Controlling an Active Suspension using Methods of Optimal Control

Sebastian Spirk and Boris Lohmann

Abstract—This paper deals with a control strategy for the vertical dynamics of a passenger quarter-car equipped with an active suspension system. The control objectives are formulated in optimal control problems with different complexity, regarding inequality constraints and nonlinear cost functions. The influence of the different formulations of the optimal control problem on the controlled system is discussed regarding the resulting optimal trajectories in comparison to the passive system. For the favored cost functions the offline calculated solutions for the discretized state-space are then used to generate a state-feedback control. An optimization of calculation time is sought applying dynamic programming and modal synthesis. The developed controllers are implemented and their performance is analyzed in simulation and at a quarter-car test rig equipped with a hybrid suspension system.

I. INTRODUCTION

The three main objectives of vertical suspension control are to ensure ride safety in terms of good contact between tire and road, to largely isolate passenger from the road induced vibrations and to keep the suspension travel within the construtional limits. To better ease the conflict of objectives semi-active suspensions have been widely introduced in passenger cars from luxury class up to compact class in newer days. Despite the increasing distribution control concepts in series cars are still dominated by the simple skyhook control law combined with passive damping, [1]. But control concepts for semi-active suspensions are steadily enhanced and active suspensions are fighting their way to series, starting up with slow-active suspensions (see [2]) in series production since 1999 and fully active systems tested for automotive application [3]. In [4], [5] a new hybrid suspension concept with a novel combination of two actuators using stock components has been presented closing the gap between slow-active and fully active suspensions regarding performance, system costs and energy consumption.

Regarding passive suspensions the target of maintaining suspension travel limitations for all possible excitations and chassis mass variations especially leads to high necessary stiffnesses between wheel and chassis beyond pareto-optimal configurations regarding ride safety and comfort, thus producing undesired discomfort for the passengers due to bad vibration isolation. Comfort differences between luxury vehicles and compact cars accrue mainly from the significantly higher rattle space at these cars, [1]. The right treatment of this hard control objectives in (semi-)active suspension control concepts is therefore an important potential of reaching further enhancements in ride comfort. The most commonly used modern control concept for suspension control, the LQR, e.g. in [6], [7], [8], weights the rms-values of suspension deflection (handled as a weak control objective) and is not able to map the inequation of the hard objective of keeping suspension deflection within its limits.

Therefore control concepts using more complex cost functions or adaptation logics show more promise for reaching significant improvements in ride comfort. Adaptation based concepts have been presented in [9], [10], [11], [8] and recently in [4], [5]. With some assumption and a simple nonintegrative cost function a comfort-optimal semi-active control strategy is derived in [12], leading to an implementable chassis acceleration based optimal control strategy. A so called optimal-predictive control, for known road profile and comfort cost function only, is developed in [13] as offline calculated benchmark for semi-active control strategies. For two-state and continuously variable damping in [14] the comfort can be improved for the special case of a sinusoidal excitation at the frequency of the damping invariant point of the chassis acceleration amplification function by an optimal control strategy. A first approach of adding an optimal control component to an existing adaptive reference model based control [4] for a single event problem of a controlled system has been presented by the authors in [15].

Using methods of optimal control allows the application of a nonlinear model, abstract nonlinear cost functions and consideration of state and actuating constraints for control design [16], [17]. The problem of using model predictive control with nonlinear complex cost functions is the online calculation of the resulting optimal control force. For high sample rates, complex cost functions, state or actuator constraints finding the solution of the optimal control in each step size is usually not realtime-capable. The presented method is therefore based on the offline calculation of the optimal solution for all starting points in the discretized state-space. Due to the possibility of nonlinear cost functions the maximum value of chassis acceleration can be weighted and its minimization can be taken into account as a control objective. Especially the correct consideration of the hard control objectives of the suspension deflection as inequality constraint offer the potential of massive improvements in ride comfort.

