Second order sliding mode output feedback control: impulsive gain and extension with adaptation

Antonio Estrada, Franck Plestan

Abstract—This paper proposes a discontinuous-impulsive output feedback control which is a sampled based second order sliding mode controller. First, constructive convergence conditions are established for the controller, in the unperturbed and perturbed case. An adaptation mechanism is based on the aforementioned convergence conditions and allows to reduce gain magnitude.

I. INTRODUCTION

Sliding mode (SM) and high order sliding modes (HOSM) are nonlinear control strategies that provides robustness with respect to uncertainties and perturbations. The main common properties of SMC/HOSM control laws are robustness (insensitivity) to the bounded disturbances matched by control, and finite convergence time. The main drawback of the

The main drawback of HOSM control laws is the necessity of high order derivatives of the sliding variable [5], [3], which are obtained using HOSM differentiators [5]. Differentiation yields a performance degradation of the controlled system due to the presence of measurement noise. Then, there is a real interest to propose high order sliding mode controllers with a reduced number of time derivatives of the sliding variable.

Concerning the second order sliding mode controllers, a popular second order sliding mode output feedback is the supertwisting algorithm [5]. In fact, this controller allows the establishment of a second order sliding mode with respect to the sliding variable, in a finite time, by using only the measurement of the sliding variable. However, this algorithm is intended to be applied to systems whose relative degree of the sliding variable equals 1. An alternative strategy, which do not have the previous drawback, has been proposed in [8]: it was a discrete-continuous output feedback control algorithm that requires finite sampling time for its analysis and implementation. The basic idea consists in change the gain magnitude in an impulsive manner, at some precise time instants. The gain is then evolving between a small value (fixed in order to counteract the effects of uncertainties and perturbations) and a larger one, which defines the size of the impulsion, this latter having a duration equal to a sampling period. These gain “jumps” allow to ensure the compensation of delays induced by the sampling period, when the sliding variable is changing of sign. A drawback of this approach is that there is no constructive way to fix the value of the larger gain. The result reported in [8] has motivated [4], in which impulsive sliding mode state feedback control laws are proposed for a larger class of system (sampling period can be equal to 0).

In the current paper, the first result consists in improving the main result of [8], by given a formal condition on the larger gain, in order to get a constructive way to design the second order sliding mode output feedback controller. The approach is presented firstly for unperturbed systems, then for perturbed ones. For this latter class of systems, the output feedback controller is proposed in an adaptive version which allows to reduce the gain when system trajectories are close from the origin. Note that the adaptation of gain in sliding mode context [9], [10] allows to reduce the magnitude of the control input, allowing a reduction of the chattering.

II. PROBLEM STATEMENT

Second order sliding mode control of uncertain nonlinear system is equivalent to the finite time stabilization of the following system

\[ \begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= u + \omega
\end{align*} \]  

(1)

where \( z = [z_1 \ z_2]^T \) is the state vector, \( u \) the control input and \( |\omega| \leq \delta \) (\( 0 \leq \delta \)) a bounded perturbation. The system (1) is viewed as an uncertain nonlinear continuous one, whereas the control law \( u \) is supposed to be evaluated at a sampling period \( T_e \) which yields

\[ u = -K(t)\text{sign}(z_1(kT_e)) \]  

(2)

where \( K(t) > 0 \) and \( k \in \mathbb{N} \) (\( k \) can be viewed as a time counter). The gain \( K(t) \) is constant on the time interval \( t \in [k \cdot T_e, (k + 1) \cdot T_e] \), and \( k(0) = 0 \). Note that the control input depends only on the output \( z_1 \). Then, the objective is to propose an output feedback (2) which allows to reach, in a finite time, a vicinity of the origin of (1) in spite of perturbation \( \omega \).

III. SECOND ORDER SLIDING MODE OUTPUT FEEDBACK CONTROL OF THE UNPERTURBED CASE

The result displayed in the sequel allows to ensure the finite time establishment of a second order sliding mode in system (1)-(2), when there is no perturbation (\( \omega = 0 \)).

