective
of other NOILC methods to include information of state errors
over the whole time interval of each iteration, which will lead
to the matrices with large dimension in controller and heavy
computational load. The cost objective of method proposed
here only contains information at a certain sample time, which
will avoid huge calculation. Last but not the least, accurate
system model is not required by the proposed NOILC
approach, only coefficient in the control input, $b$, is needed,
and its unknown nonlinear part $f(x_n(k))$ will not affect
controller design and convergence.

V. NUMERICAL SIMULATIONS

In this section, we will verify the validity of the proposed
NOILC approach and its superiority to PID and D-type ILC
approaches through numerical simulations. The train motion
dynamic model is merely used for the generation of the train
operation by MATLAB.

The chosen railway track is 36.28 km long with an upgrade
of 9.68 km in length. Sampling period is chosen as 0.01s. Fig.
1 shows the vertical profile of route and its additional resistance,
a piecewise continuous function of displacement. The actual parameters of the train are listed below, which are
only for simulation of train motion rather than controller
design. $c_a = 1.5 \times 10^{-6}$, $c_v = 7.5 \times 10^{-3}$, and $c_b = 0.001$. These actual parameters are dependent on the type of the train,
and can be obtained from the train design manual. The desired state profiles, i.e., pre-specified speed and position profiles, is
given in Fig. 2.

Three cases are simulated: Case I, PID; Case II, D-type ILC; Case III, NOILC with traction/braking force constraint.

Case I—PID: Through substantial simulations, coefficients of
proportional, integral, and derivative terms are chosen to be
$k_p = [0.3,0]^T$, $k_i = [0.001,0]^T$, and $k_d = [0.02,0]^T$, yielding a
pretty reasonable performance. This PID controller will also
be applied in the initial iteration of Case II and Case III.

Case II—D-type ILC: The D-type ILC applied here is

$$u_n(k) = u_{n-1}(k) + \beta^T \cdot (\delta x_{n-1}(k+1) - \delta x_{n-1}(k)) \quad (36)$$

where learning gain is chosen as $\beta = [0.4, 0]^T$ to guarantee the
corvergence of learning errors.

Case III—NOILC with traction/braking force constraint:
Parameters in estimator (15) and (16) are chosen as $\mu_i = 50$
and $\alpha_i = 0.001$. In controller (23) and (24), two different
values of controller parameter $\lambda$ are simulated to show its
influence on control performance, that is $\lambda = 0.5$ and $\lambda = 5$.
As for input constraint, the maximum braking force on unit
mass is $u_{\text{max}} = -1.5$ N/kg, and the traction constraint is
calculated by $u_{\text{max}}(k) = p_{\text{max}}/v(k)$, where $p_{\text{max}}$ is the
maximum train power on unit mass and set to be 25W/kg.
with input constraint is formulated and solved for the first learning convergence condition of the train motion system is detailed. Moreover, when applying the proposed NOILC, solely determined by sampling period, and other unknown can observe clearly that control performance of PID stays the same without any improvement, since it belongs to pure tracking error, over the whole time interval. From Fig. 3 we have shown the feasibility of the proposed NOILC. Rigorous theoretical analyses and simulations have shown the feasibility of the proposed NOILC.

VI. CONCLUSION

The NOILC approach for input-affine nonlinear systems with input constraint is formulated and solved for the first time. By capturing and utilizing repeatability of train motion dynamics, application of NOILC to train trajectory tracking is detailed. Moreover, when applying the proposed NOILC, learning convergence condition of the train motion system is solely determined by sampling period, and other unknown nonlinear part and parameters will not affect its asymptotic convergence. Rigorous theoretical analyses and simulations have shown the feasibility of the proposed NOILC.

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