The paper is organized in 5 sections: After the modeling in section 2, the control strategy based on optimal control is developed in section 3. The resulting controls are then implemented and their benefit is analyzed in simulation and at the test rig in section 4, then the conclusions and an outlook on future works is given in section 5.
II. MODELING

Modeling only the vertical dynamics of a passenger car the so-called quarter-car model can be used (Fig. 1). It is a 2 DOF-model with the chassis mass (sprung mass) \( m_c \approx 500 \) kg consisting of one quarter of the vehicle body mass and the wheel mass (unsprung mass) \( m_w \approx 70 \) kg. Both masses are connected by the main spring and the damper, while the tire can be modelled as a stiffness and damping. The quarter car is excited by the road profile and, in the active case, can be controlled by the actuator force \( F(t) \) in parallel to the main spring. The actuator replaces the damper in the active case, so that \( d_c = 0 \) holds then. In the passive case the actuator is replaced by the damper \( F(t) = 0, \) \( d_c = d_w(z_{cw}) \). Important nonlinearities of the system are the stiffness characteristic with compression and rebound stops and nonlinear (often depressive) damping characteristics including coulomb friction. All of these characteristics are additionally influenced by the variable kinematic transmission factor of the suspension. Furthermore we have to take into account the nonlinear (quadratic portion) tire stiffness - limited by the loss of tire-road contact - and the frequency dependent tire damping. All mentioned nonlinearities are included in the simulation model and accordingly in the simulation results. The state vector

\[
\mathbf{x} = [z_c - z_w, \dot{z}_c, z_w - z_g, \dot{z}_w]^T
\]

(1)

includes the control objective signals, where \( F_{dyn} = c_w(z_g - z_w) \) denotes the dynamic wheel load. The used passive suspension configuration has the original spring stiffness including rebound and compression stops of a BMW 7 series and a fixed damping characteristic with focus on ride comfort. It can be characterized by the chassis eigenfrequency \( f_{c,p} = \frac{1}{2\pi} \sqrt{c_e/m_c} \approx 1.1 \) Hz and a damping ratio \( D_p = \frac{d_c}{2\sqrt{c_e/m_c}} \approx 0.21 \) around the equilibrium.

III. OPTIMAL CONTROL

With the known system dynamics and a free choice of cost function, we define the optimal control problem

\[
\text{Cost function: } J = \int_{t_0}^{t_e} g(x(t), u(t)) dt, \quad \min_{u(t), t \in [t_0, t_e]} J
\]

\[
\text{Side condition: } \dot{x}(t) = f(x(t), u(t))
\]

\[
\text{Start condition: } t_0 \text{ given, } x(t_0) = x_0
\]

\[
\text{End condition: } t_e \text{ fixed or } \infty, x(t_e) \text{ fixed or free}
\]

\[
\text{Inequality state constraint: } \mathbf{C}(x(t)) \leq 0
\]

with the possibility of including the state constraint

\[
\mathbf{C}(x(t)) = -x_1 - x_{max} \leq 0
\]

(5)

given by the suspension deflection control objective of staying within the maximum suspension deflection \( x_{max} \) \( (x_{max} > 0) \). As optimal control problems are often nonlinear they usually cannot be solved analytically. Using indirect methods the solution can be found solving the following boundary-value problem [16], [17]:

\[
\dot{x} = \frac{\partial H}{\partial \psi} = f(x, u, t) \quad \text{state differential eq.}
\]

\[
\dot{\psi} = -\frac{\partial H}{\partial x}
\]

(6)

\[
\dot{\psi} = -\frac{\partial J}{\partial \mathbf{x}} + (\frac{\partial f}{\partial \mathbf{x}})^T \psi \quad \text{adjoint differential eq.}
\]

\[
0 = \frac{\partial H}{\partial u} \quad \text{control equation}
\]

In this case the optimal control problem is solved using the direct method of nonlinear programming [16]. Due to the piecewise constant discrete control inputs \( u_k \) with a sample time of \( \Delta t = 10 \) ms. Nonlinearities of the tire could be included in the state equation, but they are neglected to reduce the calculation time by making available the analytical gradients of cost function, side conditions, inequality conditions and the analytical Hesse-matrix of the Lagrangian function to the used interior-point algorithm.