**Theorem 1:** Consider system (1) with \( \omega = 0 \) and controlled by (2). The gain \( K(t) \) is defined as

\[ K(t) = \begin{cases} 
K_m & \text{if } t \notin \mathcal{T} \\
K_M & \text{if } t \in \mathcal{T}
\end{cases} \]  

(3)

Antonio Estrada and Franck Plestan are with LUNAM Université, Ecole Centrale de Nantes, IRCCyN UMR CNRS 6597, Nantes, France. E-mail: xheper@yahoo.com and Franck.Plestan@irccyn.ec-nantes.fr

978-1-4673-2064-1/12/$31.00 ©2012 IEEE 5482
with \( \mathcal{T} = \{ t \mid \text{sign}(z_1(kT_e)) \neq \text{sign}(z_1((k-1)T_e)) \} \) and \( K_m > 0 \). If \( K_M \) is such that
\[
K_M \geq \gamma K_m \quad \text{with} \quad \gamma > 3,
\]
then the control law (2) ensures that there exists a time \( t_F \) such that for all \( t \geq t_F \)
\[
|z_1| < \frac{1}{2} K_m \left[ \bar{\eta}(\gamma) - 1 \right] T_e^2, \quad |z_2| < K_m \bar{\eta}(\gamma) T_e
\]
with
\[
\bar{\eta} = \frac{\gamma^2 - \gamma - 2}{2(\gamma - 3)}
\]
Proof. For the sake of clarity, consider Figure 1. A delay exists, due to the discrete sampling, between the time \( T_{sr} \) at which \( z_1 \) crosses de \( z_2 \)-axis, and the time \( T_s \) at which this crossing is detected. It has been shown in [8] that, due to this delay, switching gain \( K(t) \) is required in order to ensure convergence. If the gain of control law \( u = -K \text{sign}(z_1(kT_e)) \) is kept constant, this control law is not able to ensure the convergence of the closed-loop system trajectories: the system trajectories diverges. Defining the aforementioned delay as \( T_d = T_s - T_{sr} \), one has
\[
0 \leq T_d < T_e
\]
According to Theorem 1, the gain \( K(t) \) equals \( K_M \) for
\[ t \in [T_s, T_s + T_e] \]
In order to ensure convergence, \( K_M \) must have a sufficient magnitude in order to reach parabolic trajectories which are closer from the origin, once it detects that the single measured output \( z_1 \) has changed of sign. The proof consists, first-of-all, in determining the formal expression of \( z(T_{sr} + T_e) \) and \( z(T_s - \gamma T_e) \). Then, one has to prove that the following inequalities (see Figure 1)
\[
|z_1(T_s + T_e)| \leq |z_1(T_s - \gamma T_e)| \\
|z_2(T_s + T_e)| < |z_2(T_s - 3T_e)|
\]
are sufficient to ensure that system trajectories reach a parabola closer to the origin, and then to ensure the convergence to a vicinity of the origin. Denoting \( t_o \) as the initial time, the initial conditions are defined as \( z = [z_{1o} \ z_{2o}]^T \). For the sake of simplicity but without loss of generality, suppose that \( z_{1o} > 0 \) and \( z_{2o} > 0 \). Then, for \( t \in [t_o, T_s] \), system trajectories belong to the parabola defined by
\[
\begin{align*}
\text{(9a)} & \quad z_1(t) = -K_m \left( t - t_o \right)^2 + z_{2o}(t - t_o) + z_{1o} \\
\text{(9b)} & \quad z_2(t) = -K_m(t - t_o) + z_{2o}.
\end{align*}
\]
Furthermore, for \( t \in [T_s, T_s + T_e] \)
\[
\begin{align*}
\text{(10a)} & \quad z_2(T_s + T_e) = K_M T_e + z_2(T_s) \\
\text{(10b)} & \quad z_2(T_s) = -K_m (T_s - T_m) + z_2(T_m) \quad \text{where} \quad T_m = \gamma T_e,
\end{align*}
\]
In the sequel the trajectories evolving according to equations (9) and (10) will be referred as \( K_M \)-trajectories and \( K_m \)-trajectories respectively, regardless of its particular initial condition. For an arbitrary \( K_m \)-trajectory, let \( z(T_m) \) be the point at which the \( z_1 \)-axis is crossed, \( i.e. \) where \( z_1 \) has its maximum, \( z(T_m) = [z_{1m} \ 0]^T \). Let us express the time interval \( T_s - T_m \) in terms of the sampling time, \( T_e \), and the gain \( K_M \) in terms of \( K_m \)
\[
\eta T_e = T_s - T_m \quad \quad K_M = \gamma K_m
\]
By supposing \( t_o = T_s \), from (9b) it is obtained.
\[
\begin{align*}
\text{(11)} & \quad z_2(T_s) = -K_m (T_s - T_m) + z_2(T_m) \quad \text{where} \quad \eta, \gamma \text{ are positive parameters. From (10b), one gets}
\text{(12)} & \quad z_2(T_s + T_e) = K_M T_e + z_2(T_s) \quad \text{By supposing} \ t_o = T_s, \text{from (9b) it is obtained.}
\text{(13)} & \quad = \gamma K_m T_e + z_2(T_s)
\text{(14)} & \quad = -\eta K_m T_e
\text{(15)} & \quad = -\eta K_m T_e
\text{(16)} & \quad = \gamma K_m T_e
\text{(17)} & \quad = (\gamma - \eta) K_m T_e
\text{The gain} \ K_M \text{ applied between} \ t = T_s \text{ and} \ t = T_s + T_e \text{ generates a “jump” of} \ z_2 \text{ whose magnitude equals} \ K_M T_e = \gamma K_m T_e. \text{ This jump is required to reach a parabola closer from the origin. Given the dynamics of} \ z_2 \text{ when} \ t = T_s, \text{ means that} \ z_2(T_s + T_e) = z_2(T_s - \gamma T_e). \text{ Note that if} \ T_d = T_e, \text{ the reaching of a parabola closer to the origin implies} \ \gamma > 3, \text{ since} \ |z_2(T_s + T_e)| < |z_2(T_s - T_d - 2T_e)| \text{ must be fulfilled. From (10a), one has}
\text{(18)} & \quad z_1(T_s + T_e) = \frac{1}{2} K_M T_e^2 + z_2(T_s + T_e) \cdot T_e + z_1(T_s)
\text{From (9a), and by supposing} \ t_o = T_{sr} \text{,}
\text{(19)} & \quad z_1(T_s) = -\frac{K_m}{2} (T_s - T_{sr})^2 + z_2(T_{sr}) \cdot (T_s - T_{sr}) + z_1(T_{sr})
\text{Given that} \ z_2(T_{sr}) = -K_m (T_{sr} - T_m), \text{ then}
\text{(19)} & \quad z_1(T_s) = -\frac{K_m}{2} T_d^2 - K_m (T_{sr} - T_m) \cdot T_d
Finally, \( z_1(T_s + T_e) \) can be expressed as
\[
z_1(T_s + T_e) = \frac{1}{2} K_m T_e^2 - \eta K_m T_e - \frac{K_m}{2} T_d^2 \\
- K_m (T_s - T_m) \cdot T_d
\] (20)

It is obvious that \( |z_1(T_s)| \) is reaching a maximum value (which is the worst case) when \( T_d = T_e \), which gives
\[T_s + T_m = T_s - T_d - T_m = (\eta - 1)T_e.
\]

Then,
\[
z_1(T_s + T_e) = \frac{1}{2} \gamma K_m T_e^2 - 2\eta K_m T_e + \frac{K_m}{2} T_e^2
\] (21)