A. Comparing single solutions

Choosing a challenging start vector

\[
\mathbf{x}_0 = [0.040 \text{ m}, -0.8 \text{ m/s}, -0.005 \text{ m}, 1.0 \text{ m/s}]^T
\]

which results in a large compressive suspension deflection of \( \min z_{cw}(t) = -7.83 \) cm for the time response of the passive suspension (Fig. 2), the resulting optimal trajectories
For different cost functions are compared in the following. For all discussed cost functions a weighting of the control variable is not necessary as there exists a direct feedthrough from control variable to chassis acceleration output.

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The first optimal control result from Fig. 3 with quadratic weighting of chassis acceleration, dynamic wheel load and suspension deflection and no state constraints is similar to a LQR (see III-B) feedback solution. It leads to a large reduction of the rms\(^2\)-values of dynamic wheel load and chassis acceleration, however the maximum value for the acceleration increases and we have to keep in mind that this LQ-control will not prevent suspension travel from violating the limits for more intense excitations.

To ensure the adherence to the maximum suspension deflection, the inequality constraint \(-x_1 - 0.08 \text{ cm} \leq 0\) is introduced. The control will now ensure that the suspension travel stays within the limit at all time, but, as you can see in Fig. 4, it will also exploit the maximum deflection to improve comfort. As the suspension deflection is now omitted in the cost function, the controlled quarter-car has no incentive to return to equilibrium. (For the numerical solution \(t_e \rightarrow \infty / x(t_e)\) free is approximated by \(t_e = 5 \text{ s} / x(t_e) = 0\) which leads to a slight return to equilibrium.)

**Cost function:** \(J = \frac{1}{2} \int_0^{t_e} (\dddot{x}_c^2 + bF_{dyn}^2)dt\)

**End condition:** \(t_e = \infty, x(t_e)\) free

**Ineq. constr.:** \(-x_1 - 0.08 \text{ cm} \leq 0\)

![Fig. 2. Time plot of the passive quarter-car for a certain start vector](image)

![Fig. 3. LQ-controlled quarter-car](image)

![Fig. 4. Controlled behaviour when adding the inequality constraint](image)
Cost function:  \[ J = \frac{1}{2} \int_0^{t_e} \left( \ddot{z}_c^2 + bF_{dyn}^2 \right) dt \]
End condition:  \[ t_e = 1 \text{ s}, \ x(t_e) = 0 \]
Ineq. constr.: \[ -x_1 - 0.08 \text{ cm} \leq 0 \]

### B. State-feedback Control using Methods of Optimal Control

To create a feedback control, based on the discussed optimal (feedforward) control solutions, the actual state \( x(t) \) of the quarter-car can be used as start vector \( x(t) = x_0 \) of an optimal control problem (that has already been solved). The actual control force is the first value \( u_0 = u(t=0) \) of the solution \( u(t) \) (or \( u = [u_0, u_1, \ldots, u_n] \) in case of the discrete solution) of the optimal control problem (6). For a linear model, linear weighting function and infinite time horizon the resulting Riccati-equation can be solved analytically leading to a constant state-feedback controller [20], known as LQR (Linear-Quadratic regulator). Not being able to solve this problem analytically or numerically online, you can calculate the solutions for all possible points in the discretized state-space from (7) in advance and store all start values \( u_0(x) \) in a \( n \)-dimensional matrix (Fig. 7). This procedure allows the use of nonlinear cost functions, nonlinear side conditions (system dynamics) and constraints for states or control values without having the demand for being real-time capable.

Using the state-space discretization

\[
\begin{align*}
  x_1 &\in [-0.08 : 0.01 : 0.08] \text{ [m]}, \quad (N_1 = 17) \\
  x_2 &\in [-0.5 : 0.10 : 0.5] \text{ [m]}, \quad (N_2 = 11) \\
  x_3 &\in [-0.02 : 0.0025 : 0.02] \text{ [m]}, \quad (N_2 = 17) \\
  x_4 &\in [-2.0 : 0.25 : 2.0] \text{ [m]}, \quad (N_4 = 17)
\end{align*}
\]

leads to \( N_{tot} = \prod_{i=1}^{4} N_i = 54043 \) start-vectors for which you have to find the solution for the optimal control problem.