Now, in order to analyze (8), it is necessary to evaluate \( z_1(T_s - \gamma T_e) \). Considering \( t_0 = T_s - \gamma T_e \), with \( T_d = T_s - T_{sr} \), one has
\[
z_1(T_{sr}) = \frac{1}{2} K_m (\gamma T_e - T_d)^2 + z_1(T_s - \gamma T_e)
\]
\[+ (\gamma T_e - T_d) \cdot z_2(T_s - \gamma T_e)
\]
with (from (9) and with \( t_0 = T_m \))
\[
z_2(T_s - \gamma T_e) = -K_m (T_s - \gamma T_e - T_m)
\]
\[= -K_m (\eta T_e - \gamma T_e)
\] (22)

it gives
\[
z_1(T_s - \gamma T_e) = \frac{1}{2} K_m (\gamma T_e - T_d)^2
\]
\[+ K_m (\eta T_e - \gamma T_e)(\gamma T_e - T_d).
\]

As previously, by considering \( T_d = T_e \), one gets
\[
z_1(T_s - \gamma T_e) = \frac{1}{2} K_m [\gamma - 1]^2 T_e^2 + K_m [\eta - \gamma] [\gamma - 1] T_e^2.
\] (23)

From inequality (8), one has
\[
|z_1(T_s - \gamma T_e)| - |z_1(T_s + T_e)| > 0.
\] (24)

By using (21)-(24) in (25) and multiplying by \( \frac{2}{K_m T_e^2} \), the next expression is obtained
\[
2\eta[\gamma - 3] - \gamma^2 + \gamma + 2 > 0.
\] (26)

Then, inequality (26) allows to conclude that, given a fixed \( \gamma \), there exists some \( \bar{\gamma}(\gamma) \) such that (26) holds for any \( \eta > \bar{\gamma}(\gamma) \); furthermore, given the definition of \( \eta \), \( \eta > 0 \). It means that if \( T_s - T_m > T_e \cdot \bar{\gamma}(\gamma) \) then trajectories will be forced to converge to a vicinity of the origin, in a finite time. By supposing that the system trajectories are initially far from origin, it is obvious that \( \eta \) is large. It yields a convergence of the trajectories to a parabola closer from the origin, until \( \eta \) becomes lower than \( \bar{\gamma} \). Of course, a small \( \eta \) means that the system is evolving around the origin: the system has converged to a domain defined as
\[
z_1 \leq \frac{1}{2} K_m [\bar{\gamma}(\gamma) - 1]^2 \cdot T_e^2, \quad z_2 \leq K_m \bar{\gamma}(\gamma) \cdot T_e
\]
with \( \bar{\gamma}(\gamma) = \frac{\gamma^2 - \gamma - 2}{2(\gamma - 3)}. \)
IV. A SOLUTION FOR PERTURBED CASE

In this section, a robust output feedback controller is proposed for system (1) with \( \omega \neq 0 \). Nevertheless, the global validity of Theorem 1, for a single lower bound condition on the \( \gamma \) which relates \( K_M \) to \( K_m \), cannot be resembled for the perturbed case. The reason is that, when perturbation is different from zero in system (1), the minimum required value for \( K_M \) that ensures convergence, increases when trajectories are far from the origin.

**Theorem 2:** Consider system (1) controlled by (2)-(3). If the next inequalities are fulfilled (with \( \gamma > 3 \))

\[
\omega < \delta < K_m \quad (27)
\]

\[
\gamma K_m + (\gamma + 1) \delta < K_M < \infty \quad (28)
\]

then, a real second order sliding mode with respect to \( z_1 \) is established provided that initial conditions are inside the region of the \((z_1, z_2)\)-phase plane delimited by

\[
z_{2b}^2(t) = 2(K_m - \delta)(z_{1m} + \nu \cdot z_1(t)) \quad (29)
\]

with

\[
\nu = \begin{cases} 
1 & \text{for } z_1 \in [-z_{1m}, 0), \\
-1 & \text{for } z_1 \in [0, z_{1m}] 
\end{cases}
\]

and

\[
z_{1m} = \frac{(K_M - 3K_m)^2 \cdot T_c^2}{4(K_m - \sqrt{K_m^2 - \delta^2})} \quad (30)
\]