### C. Dynamic Programming

With dynamic programming (see [16], [17], [21], [22]) the calculation time for the whole set of optimal control solutions can be significantly decreased as the solutions for the whole state-space are gained all at once. It also instantly includes the solutions for arbitrary end times \( t_e \leq t_{e,max} \) smaller than the maximum calculated time horizon \( t_{e,max} \). But, it is only applicable if the given cost function is additive. A cost function is called additive when the formed optimal control problem satisfies the Principle of Optimality [21]. With the optimal control- and state-trajectories \( u^*(t) \) and \( x^*(t) \), \( t \in [0, t_e] \), the Principle of Optimality means that each remaining trajectory \( u^*(t), \ t \in [t_1, t_e], \ 0 \leq t_1 \leq t_e \) of the optimal trajectory \( u^*(t), \ t \in [0, t_e] \) must also be optimal with regard to the resulting state \( x^*(t_1) \) for transfer to the final state \( g[x(t_e), t_e] = 0 \).

\[
J_k^*(x_k) = \min_{u_k} \left\{ J_k(x_k, u_k) + J_{k+1}^*(x_{k+1}) \right\}
\]

Among the discussed cost functions only those containing the maximum value of the chassis acceleration \( \max(\ddot{z}_c) \) are nonadditive.
With dynamic programming a more closely meshed grid can be used while still reducing calculation time³, as this comparison of Table I and Table II shows:

**TABLE I**  
**Discretization for Dynamic Programming that leads to a calculation time of 25.0 h for the complete set (as well as for one single solution)**  

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0 : 0.01 : 1]$</td>
<td>$[-0.08 : 0.01 : 0.08]$</td>
<td>$[-0.5 : 0.05 : 0.5]$</td>
<td>$[-0.02 : 0.001 : 0.02]$</td>
<td>$[-2.0 : 0.05 : 2.0]$</td>
</tr>
</tbody>
</table>

$(N_t = 100)$, $(N_1 = 161)$, $(N_2 = 21)$, $(N_3 = 41)$, $(N_4 = 81)$

**TABLE II**  
**Discretization for Nonlinear Programming that leads to a calculation time of 43.5 h for the complete set (3 s are needed for a single solution)**  

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0 : 0.01 : 1]$</td>
<td>$[-0.08 : 0.01 : 0.08]$</td>
<td>$[-0.5 : 0.10 : 0.5]$</td>
<td>$[-0.02 : 0.0025 : 0.02]$</td>
<td>$[-2.0 : 0.25 : 2.0]$</td>
</tr>
</tbody>
</table>

$(N_t = 100)$, $(N_1 = 17)$, $(N_2 = 11)$, $(N_3 = 17)$, $(N_4 = 17)$

D. Modal Synthesis for Order Reduction

Another possibility to decrease the calculation effort is to reduce the number of system states $n$ by reducing the double-mass oscillator (DMO) system dynamics to a one-degree-of-freedom (1-DOF) model. For an equal discretization of Number $N$ of each state the calculation effort is proportional to $N^n$. The dynamics of the linear 2-DOF model $\dot{x} = Ax + Bu$ are, in the active case, characterized by the two modes with the eigenvalues $\lambda_{1,2} = \pm 77i$ (wheel mass) and $\lambda_{3,4} = \pm 6.7i$ (chassis mass). By neglecting the dynamics of the wheel mass the model order reduces from 4 to 2 states, leading to a radical reduction of the calculation time needed to gain all solutions for the discretized state-space. With the normed eigenvectors as columns of the transformation matrix

$$E_{dmo} = \begin{bmatrix} 0.0142i & -0.0142i & -0.1382i & 0.1382i \\ -0.0095 & -0.0095 & 0.9879 & 0.9879 \\ -0.0140i & +0.0140i & -0.0105i & 0.0105i \\ 0.9998 & 0.9998 & 0.0697 & 0.0697 \end{bmatrix}$$

a modal transformation to the new states

$$z_{dmo} = E_{dmo}^{-1}x$$

is done. With the new state vector $z_{dmo}$ the transformed system

$$\dot{E}z = AEz + Bu$$
$$\dot{z} = E^{-1}AEz + E^{-1}Bu$$

has two uncoupled modes, so that the mode of the chassis mass can be separated. Though the dynamic matrix $A = E^{-1}AE$ now has complex entries. Therefore equivalent single-mass oscillator (SMO) dynamics $\dot{x}_{smo} = A_{smo}x_{smo} + b_{smo}u$ with the same pair of eigenvalues and a dynamic matrix

$$A_{smo} = \begin{bmatrix} 0 & 1 \\ -44.5 & 0 \end{bmatrix}$$

are formulated for the optimal control design. With the transformation matrix

$$E_{smo} = \begin{bmatrix} 0.1483i & -0.1483i \\ 0.9889 & 0.9889 \end{bmatrix}$$

the modal states $z_{dmo,3}$ and $z_{dmo,4}$ can be transformed into the states of the single-mass oscillators:

$$x_{smo} = E_{smo} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} z_{dmo}$$
$$= E_{smo} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{dmo}^{-1}x$$

$$= \begin{bmatrix} 0.9651 & 0 & 0.9743 & 0 \\ 0 & 1.0011 & 0 & 0.0095 \end{bmatrix} x$$

The proposed optimal control design can now be done for the single-mass oscillator dynamics. Using (13) the SMO-states are calculated for the state feedback control. In addition to this state feedback control of the chassis mode, it is necessary to control the wheel dynamics. This is realized using the so-called groundhook damping [1], a damping of the wheel velocity with the proportional force $F_{GH} = d_g \dot{z}_w$. This force is set in addition to the optimal control force for the chassis.

E. Favoured cost functions

Based on the in subsection III-A discussed cost functions, further analysis focus on the controlled behaviour resulting from the cost functions considered in figure 5 and 6 with inequality constraint for the suspension deflection, fixed end time $t_e = 1$ s and fixed end point $x(t_e) = 0$. Both, the variant with pure quadratic measure (V1) and the one additionally considering the maximum value of the chassis acceleration (V2) are implemented and applied to the system. Also the control strategy with optimal control solutions for the single-mass oscillator as described in subsection III-D with quadratic measure in combination with a groundhook control (V1-SMO) is tested and compared to the double-mass oscillator control.

IV. SIMULATION AND MEASUREMENT RESULTS

A. Simulation Results

In order to analyze the performance of the designed controllers, the passive and the actively controlled system are compared by exciting the fully nonlinear model described in section II with the road profile of a bad country road (see Fig. 8). Moreover the results of an appropriate LQR feedback control are given. For the LQR control design a fully active linear model (cf. fig. 1, $d_e = 0$) is used and the resulting control force $u = -K_e^T \ddot{x}$ is applied to the actuator. Diagonal weighting matrices of the control output vector $y$ are then adapted in each case, so that the LQ-controlled results match the active results in terms of dynamic wheel
The results of the controlled system using a one-DOF model for a reduced optimal control with quadratic cost function (V1-SMO) as proposed in subsection III-D is compared in table V. The advantages of the more closely meshed grid compensate for the neglect of the wheel dynamics in the optimal control problem formulation, so that the comfort level can be further improved, compared to V1. The combination with the groundhook control taking care of the wheel dynamics results in a slight decrease in the rms-value of the dynamic wheel load compared to the passive system, but it cannot reach the ride safety level of control V1 especially regarding the minimum value of the dynamic wheel load. However the additional groundhook control allows an easy adaptation towards more ride safety, as table VI shows. By increasing the groundhook damping from $d_{GH} = 300$ Ns/m to $d_{GH} = 2000$ Ns/m the wheel load deviations can be significantly decreased down to -20.2 % for the rms-value and -20.8 % for the minimum value, taking into account a loss of comfort by 43.8 percentage points in rms and 34.2 percentage points for the maximum value.

TABLE IV
RESULTS OF THE CONTROLLED SYSTEM WITH EXTENDED NONLINEAR COST FUNCTION (V2).