**Proof.** Without loss of generality, consider a \( K_m \)-trajectory which starts at a point \( z(T_m) = [z_{1m}, 0]^T \) (see Figure 6). In order to ensure convergence, the \( K_M \)-trajectory should finish above the symmetric of the inner parabola, even if (the “so-called” worst case) the mentioned trajectory was evolving on the external parabola. Furthermore, by a similar way than the unperturbed case, one will consider the worst case with respect to the delay of the \( z_1 \)-sign change detection, i.e. \( T_d = T_c \), for both the external and inner bounding parabolas. These two latter parabola delimit the domain in which the system trajectories are going to evolve: for each of these two parabolas, \( z_2(t) \) is defined as (as previously, \( t_o \) is defining the considered initial time)

**Inner parabola.**

\[
z_{22}(t) = -(K_m - \delta) \cdot (t - t_o) + z_2(t_o) \quad (31)
\]

**External parabola.**

\[
z_{22}(t) = -(K_m + \delta) \cdot (t - t_o) + z_2(t_o) \quad (32)
\]

Define \( T_{srp} \) and \( T_{srb} \) as the time intervals, for the inner and external parabolas respectively, between \( T_{m} \) and the time instant for which \( z_1 = 0 \) (a similar notation has been used in Figure 1). It is obvious that the lower value taken by \( z_2(t) \) (see Figure 6), denoted \( z_{2b} \), is obtained from the external parabola by supposing that the delay \( T_d \) to detect \( z_1 \)-sign change is maximum, i.e. \( T_d = T_c \). By choosing \( t_o = T_m \), one gets

\[
z_{2b} = -(K_m + \delta) \cdot (T_{srp} + T_c) \quad (33)
\]

Then, from (32), one derives

\[
z_1(T_{srb}) = -\frac{1}{2}(K_m + \delta) T_{srb}^2 + z_1(T_m) = 0 \quad (34)
\]

It yields

\[
T_{srb} = \sqrt{\frac{2z_1(T_m)}{K_m + \delta}} \quad (35)
\]

Finally, the minimal value \( z_{2b} \), denoted \( z_{2b} \), obtained for a given \( z_1(T_m) \) reads as

\[
z_{2b} = -(K_m + \delta) \cdot \sqrt{\frac{2z_1(T_m)}{K_m + \delta} + T_c} \quad (36)
\]
The variable \( z_{2p} \) (see Figure 6) is defined as the value over which the system trajectories need to “jump” when \( K_M \) is applied, because it ensures that the system has reached a parabola closer from the origin. In order to fulfill this latter constraint (even in the worst case), it is trivial that \( z_{2p} \) reads as (from (32) with \( t_o = T_m \))

\[
\begin{align*}
\quad z_{2p} &= -(K_m - \delta)(T_{srp} - 2T_e), \\
\text{the time } T_{srp} \text{ being derived by the similar way than } T_{srp};
\end{align*}
\]

one has

\[
T_{srp} = \sqrt{\frac{2z_1(T_m)}{K_m - \delta}}
\]

(38)

As depicted by Figure 6, the \( K_M \)-gain “jump” has to be sufficient in order to reach a parabola closer, which yields

\[
(K_M - \delta)T_e > |z_{2b} - z_{2p}|
\]

(39)

From equations (36)-(37)-(39), the next condition is obtained

\[
(K_M - \delta)T_e > \sqrt{2z_1(T_m)} \left[ \sqrt{K_m + \delta} - \sqrt{K_m - \delta} \right] + 3K_mT_e - \delta T_e
\]

(40)