<table>
<thead>
<tr>
<th>Value:</th>
<th>Passive rms max min</th>
<th>Active rms max min</th>
<th>Comparison [%]</th>
<th>Passive rms max min</th>
<th>Active rms max min</th>
<th>Comparison [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_c$ [cm]</td>
<td>1.76 7.51 -5.29</td>
<td>0.28 1.04 -0.83</td>
<td><strong>-83.9</strong></td>
<td>1210 5877 -3927</td>
<td>-8.4</td>
<td><strong>-84.2</strong></td>
</tr>
<tr>
<td>$F_{dyn}$ [N]</td>
<td>162 5049 -3448</td>
<td>2.61 7.84 -7.67</td>
<td>-47.3</td>
<td>2.13 -6.19 -7.5</td>
<td><strong>-21.6</strong></td>
<td></td>
</tr>
<tr>
<td>$s_{cw}$ [cm]</td>
<td>-25.0</td>
<td>-20.2</td>
<td><strong>-33.0</strong></td>
<td>-41.8</td>
<td><strong>-26.3</strong></td>
<td><strong>-34.6</strong></td>
</tr>
<tr>
<td>Appropriate LQR Feedback Control</td>
<td>Comparison [%]</td>
<td>Passive rms max min</td>
<td>Active rms max min</td>
<td>Comparison [%]</td>
<td>Passive rms max min</td>
<td>Active rms max min</td>
</tr>
<tr>
<td>$x_c$ [cm]</td>
<td>1.52 4.72 -4.79</td>
<td>0.56 1.93 -1.75</td>
<td>-68.2</td>
<td>1210 5638 -4098</td>
<td>-4.2</td>
<td>11.7</td>
</tr>
<tr>
<td>$F_{dyn}$ [N]</td>
<td>-41.8</td>
<td>-39.8</td>
<td><strong>-37.5</strong></td>
<td>1.97 5.79 -5.02</td>
<td>-24.5</td>
<td>-26.3</td>
</tr>
<tr>
<td>$s_{cw}$ [cm]</td>
<td>-41.8</td>
<td>-39.8</td>
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</tr>
</tbody>
</table>

TABLE V
PERFORMANCE OF OPTIMAL CONTROL GAINED WITH THE ONE-DOF MODEL (V1-SMO) COMBINED WITH A GROUNDHOOK-CONTROL ($d_{GH} = 300$ Ns/M) OF THE WHEEL DYNAMICS.

<table>
<thead>
<tr>
<th>Value:</th>
<th>Passive rms max min</th>
<th>Active rms max min</th>
<th>Comparison [%]</th>
<th>Passive rms max min</th>
<th>Active rms max min</th>
<th>Comparison [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_c$ [cm]</td>
<td>1.62 4.67 -6.26</td>
<td>0.77 2.50 -2.25</td>
<td>-56.3</td>
<td>2.56 7.84 -7.67</td>
<td>-20.8</td>
<td>-21.6</td>
</tr>
<tr>
<td>$F_{dyn}$ [N]</td>
<td>162 5049 -3448</td>
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<td>-34.2</td>
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</tr>
<tr>
<td>$s_{cw}$ [cm]</td>
<td>-2.3</td>
<td>-2.3</td>
<td><strong>-21.6</strong></td>
<td>1.52 4.72 -4.79</td>
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<tr>
<td>Appropriate LQR Feedback Control</td>
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<td>Active rms max min</td>
<td>Comparison [%]</td>
<td>Passive rms max min</td>
<td>Active rms max min</td>
</tr>
<tr>
<td>$x_c$ [cm]</td>
<td>1.81 5.00 -3.07</td>
<td>1.32 3.96 -4.15</td>
<td>-56.3</td>
<td>2.56 7.84 -7.67</td>
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<tr>
<td>$s_{cw}$ [cm]</td>
<td>-2.3</td>
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<td><strong>-21.6</strong></td>
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</tr>
</tbody>
</table>

TABLE VI
PERFORMANCE OF OPTIMAL CONTROL GAINED WITH THE ONE-DOF MODEL (V1-SMO) AND INCREASED GROUNDHOOK DAMPING ($d_{GH} = 2000$ Ns/M).