From the above inequality, it yields that the right hand side is proportional to \( \sqrt{z_1} \). Next, one gives an estimation of the region of attraction for a given \( K_M \); solving (40) versus \( z_{1m} \), one gets

\[
z_{1m} < \frac{(K_M - 3K_m)^2 \cdot T_e^2}{4 \left( K_m - \sqrt{K_m^2 - \delta^2} \right)}
\]

(41)

Equation (29) corresponds to the equation of the inner parabola in the \((z_1, z_2)\)-phase plane. In fact, in the right half of the phase plane, the inner parabola corresponds to the following \( z_1(t) \)-\( z_2(t) \) functions (with \( z_o = [z_{1m}, 0]^T \))

\[
\begin{align*}
z_1(t) &= -\frac{1}{2}(K_m - \delta)(t - t_m)^2 + z_{1m} \\
z_2(t) &= -(K_m - \delta)(t - t_m)
\end{align*}
\]

(42)

From the second line of (42), one gets \( t - t_m = \frac{-z_2(t)}{K_m - \delta} \).

Substitution of the term \( t - t_m \) in the first equation of (42) gives

\[
z_1(t) = -\frac{z_2^2(t)}{2(K_m - \delta)} + z_{1m}
\]

\[
\Rightarrow z_2^2 = 2(K_m - \delta)(z_{1m} - z_1)
\]

(43)

A similar equation can be obtained for the left hand-side of the phase plane which gives (29). For the perturbed case, \( K_M - \delta \) is the smallest magnitude of \( z_2 \)-dynamics along \( K_M \)-trajectory, whereas \( K_m + \delta \) is the largest magnitude of \( z_2 \)-dynamics along \( K_m \)-trajectory. Then, supposing that the gain for \( K_M \)-trajectories equals \( K_M - \delta \), and the gain for \( K_m \)-trajectories equals \( K_m + \delta \), one gets from (4)

\[
K_M - \delta > \gamma(K_m + \delta) \Rightarrow K_M > \gamma K_m + (\gamma + 1) \delta
\]

(44)

V. IMPULSIVE ADAPTIVE GAIN

In the previous section, Theorem (2) establishes that the region of attraction of the impulsive controller is proportional to \( K_M \). On the other hand, a large value for \( K_M \) implies more chattering and less accuracy. In this section, a strategy for reducing \( K_M \), up to the lower bound given in Theorem (2), is proposed. In Figure 6, \( T_s \) represents the time at which the commutation of \( z_1 \) is detected, and \( T_m \) the time at which \( z_1 \) has its maximum values for the trajectory under consideration. Now, let us define \( T_{m}^i \), \( T_s^i \), \( i \in \mathbb{N} \) as follows: \( T_{m}^i \) are the sampling instants of detection of the \( i \)-th peak of \( z_1 \), and \( T_s^i \) the sign commutation detection sampling instants which are just following the corresponding \( T_m^i \).

Our proposal is based on a procedure for computing an upper bound for \( |z_2(T_s^i)| \) based on \( z_1(T_m^i) \). To summarize, the question is: what is the largest possible value \( |z_2(T_s^i)| \), given that \( z_1(T_m^i) \) is known? From this answer, an “estimation” (which is probably overevaluated) of \( |z_2(T_s^i)| \) allows to fix the next \( K_M \) applied from \( T_s^i \) during a single sampling period \( T_e \).

As depicted in Figure 6 (and in the same conditions, without loss of generality), at \( t = T_s^i \), the gain is switching; it yields

\[
z_2(T_s^i + T_e) = (K_M + \omega)T_e + z_2(T_s^i)
\]

(45)

Suppose that the gain \( K_M \) has been stated at some value \( K_M^i \), such that, for this choice of gain, \( z_2(T_s^i + T_e) = 0 \). One gets

\[
K_M^i = -\frac{z_2(T_s^i)}{T_e} + \omega
\]

(46)