<table>
<thead>
<tr>
<th>Value:</th>
<th>Passive rms max min</th>
<th>Active rms max min</th>
<th>Comparison [%]</th>
<th>Passive rms max min</th>
<th>Active rms max min</th>
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</tr>
<tr>
<td>Appropriate LQR Feedback Control</td>
<td>Comparison [%]</td>
<td>Passive rms max min</td>
<td>Active rms max min</td>
<td>Comparison [%]</td>
<td>Passive rms max min</td>
<td>Active rms max min</td>
</tr>
<tr>
<td>$x_c$ [cm]</td>
<td>1.62 4.67 -6.26</td>
<td>1.32 3.96 -4.15</td>
<td>-56.3</td>
<td>2.56 7.84 -7.67</td>
<td>-20.8</td>
<td>-21.6</td>
</tr>
<tr>
<td>$F_{dyn}$ [N]</td>
<td>162 5049 -3448</td>
<td>2.61 7.84 -7.67</td>
<td>-34.2</td>
<td>1.62 4.67 -6.26</td>
<td>-20.8</td>
<td>-21.6</td>
</tr>
<tr>
<td>$s_{cw}$ [cm]</td>
<td>-25.0</td>
<td>-20.2</td>
<td><strong>-33.0</strong></td>
<td>-41.8</td>
<td><strong>-39.8</strong></td>
<td><strong>-37.5</strong></td>
</tr>
</tbody>
</table>
This high potential of adaptation between ride safety and ride comfort enables this control variant to be used in combination with adaptation logics as presented in [4], [5], [8]. An adequate modification of the cost functions for a higher weighting of ride safety – as it would be necessary for variant V1 – would require a new calculation of the optimal control solutions and allows no continuously adaptation.

B. Measurement Results at the Test Rig

First validations at a test rig equipped with an active suspension system confirm the simulation results. The test rig, which is described in more detail in [4], [5], [23], is a passenger quarter-car (of an upper class limousine) enhanced with a hybrid suspension system consisting of the combination of a continuously variable damper and a hydraulic actuator in series to the main spring. This combination approximates the performance potential of a fully active suspension while having an acceptable energy consumption. Beside measurement noise, the limitations of the used actuators in dynamics and saturation lead to a loss of performance: Compared with the simulation results of the applied control strategy V1 the measurement results have 17.7 pp lower improvement in rms-value of chassis acceleration and loss of 34.7 pp for the maximum value, resulting in an approximate benefit of about -50 % compared to the passive suspension. A reduction of the dynamic wheel load deviations of -4.0 % could be held up while the minimum value could be actually decreased by -11.3 %.

**TABLE VII**

<table>
<thead>
<tr>
<th>Value</th>
<th>Passive</th>
<th>Active</th>
<th>Comparison [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{dV} [\text{N}] )</td>
<td>1189 4081 -3517</td>
<td>1142 4276 -3119</td>
<td>-4.0 4.8 -11.3</td>
</tr>
<tr>
<td>( F_{dC} [\text{N}] )</td>
<td>2.4 6.52 -7.14</td>
<td>2.3 5.60 -6.05</td>
<td>-5.0 -14.1 -15.3</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS AND FUTURE WORKS

Defining an optimal control problem with varying cost functions, a control strategy could be developed using the offline calculated solutions of optimal control problems with nonlinear measure and inequality constraint. A comparison of the controlled behaviour resulting from cost functions with different complexity led to controllers that could improve comfort by more than 80 % compared to the passive reference. By modal synthesis of the system the optimal control design could be applied to a reduced one-DOF model allowing a finer discretization of state-space and more precise optimal control solutions. A more ride safety oriented controller design then resulted in a comfort improvement by almost 40 % while additionally improving ride safety by about 20 %. A first validation of the control strategy at a quarter-car test rig equipped with an active suspension confirmed the simulation results.

Further validations at the test-rig are necessary to confirm the performance for all of the discussed control strategies and different road profiles or single excitation events. As it can deal with highly nonlinear state-dependent actuator constraints, this control strategy enables the integration of force restrictions in semi-active suspensions due to the characteristics of the used variable dampers, that are mostly neglected in control design so far.

**REFERENCES**