Both, \( z_2(T_s^i) \) and \( \omega \) are unknown and an upper bound of the both will be used in order to select \( K_M^i \). An upper bound for \( |z_2(T_s^i)| \) may be obtained trough the next equation

\[
|z_2(T_s^i)| = (K_m + \delta)(T_s^i - T_m^i) + z_2(T_m^i) \leq (K_m + \delta)(T_s^i - T_m^i) + |z_2(T_m^i)|
\]

(47)

Nevertheless, the larges \( T_s^i \) occurs for the smallest gain applied to \( z_2 \)-dynamics, i.e. \( |K_m - \delta| \), which leads to unnecessary over estimation of \( |z_2(T_s^i)| \). Then, instead of using the available \( T_s^i \), our upper bound for \( |z_2(T_s^i)| \) is computed as described next. Assume the initial conditions

\[
z(T_{sr}^i) = [z_1(T_{sr}^i), 0]^T,
\]

(48)

then

\[
z_1(T_{sr}^i) = 0 = \frac{1}{2}(K_m + \omega)[T_{sr}^i - T_m^i]^2 + z_1(T_{sr}^i)
\]

\[
\Rightarrow T_s^i - T_m^i \leq \left( \frac{2z_1(T_{sr}^i)}{K_m - \delta} \right)^{\frac{3}{2}}
\]

(49)

Define \( \eta^i \) as

\[
\eta^i = \frac{\sqrt{\frac{2z_1(T_{sr}^i)}{K_m - \delta}}}{T_e} + 1
\]

(49)
Then, one gets
\[ T_{sr}^i - T_{sr}^{i_0} \leq \eta^i \cdot T_e \quad (50) \]
From this latter inequality, it comes
\[ T_{sr}^i - T_{sr}^{i_0} \leq (\eta^i + 1) \cdot T_e \quad (51) \]
From (47), one has
\[ |z_2(T_{sr}^i)| \leq (K_m + \delta) \cdot (\eta^i + 1) \cdot T_e \quad (52) \]
Then, from (46), one has
\[ K_M^i = (\eta^i + 1)(K_m + \delta) + \delta. \quad (53) \]
From Theorem 2, by defining \( K_{\text{Min}} \) as
\[ K_{\text{Min}} = \gamma K_m + (\gamma + 1) \delta \]
with \( K_m > \delta \) and by supposing that the gain has an upper bound equal to \( K_{\text{Max}} \) (due to practical constraints - saturation of the control input applied to an actuator, for instance), the \( K_M^i \)-gain adaptation law of the controller (2)-(3) reads as
\[ K_M = \begin{cases} K_{\text{Max}} \quad & \text{if} \quad K_M^i \geq K_{\text{Max}} \\ K_m^i \quad & \text{if} \quad K_{\text{Min}} \leq K_M^i < K_{\text{Max}} \\ K_{\text{Min}} \quad & \text{if} \quad K_M^i < K_{\text{Min}} \end{cases} \quad (54) \]

**Example.** Simulations have been made with system (1) such that \( \omega = 0.6 \text{sign}(z_2), \) \( K_{\text{Max}} = 100, \) \( K_m = 1 \) and \( T_e = 0.01 \text{ sec}. \) The initial conditions are \( z = [1.1 \quad 0.1]^T. \)
Figure (7) shows the behavior of the control signal with the above adaptation for \( K_M, \) whereas Figure (8) depicts the corresponding phase portrait and shows the convergence to a vicinity of the origin. Note that the gain \( K_M \) is starting with large values and, when trajectories become closer from the origin, it starts to be reduced and goes to its minimum value.

![Control input u versus time (sec) for a system (1) with \( \omega = 0.6 \text{ sin}(z_2). \)](image)

**VI. CONCLUSION**

This paper proposed output feedback scheme based on second order sliding mode theory and impulsive gain. The solutions have been presented in case of certain and uncertain systems, and a first adaptation law for the impulsion magnitude has been given. Future works consist in applying this class of controllers on real systems, and in the extension for systems with unknown bounds of uncertainties and perturbations.

**REFERENCES**